PHYSICAL REVIEW A VOLUME 46, NUMBER 12 15 DECEMBER 1992

Dynamical model of an earthquake fault with localization

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(Received 28 February 1992; revised manuscript received 6 July 1992)

We consider the one-dimensional dynamical Burridge-Knopoff stick-slip model for an earthquake fault with spatially inhomogeneous friction. The model is self-organizing under the externally imposed constraint of a spatial irregularity of strength which is designed to simulate residual damage to the system due to earlier earthquakes. The dissipation due to seismic wave radiation is adjusted so that the system is asymptotic to elasticity at all wavelengths. The model yields a self-organizing, spatially localized sequence of seismic events constrained by the spatial fluctuations. Repetitive, localized patterns of seismicity are erratically interrupted due to dynamical breaching of friction barriers. The model simulates an earthquake phenomenology that includes (1) the usual Gutenberg-Richter power-law energy-rate distribution at low energies with a rolloff at large energies, (2) spatial localization of large events on parts of a fault system and small events in the other parts of the system, (3) an ability to radiate seismic energy to distances far from the fault, and (4) a set of fractures whose lengths are never equal to the dimensions of the lattice or that intersect a free edge.

PACS number(s): 05.40.+j, 91.30.Px, 62.20.Mk, 05.45.+b

The scale independence implied by the Gutenberg-Richter frequency law for the number rate of occurrence of earthquakes with energies E, $\dot{N}(E)dE \sim E^{-p-1}dE$ [1] suggests that the physical process for earthquake occurrence should also be scale independent. As a consequence, a number of models of the self-organization of fracture events on individual, isolated faults have been developed for a geometry-independent, i.e., homogeneous landscape, that yield this relationship. These models involve a globally uniform rate of loading of the system, a stick-slip model of fracture, and stress redistribution after fracture. Among the models are a number of variations: some include dynamics in one dimension (1D) or 2D without instantaneous healing [2], some involve static fracture criteria in 2D or 3D with instantaneous healing [3], and there is a static 2D model without instantaneous healing [4]. In these homogeneous models, the stress field on a fault shows fluctuations on all scales; the fluctuations arise because of the history of stress drops on the fractured segments and the redistribution of stresses to neighboring locations. The final size of any growing fracture is defined by its encounter with local fluctuations in stress. Many of these models are variations of a model due to Burridge and Knopoff [5].

If the rate of energy released in earthquakes $\int E\dot{N}(E)dE$ is to be finite, then the upper limit on the integral cannot be infinity [6] since $p \sim \frac{2}{3}$. Hence, the scale independence implied by the G-R law cannot extend to the largest earthquakes, and there must be a cutoff to the size spectrum at the large-energy end, with impact on the arguments for homogeneity at all scales.

From scaling arguments that relate the dimensions of fractures to the energies released [7], the existence of a maximum energy cutoff argues in favor of a maximum fracture length. Thus we suppose that significant barriers of fracture strength exist on the faults that confine fracture growth. If these barriers are spatially fixed, then the largest earthquakes must be localized and recur on the same fault segments repeatedly; this is the seismologists' characteristic earthquake model [8].

The spatial distribution of earthquakes strongly suggests a model favoring significant long-term spatial localization. Although the power-law distribution for the rate of energy released describes seismicity on a broad two-dimensional network of faults, it is rare, if ever, that the power-law energy-rate distribution has been observed for an individual fault segment. Seismicity on the network of faults is highly irregular spatially, with large activity of small earthquakes on some faults of the system, while on a restricted subset of the fault system, only the largest earthquakes seem to occur. Thus the regional energy-rate power-law distribution that is a target for modeling is a large-scale spatial average of the seismicity over an extremely heterogeneous but localized geometry. Temporally the distribution density also shows fluctuations on the time scale of occurrence of aftershocks, which is typically a small number of years; but on longer time scales the spatially averaged time-rate distribution may be stable.

