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Eckhaus instability for traveling waves

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The mechanism by which a one-dimensional pattern of traveling waves changes wavelength (i.e., the Eckhaus instability) is studied in a binary fluid mixture. Long-lived transient phase modulations connect states of uniform wave number. The dynamics of wave-number increases and wave-number decreases are found to be qualitatively different due to a strong dependence of the group velocity on wave number. Relevant theoretical models are discussed.

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A fundamental question in pattern selection and dynamics in nonequilibrium systems involves understanding the mechanism by which a periodic pattern changes wavelength in response to changes in a control parameter. This can occur through the growth of long-wavelength phase modulations, conventionally referred to as the Eckhaus instability [1]. In the case of stationary patterns, this instability has been studied in a number of physical systems [2,3]. In contrast, the case in which the underlying pattern consists of traveling waves has received very little attention. As we show below, the fact that pattern modulations in such a system travel at the group velocity of the waves leads to a wide range of dynamical phenomena. The one previous experiment to consider this question, by Janiaud and co-workers [4,5], studied wavelength changes at a secondary instability (the oscillatory instability) in Rayleigh-Bénard convection. They examined unstable waves at two different wave numbers which evolved to higher wave-number states via the Eckhaus instability.

In this paper, we present the results of a study of the dynamics associated with the Eckhaus instability in traveling-wave convection in a binary fluid mixture. The fluid is confined in a long, narrow annular channel which approximates a one-dimensional system with periodic boundary conditions. The bifurcation to traveling-wave convection is subcritical, and stable patterns with different numbers of roll pairs in the cell can be created. The wavelength of the pattern is the extent of a roll pair in the direction of roll propagation. As the Rayleigh number (i.e., the temperature difference across the fluid layer) is decreased, an Eckhaus instability is encountered in the region near the saddle-node bifurcation to the conducting state. We find that wavelength changes occur by the growth of wavelength compressions and dilations which grow in amplitude until they nucleate the annihilation or creation of new roll pairs. Compression and dilation pulses propagate at approximately the measured group velocity v_g which varies by an order of magnitude for wave numbers near the minimum of the Eckhaus stability curve. This instability results in either stable patterns or conduction, but can evolve by long-lived transients with the successive nucleation and annihilation of many roll pairs.

The apparatus has been described previously in Ref. [6]. The fluid is confined between a sapphire top plate and a copper bottom plate with a separation of $d=0.309$ cm. Sidewalls of Ultem plastic form an annular channel with a rectangular cross section of width $1.288d$ and a mean circumference of $67.09d$. The top plate is regulated at $25.000\pm0.0007\,^\circ$C. The fluid is 8.00% (by weight) ethanol in water. At the onset of convection, the average temperature is $27.53\,^\circ$C, and the fluid parameters are $\psi=-0.26$, $P=9.16$, and $L=0.008$ where ψ, P, and L are the separation ratio, Prandtl number, and Lewis number [6]. The vertical thermal diffusion time is $\tau_v=d^2/\kappa=74.2$ s, where κ is the thermal diffusivity. The shadowgraph technique, which images index of refraction variations in the convecting fluid, is used to visualize the pattern [6]. The resulting optical signal is measured with a 720-element annular-array CCD camera.

The technique of complex demodulation, which can be regarded as a local version of harmonic analysis, is used to obtain the local amplitude and phase of the nearly harmonic signal [7]. The wavelength profile $k(\theta,t)$ in the spatial coordinate θ at time t is obtained from the demodulation in t. The wave-number profile is further refined by applying a local scale factor to correct for optical distortions. This factor is computed using the wave-number profile of a uniform state traveling at constant phase velocity. The result is $k=k(\theta,t)$.

