# Cloud of virtual photons surrounding a two-level atom driven by an external field

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The distortions of the cloud of virtual photons surrounding a ground-state two-level atom, induced by an external field driving the atom, are investigated. It is first shown that if the driving field is coupled to the atom via diagonal dipole matrix elements, then a region of quasistability for the virtual cloud exists. Then the number of photons in this virtual cloud is evaluated by perturbation theory, as a function of the intensity of the external driving field. Finally, the analytical expression obtained is discussed using approximations appropriate to different ranges of the intensity of the driving field.

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It has been known for a long time that the sources of a quantum field, in the ground state of the coupled source-field system, are generally surrounded by a cloud of virtual quanta of the field itself [1]. In this case one can profitably describe the physical properties of the source in terms of a dressed source that is the composite object constituted by the bare source and by the virtual cloud of field excitations [2]. The shape of the virtual cloud is expected to depend on the internal dynamical structure of the source, since the quantum fluctuations yield correlated transitions between the internal states of the source and the virtual states of the field that contribute to the cloud [3]. In QED in particular the shape of the virtual cloud has been explicitly evaluated for ground-state neutral hydrogen [4], which has been shown to be influenced by the presence of a static electric field capable of inducing atomic polarization [5]. The aim of this paper is to present preliminary results on the dependence of the virtual cloud from a strong time-dependent electromagnetic (e.m.) field acting on the atom. Part of the interest of this problem arises from the recent intense research activity in connection with atoms in strong fields and related effects such as above-threshold ionization [6] and high-order harmonic generation [7], although the model adopted in this paper is capable of giving only qualitative suggestions in connections with these phenomena.

The model we have adopted in order to discuss dressed atoms under irradiation is that of a two-level atom in the pseudospin $S=\frac{1}{2}$ formalism, interacting with a monochromatic field of frequency $\omega$ excited by an external source, and in the presence of the vacuum modes of the e.m. field. This model can be realistically used only to describe the effects of interaction between an atom and a monochromatic field when this is is almost resonant with a particular atomic transition frequency. Furthermore, the range of validity of the results obtained is also limited by the intensity of the field, which must be low enough not to involve other nonresonant atomic transitions. In this sense the results obtained with this model can be only suggestive of strong field effects.

This model is described by the Hamiltonian

$$H=H_0+V\ ,$$

$$H_0=\hbar\omega_0 S_z+\hbar\omega a^\dagger a+\sum_k \hbar\omega_k a_k^\dagger a_k+\varepsilon S_z(a+a^\dagger)\ , \tag{1}$$

$$V=\sum_k \varepsilon_k(S_+ + S_-)(a_k+a_k^\dagger)\ ,$$

where $\omega$ is the atomic natural frequency, $S_i$ $(i=\pm,z)$ are the usual $S=\frac{1}{2}$ operators, $a$ and $a^\dagger$ are the destruction and creation operators for the intense external field, while $a_k$ and $a_k^\dagger$ play the same role for the vacuum mode $k$ of frequency $\omega_k$, $k$ being representative of an appropriate set of indices which identify the mode in question. Further, $\varepsilon$ is the external-field atom-coupling constant and $\varepsilon_k$ is the corresponding quantity for the $k$th vacuum mode-atom coupling. It should be noted that while the latter coupling form is conventional and typical of a Dicke Hamiltonian, the former implies that parity of the atomic base states is not defined, since the interaction with the driving field takes place through $S_z$ rather than through $S_+$ and $S_-$. This feature of the model is indeed essential for our aims, since it is easy to realize that a conventional coupling would lead to instability of the cloud, which would become real and whose photons would be emitted in the form of fluorescent radiation. On the contrary we will demonstrate the existence of a quasistability region (QSR) in the eigenvalue spectrum of $H$, where a fairly well-defined virtual cloud can be formed, due to the $S_z$ form of the coupling selected. Moreover, an atom-field coupling such as that appearing in $H$ is certainly appropriate for atomic magnetic levels or for situations where a permanent electric moment is present [8]. We take both $\varepsilon$ and $\varepsilon_k$ real for simplicity of notation.