The localization of earthquakes is in contrast with expectations from models of uniform faults on which fractures can take place with equal probability along their length [2–4]. Carlson and Langer [2] have described a dynamical model for developing an energy-rate distribution with an upper cutoff, but because of homogeneity in the model, long-term spatial localization is absent. In [2], the deviations from self-similar scaling at large energies arise because of a characteristic distance scale in units of the lattice spacing. We use a lattice model that has no characteristic distance scale that depends on the internal lattice physics of a homogeneous system, but instead develops the cutoff in the distribution at large fracture sizes as a consequence of an assumption that there are geometrical fluctuations in the physical properties of the system. We show that this model produces long-term average spatial distributions of events that are locally restricted in their size range, but averaged over a large space scale, have the power-law property for small enough earthquakes.

We investigate the occurrence of earthquakes as a self-organizing process on a model system that we assume has the strong imprint of damage due to a long history of earlier earthquakes. We assume the earlier events have left the fault system in a state of considerable irregularity that represents a set of externally imposed constraints on the subsequent self-organization of the system under its more usual internally expressed rules. Our point of departure in these simulations is a 1D Burridge-Knopoff model (Fig. 1) that is homogeneous in all respects except for the distribution of static frictions. We adopt a suggestion that geometrical irregularities of fault surfaces and the geometry of fault networks can be described by heterogeneities of strength or friction [9]. We assume that we can map the 3D geometry of a real fault system into a 1D model with fluctuations in frictional breaking strengths. We assume that the distribution of breaking strengths will be fractal with exponent 1.2 or 1.3 [10] to correspond to the fractal exponent of the surface trace of the San Andreas Fault, and hence that barriers to extension of fractures are to be found not only at the largest, but at all scales [11]. We have no confirmation of the correlation between the exponents.

The model consists of a 1D lattice of N identical particles; each is coupled to a massive block via a frictional contact, and connected to its nearest neighbors by identical linear longitudinal springs with constants k and to a second massive block by identical linear transverse springs with constants l. The two blocks move relative to one another with a slow, constant velocity v; the force in the transverse springs increases slowly at the constant rate lv. The distributions of static frictions are generated by standard methods for fractals, and to ensure self-affinity, we add a bias constant to all values; all frictions are positive.

If the force F_n on a mass is equal to the local static friction B_n, the frictional resistance drops instantly to a lower value f (Fig. 2) and sliding begins. The motion of a particle contributes a force to its nearest neighbors. If the motion of a neighbor raises the total force on an unbroken lattice site to the level of its static friction, then this site breaks and the "crack" grows. A particle stops when its velocity becomes zero and the static friction is reestablished at this moment. We solve the coupled linear equations for the relative motion u_n describing sliding between the walls of the fault,

$$m\ddot{u}_n + k(2u_n - u_{n-1} - u_{n+1}) + lu_n = \tau_n = F_n - \phi(\dot{u}_n)\ , \quad (1)$$

where τ_n is the dynamical force drop, F_n is the force on the nth particle at the instant that the first particle begins to move in an individual event, and ϕ is the dynamical friction. In the interval between events, $dF_n/dt = lv$, $v \ll c$, where c is the velocity of sound on the lattice. The force F_n is reset to the value of the force on the nth particle immediately after each event. Immediately after a fracture, the forces F_n on a broken segment can have values less than $\phi_n(0)$ due to dynamical overshoot of the equilibrium position. The particles stop progressively with the freezing of one particle and the triggered freezing of neighbors, with dynamically derived time delays. The force drop across the advancing tearing edge between broken and unbroken sites due to the dynamical state of the broken sites is mirrored by a force drop across the contracting freezing edge between adjacent broken and unbroken sites, accounting for the overshoot in force.

In our model $\phi = f + \alpha\dot{u}_n$, i.e., the friction drops abruptly upon fracture to the lower value f. As the mass accelerates, the sliding friction increases by an amount proportional to the velocity, corresponding to radiation damping energy loss by seismic waves [5].

