

Dispersive properties of electromagnetically induced transparency

S. E. Harris, J. E. Field, and A. Kasapi

Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305

(Received 20 January 1992)

An atomic transition that has been made transparent by applying an additional electromagnetic field exhibits a rapidly varying refractive index with zero group velocity dispersion at line center. A 10-cm-long Pb vapor cell at an atom density of 7×10^{15} atoms/cm³ and probed on its 283-nm resonance transition has a calculated optical delay of 83 ns [(c/V_G)=250].

PACS number(s): 42.50.Hz, 32.70.-n, 42.25.Bs, 42.65.An

It has recently been demonstrated that an optically thick transition may be made nearly transparent to light at its resonance frequency [1,2]. This is done by applying an electromagnetic field (Fig. 1) which dresses the upper state of the transition and thereby creates a quantum interference at a probe wavelength. The applied electromagnetic field may be another laser or a microwave or dc field. The transition may be broadened by autoionization, radiative decay, and, in certain cases, by collision.

In this Rapid Communication we calculate the dispersive properties of such a media. The real and imaginary parts of the susceptibility as functions of the probe frequency are shown in Fig. 2. Because of the absorptive interference and the symmetry of the dressed states, the probe, when tuned to the position of bare state [3], experiences a linear rapidly varying refractive index with very slow group velocity and zero group-velocity dispersion. This slow group velocity is the result of the slope and not of the magnitude of the refractive index, which remains

nearly unity.

Tewari and Agarwal [3] and Harris, Field, and Imamoglu [4] have noted that the dispersive properties of a macroscopic medium may be modified by a strong (dressing) electromagnetic field. In related work, Scully [5] has noted the possibility of using coherence to allow an increased refractive index. The basic phenomenon which creates this transparency is termed as population trapping and has been studied extensively [6-9].

We work with the probe envelope quantities $E(t)$ and $P(t)$ with Fourier transforms $E(\omega)$ and $P(\omega)$. We take the probe to have a center frequency ω_0 , expand the susceptibility of the dressed atom to third order about this value, and Fourier transform. With $P(\omega - \omega_0) \equiv \epsilon_0 \chi(\omega - \omega_0)E(\omega - \omega_0)$

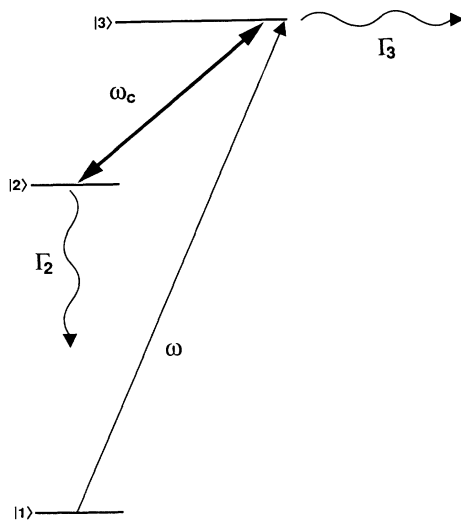


FIG. 1. Energy-level diagram for the transparency process. When a strong field of frequency ω_c is tuned to line center of the $|2\rangle \rightarrow |3\rangle$ transition, where state $|2\rangle$ is metastable, the medium becomes transparent to a field of frequency ω tuned to line center of the $|1\rangle \rightarrow |3\rangle$ transition.

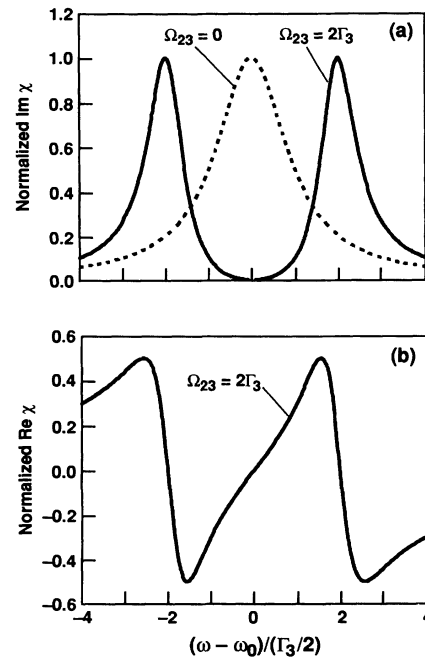


FIG. 2. (a) Imaginary and (b) real parts of the susceptibility of a probe frequency ω in the presence of a strong-coupling field ω_c . The dotted curve of (a) is the imaginary part of the susceptibility in the absence of the coupling field. Normalization is to the peak value of the imaginary part of the susceptibility.