We begin by performing the exact diagonalization of

 

$H_0$. We label the eigenstates of

$$H_0^{(0)}=\hbar\omega_0+\hbar\omega a^\dagger a+\sum_k \hbar\omega_k a_k^\dagger a_k \ , \tag{2}$$

which is the free part of $H_0$, by the eigenvalues of the operators present in (2). Thus $|\sigma,n,\{n_k\}\rangle$ is an eigenstate of $H_0^{(0)}$ describing the atom in its upper or lower state ($\sigma=\pm1$), the external field having $n$ photons and the fluorescent field having a photon distribution $\{n_k\}$ over all its possible vacuum modes. The corresponding eigenvalue is

$$E(\sigma,n,\{n_k\})=\tfrac{1}{2}\hbar\omega\sigma+n\hbar\omega+\sum_k n_k\hbar\omega_k \ . \tag{3}$$

Clearly $|\sigma,n,\{n_k\}\rangle$ is not an eigenstate of $H_0$ due to the presence of the coupling term. These eigenstates of $H_0$, however, can be easily obtained by performing the unitary transformation induced by

$$T=\exp\left[\frac{\varepsilon}{\hbar\omega}S_z(a-a^\dagger)\right] \ . \tag{4}$$

This leads to the new Hamiltonian

$$\tilde{H}_0=T^{-1}H_0T=\hbar\omega_0S_z+\hbar\omega a^\dagger a+\sum_k \hbar\omega_a a_k^\dagger a_k-\frac{\varepsilon^2}{4\omega\hbar} \ , \tag{5}$$

which is diagonal in the basis of the eigenstates of $H_0^{(0)}$. Since $T$ is unitary,

$$\begin{aligned}\langle\sigma,n,\{n_k\}|\tilde{H}_0|\sigma',n',\{n_k'\}\rangle &=\langle\sigma,n,\{n_k\}|T^\dagger H_0T|\sigma',n',\{n_k'\}\rangle\\ &=\tilde{E}(\sigma,n,\{n_k\})\delta_{\sigma\sigma'}\delta_{nn'}\delta_{\{n_k\}\{n_k'\}} \ ,\end{aligned} \tag{6}$$

and the eigenstates of $H_0$ are

$$\begin{aligned}T|\sigma,n,\{n_k\}\rangle &=\exp\left[\frac{\varepsilon}{2\hbar\omega}\sigma(a-a^\dagger)\right]|\sigma,n,\{n_k\}\rangle\\ &\equiv|\sigma,\tilde{n}_\sigma,\{n_k\}\rangle \ ,\end{aligned} \tag{7}$$

where

$$|\tilde{n}_\sigma\rangle=\exp\left[\frac{\varepsilon}{2\hbar\omega}\sigma(a-a^\dagger)\right]|n\rangle \ . \tag{8}$$

Moreover, the corresponding eigenvalue is

$$\tilde{E}(\sigma,\tilde{n}_\sigma,\{n_k\})=\tfrac{1}{2}\hbar\omega_0\sigma+n\hbar\omega+\sum_k \hbar\omega_k n_k-\frac{\varepsilon^2}{4\hbar\omega} \ .$$

Thus the eigenvalue spectrum of $H_0$ is identical to that of $\tilde{H}_0$, albeit shifted by the amount $-\varepsilon^2/4\hbar\omega$.

Next we evaluate the matrix elements of $V$ in the basis of the eigenstates of $H_0$. We have

$$\begin{aligned}\langle\sigma,\tilde{n}_\sigma,\{n_k\}|V|\sigma',\tilde{n}'_{\sigma'},\{n_k'\}\rangle\\ =\langle\sigma,n,\{n_k\}|\tilde{V}|\sigma',n',\{n_k'\}\rangle \ ,\end{aligned} \tag{9}$$

where some elementary algebra shows that

$$\begin{aligned}\tilde{V}=T^{-1}VT=\sum_k \varepsilon_k S_+\exp\left[\frac{\varepsilon}{\hbar\omega}(a^\dagger-a)\right]\\ +S_-\exp\left[-\frac{\varepsilon}{\hbar\omega}(a^\dagger-a)\right](a_k+a_k^\dagger) \ .\end{aligned} \tag{10}$$

Substituting (10) in (9) and performing the calculations we obtain the dressed matrix elements. From the form of (10) it is obvious that the matrix elements vanish unless $\{n_k\}$ and $\{n_k'\}$ are identical except for one photon in one of the vacuum modes $k$. Thus we introduce a slight change of notation and indicate these nonvanishing matrix elements as $\langle\sigma,\tilde{n}_\sigma,n_k\pm1|V|\sigma',\tilde{n}'_{\sigma},n_k\rangle$. After a lengthy procedure we obtain exactly

$$\begin{aligned}\langle\sigma,\tilde{n}_\sigma,n_k+1|V|\sigma',\tilde{n}'_{\sigma'},n_k\rangle &=(1-\delta_{\sigma\sigma'})d_k e^{-x/2}x^{|n'-n|/2}\{\sigma^{n-n'}\sqrt{n'!/n!}L_{n'}^{(n-n')}(x)\Theta(n-n')\\ &\quad+(-\sigma)^{n-n'}\sqrt{n!/n'!}L_n^{(n'-n)}(x)\Theta(n'-n)\} \ ,\\ \langle\sigma,\tilde{n}_\sigma,n_k-1|V|\sigma',\tilde{n}_{\sigma'},n_k\rangle &=\frac{c_k}{d_k}\langle\sigma,\tilde{n}_\sigma,n_k+1|V|\sigma',\tilde{n}_{\sigma'},n_k\rangle \ ,\end{aligned} \tag{11}$$