The system undergoes an initial subcritical bifurcation to traveling waves at a reduced Rayleigh number $r_{co}=1.70$, where $r=R/R_c$. The Rayleigh number $R_c=1708$ is the onset Rayleigh number for convection in a laterally infinite pure fluid, with the corresponding temperature difference across the fluid layer ΔT_c calculated using the fluid parameters of the homogeneous mixture. As the Rayleigh number is increased in the convecting state, the phase velocity v_ϕ decreases continuously and goes to zero at $r^*\approx1.73$, resulting in stationary convection [6]. As r is decreased, v_ϕ increases until a saddle-node bifurcation is reached, below which the system goes into conduction. Well above the saddle node, a range of wavelengths is stable. In the work presented here, the system is prepared in uniform states of different initial wave number k_i by making sudden transitions from the conduction state to states of stationary convection above r^*.

As r is decreased, uniform traveling-wave states with wave number k_i go unstable to wavelength perturbations at a Rayleigh number $r_E(k_i)$; this Eckhaus stability boundary is shown in Fig. 1(a). Defining k_0 as the minimum in the stability curve, a parabolic fit to the lowest three data points yields $k_0=(3.03\pm0.05)d^{-1}$ and $r_E(k_0)=1.518\pm0.003$. Our data are consistent with $r_E(k_0)$ being both the minimum of the Eckhaus curve and the location of the saddle node.

As illustrated in Figs. 2 and 3, the evolution of the system differs qualitatively for $k_i>k_0$ and $k_i<k_0$. An example of the behavior for $k_i>k_0$ is shown in Fig. 2. Initially, a long-wavelength sinusoidal modulation of k begins to grow. As discussed in Ref. [5], this is consistent with the behavior expected from the Eckhaus instability. At a later time, as shown in Fig. 2(a), the modulation amplitude has grown to 15%, and the spatial profile of the modulation has changed. The dilated regions are wider in spatial extent, and the compressions are narrower. At $t\approx19.6\times10^3$ s, the loss of one roll pair occurs within the compression. The modulation travels at approximately $2.4v_\phi$, where $v_\phi=0.008d$ s^{-1}. The features near $\theta=\pi/8$ and $3\pi/8$ rad in Fig. 2 do not appear to affect the dynamics and are most likely due to inhomogeneities in the optical system.

Figure 4 is an alternate view of the phase modulation

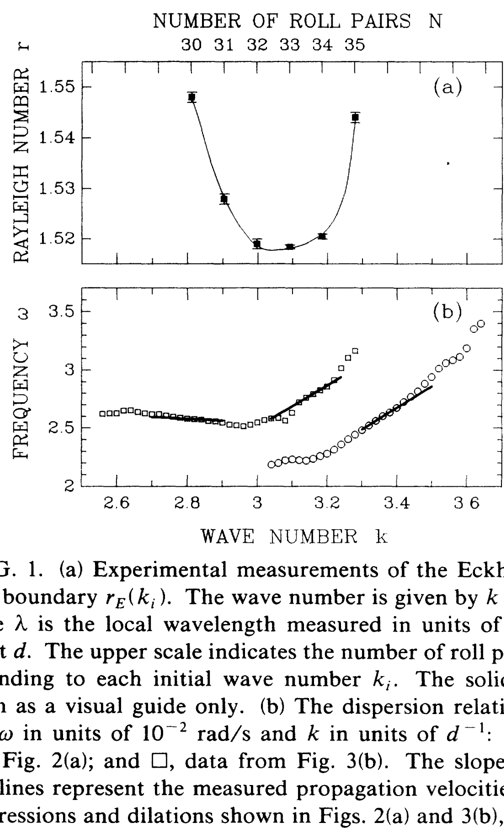

FIG. 1. (a) Experimental measurements of the Eckhaus stability boundary $r_E(k_i)$. The wave number is given by $k=2\pi/\lambda$, where λ is the local wavelength measured in units of the cell height d. The upper scale indicates the number of roll pairs corresponding to each initial wave number k_i. The solid line is drawn as a visual guide only. (b) The dispersion relation $\omega(k)$ with ω in units of 10^{-2} rad/s and k in units of d^{-1}: ○, data from Fig. 2(a); and □, data from Fig. 3(b). The slopes of the solid lines represent the measured propagation velocities of the compressions and dilations shown in Figs. 2(a) and 3(b), and the line lengths indicate the range of k in these modulations.