A fault such as the San Andreas Fault of California is significantly longer than the fracture length of even the largest earthquakes. In our models we (a) impose periodic boundary conditions to reduce 1D surface-to-volume effects, and (b) make the system sufficiently rough and stiff that all fractures are confined, i.e., so that no fracture can have a length equal to the lattice size [12]. We roughen by increasing the ratio of the peak-to-peak fluctuations to the mean of the breaking strengths; we stiffen by increasing the ratio of the transverse to the longitudinal spring constants.

Without radiation damping, particle motions in the system (1) have strong supersonic dispersion at long wavelengths in units of the lattice size [5], which can lead to rupture shock fronts for larger fracture events on this lattice [13]. However, the theory of fractures in an elastic continuum indicates that strains on the fracture surface be transmitted with the ordinary, nondispersive, sound wave speed [14] at all wavelengths; the cohesion B slows the rate of rupture below the sound wave speed [13]. The eigenfrequencies of (1) are

$$\mathrm{Re}\,\omega_p = \left\{ \frac{k}{m}\sin^2\frac{\pi p}{2N} + \frac{l}{m} - \left[\frac{\alpha}{2m}\right]^2 \right\}^{1/2} ,$$

$$p = 1, 2, \ldots, N-1 \quad (2)$$

if $N-1$ particles are in motion. If we set the attenuation factor $\alpha = (4lm)^{1/2}$, strains in the system will be transmit-

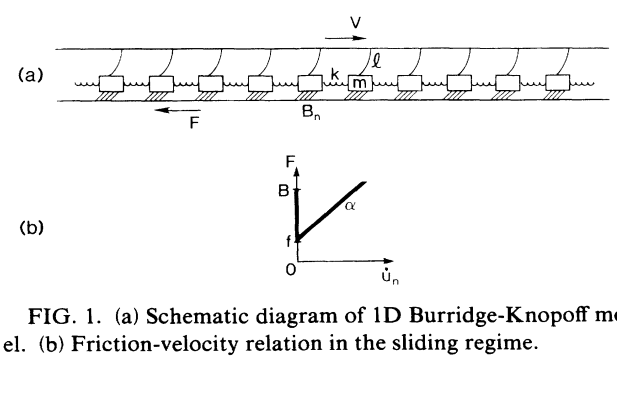

FIG. 1. (a) Schematic diagram of 1D Burridge-Knopoff model. (b) Friction-velocity relation in the sliding regime.