$$\chi(\omega - \omega_0) = \chi(\omega_0) + \left(\frac{\partial \chi}{\partial \omega} \bigg|_{\omega_0} \right) (\omega - \omega_0) + \frac{1}{2} \left(\frac{\partial^2 \chi}{\partial \omega^2} \bigg|_{\omega_0} \right) (\omega - \omega_0)^2 + \frac{1}{6} \left(\frac{\partial^3 \chi}{\partial \omega^3} \bigg|_{\omega_0} \right) (\omega - \omega_0)^3 \quad (1)$$

and therefore

$$P(t) = \epsilon_0 \chi(\omega_0) E(t) - j \epsilon_0 \left(\frac{\partial \chi}{\partial \omega} \bigg|_{\omega_0} \right) \frac{\partial E}{\partial t} - \frac{\epsilon_0}{2} \left(\frac{\partial^2 \chi}{\partial \omega^2} \bigg|_{\omega_0} \right) \frac{\partial^2 E}{\partial t^2} + j \frac{\epsilon_0}{6} \left(\frac{\partial^3 \chi}{\partial \omega^3} \bigg|_{\omega_0} \right) \frac{\partial^3 E}{\partial t^3}. \quad (2)$$

For this work we take ω_0 equal to the frequency of the bare $|1\rangle \rightarrow |3\rangle$ transition. The values of the real and imaginary parts of the dressed susceptibility χ and the pertinent derivatives evaluated at this frequency are given in Table I. The quantity ϵ_0 is the permittivity of free space. The quantity Ω_c is the Rabi frequency of the resonantly driven $|2\rangle \rightarrow |3\rangle$ transition; i.e., $\Omega_c = \mu_{23} E_c / \hbar$. The formulas, as written here, are for a lifetime-broadened system with decay rates of states $|2\rangle$ and $|3\rangle$ of Γ_2 and Γ_3 . When pressure broadening is important then, to within the accuracy of the impact approximation, Γ_2 and Γ_3 may be replaced by $\Gamma_2 + 2\gamma_{12}$ and $\Gamma_3 + 2\gamma_{13}$, respectively; the quantities $2\gamma_{12}$ and $2\gamma_{13}$ are full width at half maximum collisional Lorentzian widths. The formulas of Table I and, therefore, the results of this paper, are dependent on the assumption that the Rabi frequency of the probe is small as compared to that of the coupling laser.

The equation for the slowly varying envelope $E(z, t)$ is then

$$\frac{\partial E}{\partial z} + (a + j\beta)E + \frac{1}{V_G} \frac{\partial E}{\partial t} + j b_2 \frac{\partial^2 E}{\partial t^2} + b_3 \frac{\partial^3 E}{\partial t^3} = 0, \quad (3)$$

where

$$\alpha = -\frac{\pi}{\lambda} \text{Im} \chi(\omega_0)$$

$$\beta = \frac{\pi}{\lambda} \text{Re} \chi(\omega_0), \quad b_2 = -\frac{\pi}{\lambda} \frac{1}{2} \left(\frac{\partial^2 \chi}{\partial \omega^2} \bigg|_{\omega_0} \right),$$

$$\frac{1}{V_G} = \frac{1}{c} + \frac{\pi}{\lambda} \left(\frac{\partial \chi}{\partial \omega} \bigg|_{\omega_0} \right), \quad b_3 = -\frac{\pi}{\lambda} \frac{1}{6} \left(\frac{\partial^3 \chi}{\partial \omega^3} \bigg|_{\omega_0} \right).$$

The quantities α and β are the per length E field loss and phase shift at ω_0 , V_G is the group velocity, and b_2 and b_3 determine the pulse distortion and therefore the ability to resolve the delayed pulse.