in Fig. 2 which shows the time evolution of the local wave number at a particular position in the annulus as the modulations move past [8]. The initially sinusoidal modulation grows in amplitude, while the compressed region narrows in spatial extent and becomes a well-defined pulse. At $t\approx19.6\times10^3$ s, there is the change in roll number described above, and then the amplitude of the remaining compression pulse slowly decreases in time as the pulse continues to propagate.

The behavior for $k_i<k_0$ is different. This is illustrated by the example in Fig. 3. A slowly moving dilation develops. The amplitude of this dilation increases until a new roll pair is created at $t\approx2.5\times10^3$ s. Each creation event produces a compression pulse which travels at a velocity of approximately $2.0v_\phi$ where $v_\phi=0.009d$ s^{-1}. Later, another dilation develops and eventually leads to the creation of three more roll pairs and finally to the loss of one roll pair to bring the system back within the stable band.

During the times shown in Figs. 2 and 3, the convection amplitude is nearly constant, except very near roll pair creations or annihilations where the amplitude must go to zero in order to add or subtract phase. Thus the perturbations to the uniform pattern are almost purely phase modulations. The duration of a creation or annihilation event is quite short, corresponding to a time less than that for the rolls to move a few wavelengths.

One intriguing feature of the data shown in Figs. 2 and 3 is that some wave-number modulations appear to propagate at a large group velocity ($\sim2v_\phi$), while others move slowly. We have found that this is due to an unusually strong dependence of the group velocity $v_g=d\omega/dk$ on the local wave number. To obtain the dispersion relation $\omega(k)$, we calculate $\omega(\theta,t)$ and $k(\theta,t)$ when phase modulations are present and then average the ω's at each value of k using a bin size of $\Delta k=0.02d^{-1}$. The results are shown in Fig. 1(b) for two values of r. (The shift of the curves in ω is due to the strong dependence of ω on r.) For $k>k_0$, the group velocity is positive and faster than $v_\phi=\omega/k$, while for $k<k_0$, the group velocity is much smaller and negative. The fact that v_g changes near k_0 may reflect the intrinsic dispersion relation rather than behavior associated with the Eckhaus boundary or the saddle node. As illustrated by the solid lines in Fig. 1(b), the wave-number modulations in Figs. 2(a) and 3(b) move at $v_g=d\omega/dk$, showing that nonlinear corrections to v_g are not significant for the data shown.

The work of Janiaud and co-workers [4,5] for a traveling-wave Eckhaus instability in a pure fluid bears some similarity to that described here. They observe pulses of phase modulation which propagate at the group velocity. However, in their experiment, the wave-number increase occurs in a region where a standing wave has developed. In our system, this does not appear to occur. In Ref. [5], the authors argue that, for a supercritical bifurcation, the limiting form of phase compression pulses could be expected to be a Korteweg–de Vries soliton. The relationship between the width and amplitude of the evolving compression pulse shown in Fig. 4 is consistent with such a soliton, but several issues remained to be ad-