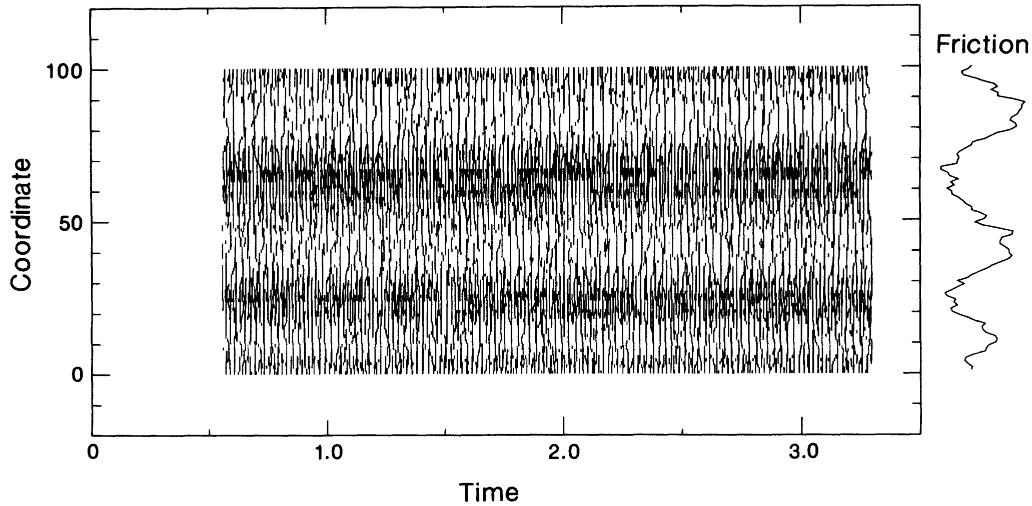

FIG. 2. Fracture history on a model fault with periodic boundary conditions. The system is homogeneous except for the friction, which is a fractal with exponent 1.3. The maximum to minimum fluctuations in friction B are in the ratio 5:1; $l/k = 0.25$. $k = m = 1$. $v = 10^{-4}$. Lattice length is 100. 5000 events are shown; initial transient is removed. The unit of time is $40B_{\min}/lv$.

ted along the fracture with the sound wave speed. The eigenfrequencies are now those of the usual linear elastic chain, and hence the system is asymptotic to elasticity in the continuum limit. The particle motions are decelerated due to the energy loss because of radiation damping. Although the system is asymptotic to elasticity in the 1D continuum limit, the model has displacements and stresses that are inconsistent with fractures in 2D or 3D.

We avoid errors due to numerical integrations by solving (1) as a piecewise linear problem, by expanding the solution in terms of the usual basis vector $\sin(\pi pn/N)$, with initial conditions determined by the coordinates and velocities of all the particles at the end of the preceding interval. The coefficients in the expansion, of course, depend on the inhomogeneous excitation stress drop vector τ_n. We trace the coordinates and velocities and increase the dimension by one if an additional particle is triggered into motion, or reduce the dimension of the array if a particle freezes.

One example of these simulations is shown in Fig. 2. The static friction is shown at the right of the diagram. The extent of a fracture, defined as an interactive set of motions on the lattice, is shown as a vertical stroke on this time scale. In general, the fractures with the smallest lengths occur near places of least stress drop, or least strength, and conversely those events with the greatest fracture lengths often begin at places of greatest stress drop, which are usually the places of greatest strength. Large fractures stop at large fluctuations in strength, while small earthquakes stop at smaller fluctuations. Thus localization of fractures of a given size and recurrence rate corresponds to the distribution of breaking strengths. Since the long-term average velocity of particles is constant, the events have an average time interval