In the (hypothetical) ideal case where state $|2\rangle$ is completely metastable ($\Gamma_2 = 0$) and the probe is tuned to ω_0 , there is complete interference and perfect transparency for all nonzero Ω_c . Noting Fig. 2 and Table I, we see that

both β and the group-velocity dispersion (the real part of b_2) are zero. For finite Γ_2 the transparency results from both the interference and the separation of the dressed states. At small Ω_c the interference dominates and the pulse distortion is the result of the nonzero loss (b_2) of off-line-center Fourier components. At sufficiently large Ω_c the medium is transparent over the pulse bandwidth and the distortion comes from the curvature of the real part of the susceptibility b_3 .

For a medium of length L the time delay relative to a pulse traveling at velocity c is $L[(1/V_G) - (1/c)]$. For a sufficiently monochromatic pulse, we take the maximum usable L as $1/2\alpha$. For $\Omega_c^2 \gg \Gamma_2 \Gamma_3$ (Table I), the time delay T_D in one attenuation length (power) is

$$T_D = \left(\frac{1}{V_G} - \frac{1}{c} \right) \left(\frac{1}{2\alpha} \right) = \frac{[(\partial \chi / \partial \omega) |_{\omega_0}]}{2\chi(\omega_0)} = \frac{1}{\Gamma_2}, \quad (4)$$

i.e., the maximum time delay for a pulse which propagates one attenuation length is equal to the decay time of metastable state $|2\rangle$.

For an exponential pulse with a characteristic rise or fall time τ , we take τ_{\min} as that time constant which produces a distortion of magnitude $1/\alpha$. Assuming either b_2 or b_3 dominates, then τ_{\min} is the larger of

$$\tau_{\min} = \left(\left| \frac{b_2}{\alpha} \right| \right)^{1/2} = 2 \left(\frac{\Gamma_3}{\Gamma_2 \Omega_c^2} \right)^{1/2} \quad (5a)$$

or

$$\tau_{\min} = \left(\left| \frac{b_3}{\alpha} \right| \right)^{1/3} = 2 \left(\frac{1}{\Gamma_2 \Omega_c^2} \right)^{1/3}. \quad (5b)$$

We define a parameter q as the ratio of the total delay to τ_{\min} . This parameter is a measure of the resolvability of the delayed pulse and for the limits of Eqs. (5a) and

TABLE I. Susceptibilities and derivatives. All quantities are to be multiplied by $|\mu_{13}|^2 N / \epsilon_0 \hbar$. To include collision broadening, Γ_2 and Γ_3 should be replaced by $\Gamma_2 + 2\gamma_{12}$ and $\Gamma_3 + 2\gamma_{13}$, respectively; $2\gamma_{12}$ and $2\gamma_{13}$ are full width at half maximum Lorentzian collisional widths.

	$\chi _{\omega_0}$	$\left(\frac{\partial \chi}{\partial \omega} \bigg _{\omega_0} \right)$	$\left(\frac{\partial^2 \chi}{\partial \omega^2} \bigg _{\omega_0} \right)$	$\left(\frac{\partial^3 \chi}{\partial \omega^3} \bigg _{\omega_0} \right)$
Real part	0	$\frac{4(\Omega_c^2 - \Gamma_2^2)}{(\Omega_c^2 + \Gamma_2 \Gamma_3)^2}$	0	$\frac{96[(\Omega_c^2 - \Gamma_2^2)^2 - \Omega_c^2(\Gamma_2 + \Gamma_3)^2]}{(\Omega_c^2 + \Gamma_2 \Gamma_3)^4}$
Imaginary part	$-\frac{2\Gamma_2}{(\Omega_c^2 + \Gamma_2 \Gamma_3)}$	0	$-\frac{16(\Gamma_3 \Omega_c^2 + 2\Gamma_2 \Omega_c^2 - \Gamma_2^2)}{(\Omega_c^2 + \Gamma_2 \Gamma_3)^3}$	0