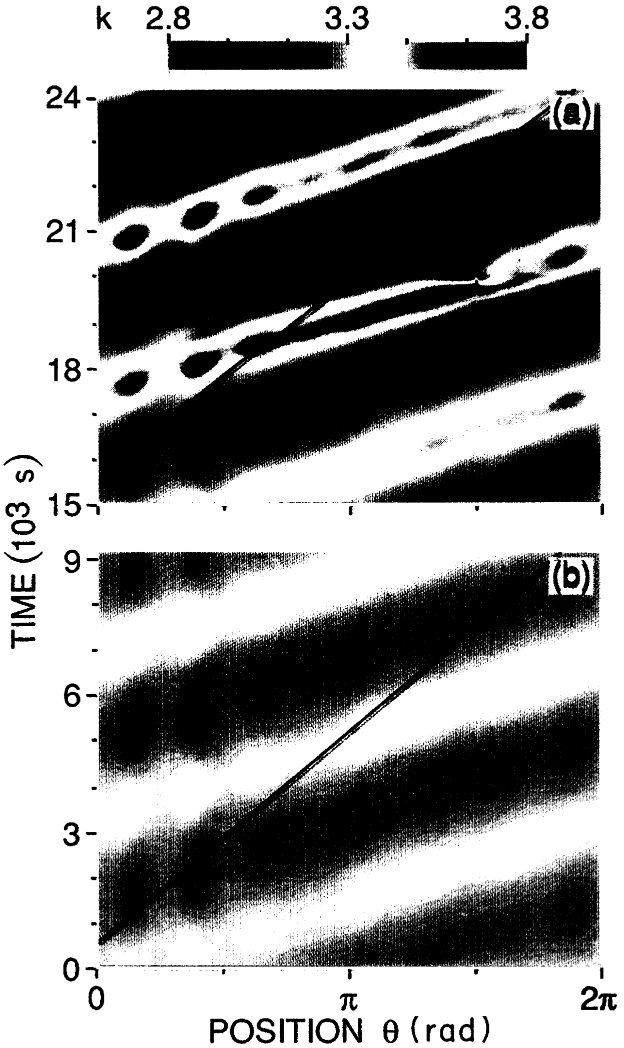

FIG. 2. Evolution of the local wave number in space and time for $k_i > k_0(k_i = 3.28d^{-1})$, at $r = 1.543$, for two time intervals in the same data set. Wave numbers is mapped to color, and the solid lines indicate the motion of a roll boundary (i.e., the phase velocity of the rolls). The annihilation of one roll pair occurs at $t \approx 19.6 \times 10^3$ s.