where

$$c_k=\varepsilon_k\sqrt{n_k},\quad d_k=\varepsilon_k\sqrt{(n_k+1)},\quad x=\left[\frac{\varepsilon}{\hbar\omega}\right]^2;$$

$$\Theta(n-n')=\begin{cases}1 & (n>n')\\ 0 & (n<n') \ ,\end{cases} \tag{12}$$

and where the generalized Laguerre polynomials are defined as [9]

$$L_n^{(\alpha)}(x)=\sum_{m=0}^{n}\frac{(-1)^m}{m!}\binom{n+\alpha}{n-m}x^m \quad (\alpha>-1) \ . \tag{13}$$

It is now appropriate to remark that $V$ does not connect states with equal values of $\sigma$. It is then easy to realize with the help of Fig. 1 that for $\omega<\omega_0$ a range of quasistability exists for the eigenstates $|\sigma=-1,\tilde{n}_{-1}<(\omega_0/\omega),\{0_k\}\rangle$ of $H_0$. These states are not connected at first order in $V$ to any other eigenstate via energy-conserving processes. In fact, the energy eigenstate of $H_0$ in the QSR is

$$\tilde{E}(-1,\tilde{n}_{-1},\{0_k\})=-\tfrac{1}{2}\hbar\omega_0+\tilde{n}_{-1}\hbar\omega-\frac{\varepsilon^2}{4\hbar\omega} \ , \tag{14a}$$

while the energy of any other eigenstate candidate for connection is

$$\tilde{E}(1,\tilde{n}'_1,\{n_k\})=\tfrac{1}{2}\hbar\omega_0+\tilde{n}'_1\hbar\omega+\hbar\omega_k-\frac{\varepsilon^2}{4\hbar\omega}\ . \quad (14b)$$

Thus the difference of the two energies is $\hbar\omega_0+(n'-n)\hbar\omega+\hbar\omega_k$, which is always positive for any $n_{-1}<\omega_0/\omega$, and transitions induced by $V$ at first order do not conserve energy. Because these states belonging to the QSR have a much longer decay time than the other states outside the same region, it makes sense to investigate the possibility that they support a relatively stable cloud of virtual photons. Thus we concentrate on the range of parameters $\hbar\omega \ll \hbar\omega_0$, where we have just seen a QSR exist, and we discuss the structure of the virtual cloud, which we describe by the average number of virtual photons in mode $k$ associated with a state $|-1,\tilde{n}_{-1},\{0_k\}\rangle$ within the QSR.

This is done in two steps, the first of which consists in evaluating the perturbative correction $|\psi^{(1)}\rangle$ to $|-1,\tilde{n}_{-1},\{0_k\}\rangle$. We find, at first order in $\varepsilon_k$, and using (11) and (14),

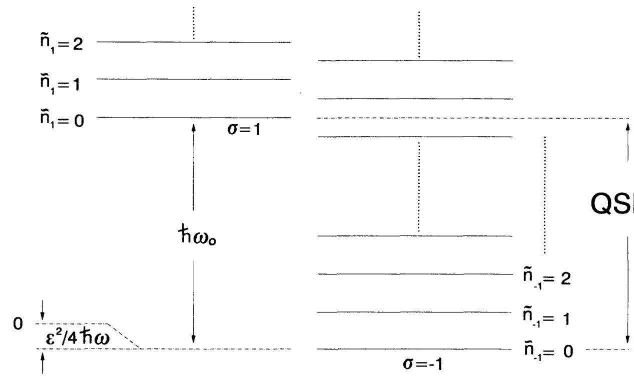


FIG. 1. Eigenvalue spectrum for the driven atom within the subspace $\{n_k\}=\{0_k\}$ and for $\omega_0>\omega$. Eigenstates of $H_0$ falling within the range of quasistability QSR are not connected by vacuum fluctuations at first order with any other eigenstate.

$$|\psi^{(1)}\rangle=\sum_{\tilde{n}'_1,k}\frac{\langle 1,\tilde{n}'_1,1_k|V|-1,\tilde{n}_{-1},\{0_k\}\rangle}{\tilde{E}(-1,\tilde{n}_{-1},\{0_k\})-\tilde{E}(1,\tilde{n}'_1,1_k)}|1,\tilde{n}'_1,1_k\rangle$$
$$=\sum_k\frac{\varepsilon_k}{\hbar}e^{-x/2}\left[\sum_{n'=0}^{n}\frac{(-1)^{n-n'}x^{(n-n')/2}\sqrt{n'!/n!}\,L_n^{(n-n')}(x)}{-\omega_0+(n-n')\omega-\omega_k}|1,\tilde{n}'_1,1_k\rangle-\sum_{n'=n+1}^{\infty}\frac{x^{(n'-n)/2}\sqrt{n!/n'!}\,L_n^{(n'-n)}(x)}{+\omega_0+(n'-n)\omega+\omega_k}|1,\tilde{n}'_1 1_k\rangle\right], \quad (15)$$

where $1_k$ indicates the photon distribution with one photon in mode $k$ and no photon in all other modes. The second step consists in evaluating the number of virtual quanta on the perturbed state $|-1,\tilde{n}_{-1},\{0_k\}\rangle+|\psi^{(1)}\rangle$.