(5b), respectively, is

$$q_2 = \frac{T_D}{\tau_{\min}} = \frac{1}{2} \left(\frac{\Omega_c^2}{\Gamma_2 \Gamma_3} \right)^{1/2} \quad (6a)$$

or

$$q_3 = \frac{1}{2} \left(\frac{\Omega_c}{\Gamma_2} \right)^{2/3} \quad (6b)$$

One may compare the dispersive properties of the dressed atom with the probe tuned to ω_0 with those of the same atom with $\Omega_c = 0$ and the probe tuned off of line center by $\Delta\omega$ so as to equalize the attenuation at the same pressure and cell length. This detuning is $\Delta\omega = [(\Gamma_3/4\Gamma_2)^{1/2}] \Omega_c$. Proceeding as above, the total time delay of the bare atom in a single attenuation length is $1/\Gamma_3$; $\tau_{\min} = (2/\Gamma_3\Delta\omega)^{1/2}$; $q_2 = (\Delta\omega/2\Gamma_3)^{1/2}$; and the ratio of the group velocities of the bare to dressed atom is Γ_3/Γ_2 . Because the group velocity dispersion of the dressed atom is zero at ω_0 , it has a much greater pulse resolvability than does a bare atom. (Instead, one may assume a hypothetical bare atom with a transition oscillator strength such that $\Gamma_2 = \Gamma_3$ and assume that both the atom density and the detuning are increased to make the loss length, the group velocity, and the pulse resolvability q the same. One finds that to do so the ratio of the atom densities of the bare-to-dressed atom must be $4q^2$, which, for reasonable pulse resolution, is impractical.)

Though this Rapid Communication is concerned with the linear response, we note that for $\Gamma_2 = 0$ and $\omega = \omega_0$, $\chi^{(3)}(-\omega, \omega, -\omega, \omega) = 0$, as do all similar terms of higher order. Nonlinear susceptibilities such as $\chi^{(3)}(-\omega, \omega_c, -\omega_c, \omega)$ are included within this formulation and, for these same conditions, are also zero.

We observe that there is an unusual type of spatial pulse compression inherent to these formulas. As a pulse enters a medium with a very slow group velocity and (approximately) unity refractive constant, its peak electric field and power density are unchanged and the pulse compresses spatially by a factor of c/V_G . The energy density is primarily determined by the slope rather than the magnitude of the susceptibility [10]

$$\begin{aligned} \langle W \rangle &= \frac{1}{4} \mu_0 |H|^2 + \frac{1}{4} \epsilon_0 \frac{\partial}{\partial \omega} (\omega \epsilon) |E|^2 \\ &\cong \frac{1}{4} \epsilon_0 \omega \left(\frac{\partial \chi}{\partial \omega} \bigg|_{\omega_0} \right) |E|^2 \end{aligned} \quad (7)$$

and the power density $(\epsilon_0 c/2) |E|^2$ equals the product of the energy density and group velocity.

The previous formulas have ignored inhomogeneous broadening. For the medium to be transparent we therefore require that the Rabi frequency of the $|2\rangle \rightarrow |3\rangle$ transition be sufficiently large that the transmission hole width $[(\Gamma_2/\Gamma_3)^{1/2}] \Omega_c$ is large as compared to the two-photon Doppler width of the $|1\rangle \rightarrow |2\rangle$ transition. (In the usual sense of Doppler-free, two-photon processes, this width

depends on the direction of propagation as well as the frequencies of the two beams; for a system where state $|2\rangle$ is above state $|3\rangle$ and the beams counterpropagate, it approaches zero as the frequencies approach each other.)

We proceed with an example: We assume a 10-cm-long ^{208}Pb vapor cell at an atom density of 7×10^{15} atoms/cm³. The probe transition is $6s^2 6p^2 \ ^3P_0 \rightarrow 6s^2 6p 7s^3 \ ^3P_1$ with a wavelength of 283 nm and an oscillator strength [11] of $gf = 0.197$. The coupling laser transition is $6s^2 6p^2 \ ^3P_2 \rightarrow 6s^2 6p 7s^3 \ ^3P_1$ with $\lambda = 405.9$ nm and $gf = 0.71$. For an atom density of 7×10^{15} atoms/cm³, the quantities $\Gamma_3 + 2\gamma_{13}$ and $\Gamma_2 + 2\gamma_{12}$ which replace Γ_3 and Γ_2 in the formulas are $4.46 \times 10^8 \text{ sec}^{-1}$ and $1.20 \times 10^7 \text{ sec}^{-1}$ [12,13], respectively. It is implicitly assumed that the linewidth of the coupling laser is narrow compared to this last value [14]. We choose the Rabi frequency of the $|2\rangle \rightarrow |3\rangle$ transition so that the power attenuation of the probe beam is unity; i.e., $1/2a = 10.0$ cm. This Rabi frequency is sufficiently large that the two-photon Doppler width $\cong 0.01 \text{ cm}^{-1}$ is unimportant. This requires an Ω_c of about 0.7 cm^{-1} , which occurs at a coupling laser power density of 283 kW/cm^2 . For these conditions $c/V_G = 250$, the total time delay is 83.3 ns, and the resolvability $q_3 = 248$. These quantities may be compared to those of the transition without the coupling laser present. With $\Omega_c = 0$ and the probe laser detuned by $\Delta\omega = 2.14 \text{ cm}^{-1}$ so as to again obtain a 10.0-cm power absorption depth, we find $c/V_G = 6.73$, a total time delay of 2.24 ns, and a ratio of total delay to minimum pulse length of $q_2 = 21.3$. (These single transition results are in the spirit of those observed by Grischkowsky [15].)