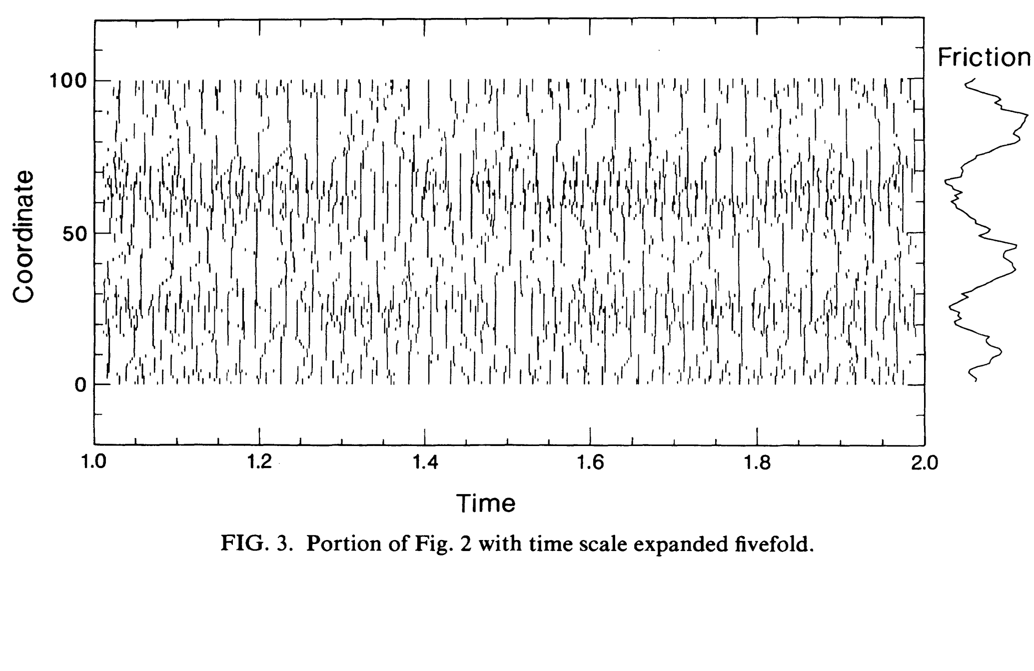

FIG. 3. Portion of Fig. 2 with time scale expanded fivefold.

between them that is proportional to their size. The spatial localization implied by these simulations is in contrast with its absence in homogeneous models, wherein events of any size can occur with equal probability along the length of a fault.

No events are wholly periodic locally (Fig. 3). After several repetitions of a sequence, seemingly regular patterns of larger events dissipate, occasionally in a sequence of smaller events. Large events often interrupt a sequence of smaller events intermittently. After dissipation or interruption, patterns form again in the same places and with similar periodicities. The interruption of locally regular patterns is due to the occasional breaching of the barriers that define localization; even locally strong barriers that delimit characteristic earthquakes must ultimately themselves rupture. Such ruptures act as gates for the diffusion of stress from one characteristic earthquake basin to another. The stress diffusion over distance appears to be episodic.

We calculate the distribution of the energies released in lattice events for comparison with the observed power-law energy-rate distribution. The near saturation at small energies in the logarithmic cumulative energy-frequency diagram (Fig. 4) is identified with finite lattice spacing effects. The rolloff at large energies is associated with the constraint that all fractures are confined and hence that there must be a maximum energy. There is a linear or scaling region for intermediate energies that is due to the distribution of sizes of localization basins for a sufficiently long lattice. We have verified the near linearity of the intermediate region by simulations with lattices of different lengths; for sufficiently short lattices the intermediate region disappears. The exponent in the intermediate-energy range is found from the differential energy-frequency distribution to avoid bias in the cumulative distribution; in the example, the exponent is 0.8. The exponent can be fine tuned to values such as $\frac{2}{3}$ by adjusting the damping coefficient or the ratio of the extreme values of the breaking strengths; there is an infinite number of combinations that yield a given exponent.

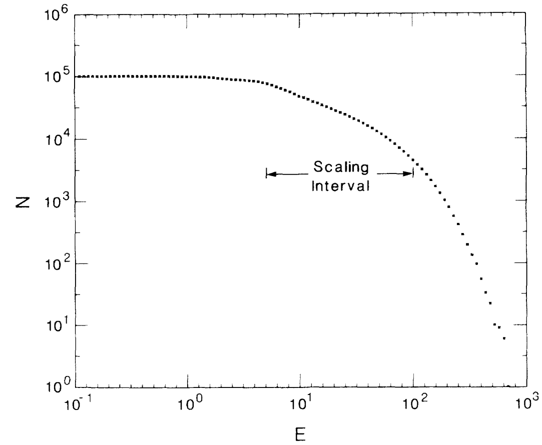

FIG. 4. Cumulative $\log_{10}N$ vs $\log_{10}E$ diagram for lattice with parameters as in Fig. 2. Lattice length is 400. Statistics of 100 000 events; initial transient is removed. The unit of energy is $B_{\min}^2/(2k+l)$.

Since the linear and the rolloff intervals are caused by the same mechanism, they should be fit by a common relation that spans both. These distributions are consistent with the gamma distribution $E^{-p-1}\exp(-E/E_{\text{ro}})$, which is asymptotic to the power law at small energies and has a large-energy rolloff scaling factor E_{ro} [15].