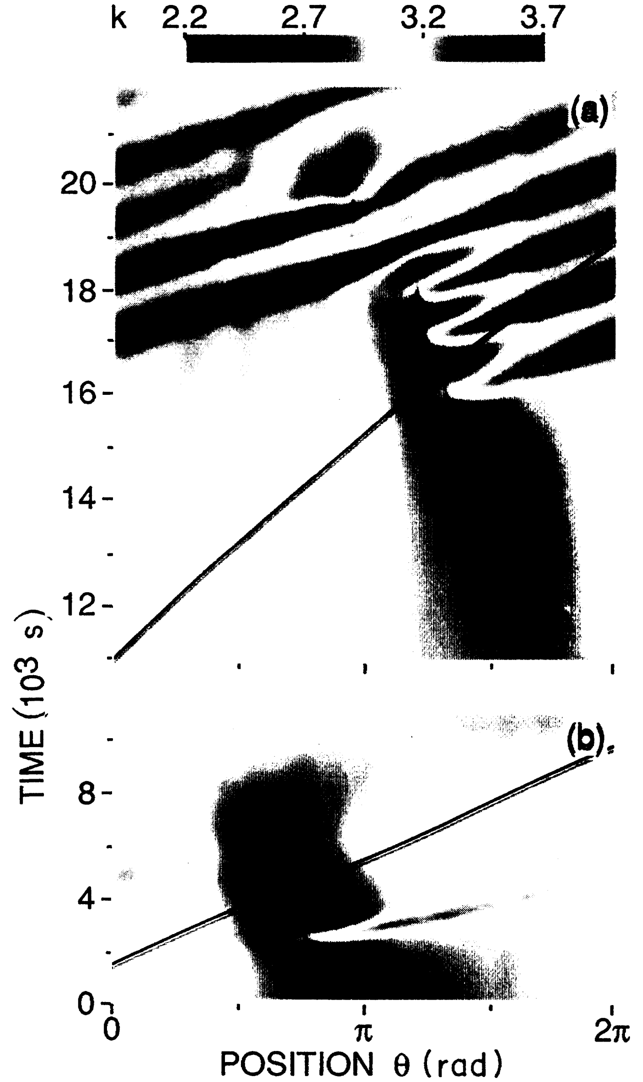

FIG. 3. Evolution of the local wave number for $k_i < k_0(k_i = 2.90d^{-1})$, at $r = 1.521$, for two time intervals in the same data set. The solid lines indicate the motion of a roll boundary. Roll pairs are created at several times, and a roll pair is lost at $t \approx 19.6 \times 10^3$ s.

dressed, including the relevance of the theory to the case of a subcritical bifurcation and the possible dependence of pulse group velocity on amplitude [5]. A separate issue raised by our experiments is the theoretical description of the propagating dilations observed for $k < k_0$.

In this paper, we have presented data for the traveling-wave Eckhaus instability in a binary fluid mixture. We find that long-lived and intricate transients connect states of uniform wave number. The dynamics of locally unstable waves is dominated by the qualitatively different propagation characteristics of modulations with $k < k_0$ as compared to those with $k > k_0$. One consequence of the strong dependence of the group velocity on k is that, as shown in Fig. 3(a), it is difficult for the pattern to increase in wave number. A dilation grows in amplitude at the Eckhaus boundary, then a roll pair is creat-

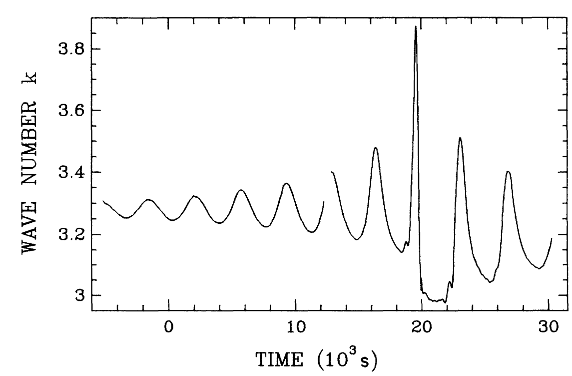

FIG. 4. Temporal evolution of the wave number k near a single spatial point, $\theta = 1.4\pi$ rad, for data which includes that shown in Fig. 2.

ed, but the resulting compression pulse propagates away leaving a dilation which can again grow and lead to more roll creations.

In the case of a supercritical bifurcation, a theoretical model of the dynamics of phase modulations of traveling waves has been proposed in terms of a complex Ginzburg-Landau equation with the limiting form of a Korteweg–de Vries equation [4,5]. Whether this framework is adequate to describe the dynamical behavior shown in Figs. 2 and 3 remains to be investigated [9]. The results presented here indicate that the study of binary fluid convection in an annular geometry can provide important insights into the mechanisms by which wavelength adjustments occur in traveling wave patterns.