From Eq. (5) we see that, for large Rabi frequency, the minimum pulse length which one may use to observe these slow group velocities is $(\Gamma_2/\Omega_c)^{2/3} (2/\Gamma_2)$. For reasonable coupling laser power the minimum pulse length in metal vapors is a few tenths of a nanosecond. The slow group velocities described here are also, in essence, a bandwidth limitation on the use of electromagnetically induced transparency to obtain large nonlinear coefficient length products for nonlinear sum and difference frequency generation [3,4,16]. For the output power of the generated signal to grow as the cell length squared, the generated pulse must be sufficiently long that it does not temporarily slip from the driving pulse in the cell or opacity length.

A device of this type may someday provide what might be termed as a group-velocity optic. For polarization in the same plane, the group velocity depends on the relative angle of the polarization of the coupling field and probe lasers and therefore varies with probe propagation angle. At the same time, the phase delay is invariant to propagation angle.

The authors acknowledge helpful discussions with A. Imamoglu and M. Jain. This work was supported by the U.S. Air Force Office of Scientific Research, the U.S. Army Research Office, and the U.S. Office of Naval Research.

- [1] K.-J. Boller, A. Imamoglu, and S. E. Harris, *Phys. Rev. Lett.* **66**, 2593 (1991).
- [2] J. E. Field, K. H. Hahn, and S. E. Harris, *Phys. Rev. Lett.* **67**, 3062 (1991).
- [3] S. P. Tewari and G. S. Agarwal, *Phys. Rev. Lett.* **56**, 1811 (1986).
- [4] S. E. Harris, J. E. Field, and A. Imamoglu, *Phys. Rev. Lett.* **64**, 1107 (1990).
- [5] M. Scully, *Phys. Rev. Lett.* **67**, 1855 (1991).
- [6] G. Alzetta, A. Gozzini, L. Mopi, and G. Orriols, *Nuovo Cimento B* **36**, 5 (1976).
- [7] H. Bachau and P. Lambropoulos, *Phys. Rev. A* **34**, 4785 (1986).
- [8] P. L. Knight, M. A. Lauder, and B. J. Dalton, *Phys. Rep.* **190**, 1 (1990).
- [9] M. B. Gornyi, B. G. Matisov, and Yu. V. Rozhdestvenskii, *Pis'ma Zh. Eksp. Teor. Fiz.* **95**, 1263 (1989) [*Sov. Phys. JETP* **68**, 728 (1989)].
- [10] Hermann A. Haus, *Waves and Fields in Optoelectronics* (Prentice-Hall, Englewood Cliffs, NJ, 1984), p. 314.
- [11] R. L. DeZafra and A. Marshall, *Phys. Rev.* **170**, 128 (1968).
- [12] W. Happer and E. B. Saloman, *Phys. Rev.* **160**, 23 (1967).
- [13] W. R. Hindmarsh and J. M. Farr, *Collisional Broadening of Spectral Lines by Neutral Atoms* (Pergamon, New York, 1972), p. 141.
- [14] G. S. Agarwal, *Phys. Rev. A* **18**, 1490 (1978).
- [15] D. Grischkowsky, *Phys. Rev. A* **7**, 2096 (1973).
- [16] K. Hakuta, L. Marmet, and B. P. Stoicheff, *Phys. Rev. Lett.* **66**, 596 (1991).