Although the mechanism of interactive fractures is deterministic, a probabilistic interpretation for predictability is difficult on this model. We have found no clear-cut way to predict the time of persistence of a locally regular pattern from observations of the local pattern alone. Since the truncation of a pattern is a consequence of the breaching of a barrier at an edge of the local region, predictions of truncation of regularity must depend on observations of the stress field at the barriers.

[1] B. Gutenberg and C. F. Richter, Bull. Seismol. Soc. Am. **46**, 105 (1956).
[2] J. Carlson and J. S. Langer, Phys. Rev. Lett. **62**, 2632 (1989); Phys. Rev. A **40**, 6470 (1989); J. M. Carlson, J. S. Langer, B. E. Shaw, and C. Tang, *ibid.* **44**, 884 (1991); J. M. Carlson, J. Geophys. Res. **96**, 4255 (1991); Phys. Rev. A **44**, 6226 (1991).
[3] P. Bak and C. Tang, J. Geophys. Res. **94**, 15 635 (1989); H. Takayasu and M. Matsuzaki, Phys. Lett. A **131**, 244 (1988); H. Takayasu, I. Nishikawa, and H. Tasaki, Phys. Rev. A **37**, 3110 (1988); K. Ito and M. Matsuzaki, J. Geophys. Res. **95**, 6853 (1990); H. Nakanishi, Phys. Rev. A **41**, 7086 (1990); **43**, 6613 (1991); K. Chen, P. Bak, and S. P. Obukhov, *ibid.* **43**, 625 (1991); H. J. S. Feder and J. Feder, Phys. Rev. Lett. **66**, 2669 (1991); S. R. Brown, C. H. Scholz, and J. B. Rundle, Geophys. Res. Lett. **18**, 215 (1991).
[4] M. Matsuzaki and H. Takayasu, J. Geophys. Res. **96**, 19 925 (1991).
[5] R. Burridge and L. Knopoff, Bull. Seismol. Soc. Am. **57**, 341 (1967).
[6] L. Knopoff and Y. Y. Kagan, J. Geophys. Res. **82**, 5647 (1977).
[7] H. Kanamori and D. L. Anderson, Bull. Seismol. Soc. **65**, 1073 (1975).
[8] D. P. Schwartz and K. J. Coppersmith, J. Geophys. Res. **89**, 5681 (1984); K. Aki, *ibid.* **84**, 6140 (1979); **89**, 5867 (1984).
[9] G. King and J. Nabelek, Science **228**, 984 (1985); G. King, Pure Appl. Geophys. **121**, 761 (1983); **124**, 567 (1986); A. Nur, *ibid.* **116**, 964 (1978).
[10] P. G. Okubo and K. Aki, J. Geophys. Res. **92**, 345 (1987).
[11] A. Nur and M. Israel, Phys. Earth Planet Inter. **21**, 225 (1980).
[12] Many of the static models in [3] make use of stress-absorbing edge conditions, which do not allow for the inward diffusion of stress into the lattice from the parts of the fault outside it. The free edge as in [2] is a reflector of stress and hence, by virtue of imaging, simulates an infinite fault with correlations, as does our assumption of

periodic boundary conditions. More important is the question whether "runaways," i.e., events whose lengths are equal to the lattice size, can appear on either the finite or the (imaged) infinite lattice. By the imaging argument, runaways create an undesirable, infinitely long fracture.

[13] J. S. Langer and C. Tang, Phys. Rev. Lett. **67**, 1043 (1991).

[14] L. Knopoff, J. O. Mouton, and R. Burridge, Geophys. J. R. Astron. Soc. **35**, 169 (1973).

[15] Y. Y. Kagan and L. Knopoff, in *Proceedings of the 8th World Conference on Earthquakes Engineering, San Francisco, 1984* (Prentice-Hall, Englewood Cliffs, NJ, 1984), Vol. 1, p. 295; Y. Y. Kagan, Geophys. J. Int. **106**, 123 (1991).