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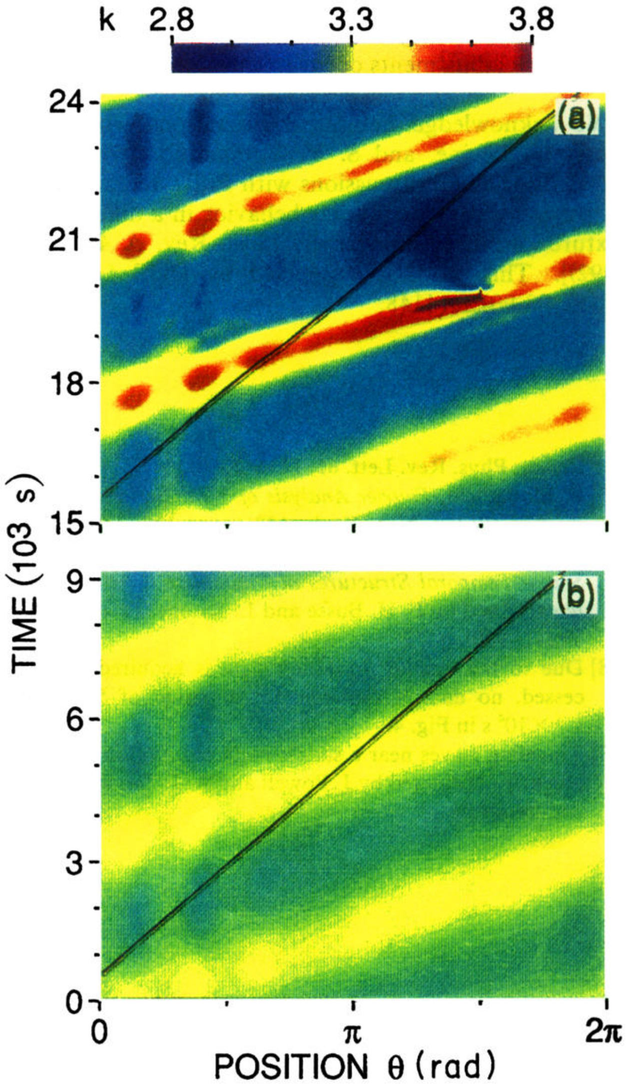
[8] Due to the manner in which data is acquired and processed, no data is available for a period of 585 s near 1.1×10^4 s in Fig. 4.

[9] Traveling waves near a saddle-node bifurcation have been recently considered by J. Powell and A. J. Bernoff (private communication).



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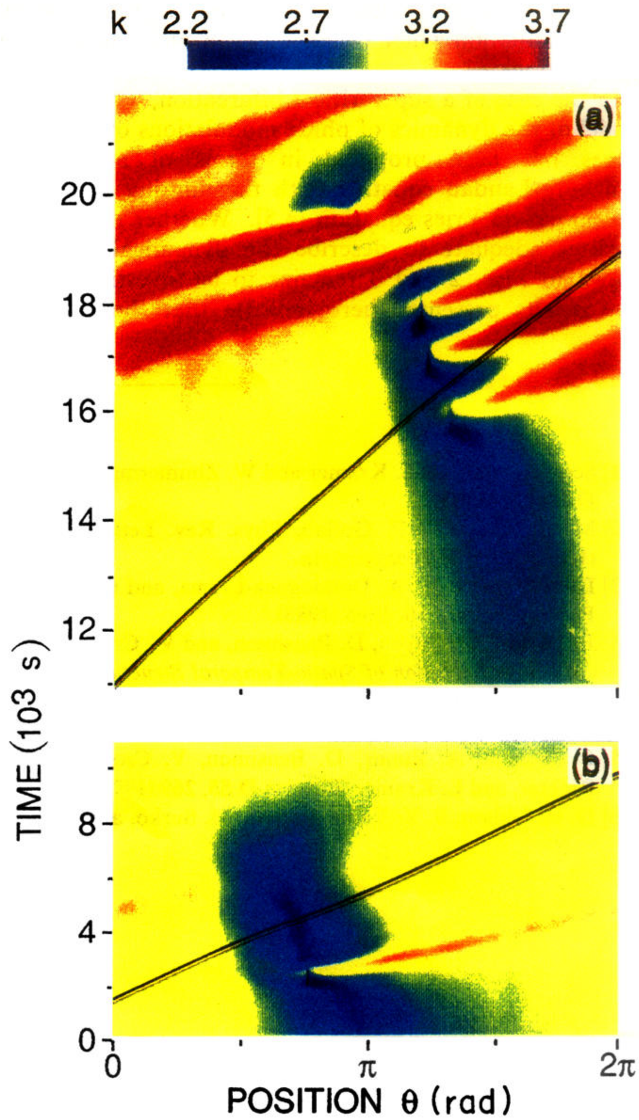

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