We obtain at second order in $\varepsilon_k$,

$$\langle a_k^\dagger a_k\rangle=\langle\psi^{(1)}|a_k^\dagger a_k|\psi^{(1)}\rangle=\frac{\varepsilon_k^2}{\hbar^2}e^{-x}\left[\sum_{n'=0}^{n}\frac{n'!}{n!}\frac{x^{n-n'}[L_{n'}^{(n-n')}(x)]^2}{[-\omega_0+(n-n')\omega-\omega_k]^2}+\sum_{n'=n+1}^{\infty}\frac{n!}{n'!}\frac{x^{n'-n}[L_n^{(n'-n)}(x)]^2}{[\omega_0+(n'-n)\omega+\omega_k]^2}\right]$$
$$(\omega_0/\omega>n)\ . \quad (16)$$

This expression for the contribution of $k$ photons to the virtual cloud, albeit exact at second order in $\varepsilon_k$ and for $n<\omega_0/\omega$, is not very transparent. In particular, the form of the dependence of $\langle a_k^\dagger a_k\rangle$ from the intensity of the external field is not easy to read out of (16). Thus we are compelled to perform some rather radical simplifications.

In order to obtain these simplifications, we assume that in the selected range of parameters it is $\varepsilon^2 n/(\hbar\omega)^2\le 1$; hence $x=O(n^{-1})$ from (12). Consequently we can apply the Tricomi asymptotic expansion [10] for the generalized Laguerre polynomials in the region near the origin as [11]

$$e^{-x/2}L_n^{(\alpha)}(x)=\left[\frac{\nu}{4x}\right]^{\alpha/2}[J_\alpha(\sqrt{\nu x})+O(n^{-1})]\ ,$$
$$\nu=4n+2\alpha+2 \quad (17)$$

where $J$ are Bessel functions of the first kind. Moreover, coherently with the approximations leading to (17), we use also [11]

$$\frac{\Gamma(b+n+1)}{\Gamma(c+n+1)}=\left[n+\frac{b+c+1}{2}\right]^{b-c}[1+O(n^{-2})]\ , \quad (18)$$

which yields

$$\frac{n'!}{n!}=\frac{\Gamma(n'-n+n+1)}{\Gamma(n+1)}=\left[\frac{n+n'+1}{2}\right]^{n'-n}[1+O(n^{-2})]\ ,$$
$$\frac{n!}{n'!}=\frac{\Gamma(n+1)}{\Gamma(n'-n+n+1)}=\left[\frac{n+n'+1}{2}\right]^{n-n'}[1+O(n^{-2})]\ . \quad (19)$$

Substituting (17) and (19) in (16) we obtain

$$\langle a_k^\dagger a_k \rangle = \frac{\varepsilon_k^2}{\hbar^2}\left[\sum_{n'=0}^{n}\frac{J_{n-n'}^2(\sqrt{2(n+n'+1)x})}{[-\omega_0+(n-n')\omega-\omega_k]^2} + \sum_{n'=n+1}^{\infty}\frac{J_{n'-n}^2(\sqrt{2(n+n'+1)x})}{[\omega_0+(n'-n)\omega+\omega_k]^2}\right] + O(n^{-1}) \ . \quad (20)$$

Thus for $\omega_0/\omega > n \gg 1$ we can approximate (20) as

$$\langle a_k^\dagger a_k \rangle = \frac{\varepsilon_k^2}{\hbar^2}\sum_{n'=0}^{\infty}\left[\frac{J_{|n-n'|}(\sqrt{2(n+n'+1)x})}{\omega_0+(n'-n)\omega+\omega_k}\right]^2 \quad \left[\frac{\omega_0}{\omega} > n \gg 1, \ \ \varepsilon\sqrt{n} < \hbar\omega\right] , \quad (21)$$

which is much more manageable than (16) and which is valid for $\varepsilon\sqrt{n} < \hbar\omega \ll \hbar\omega_0$. Keeping only terms linear in $nx$ (that is, assuming $\varepsilon\sqrt{n} \ll \hbar\omega$), (21) reduces to

$$\langle a_k^\dagger a_k \rangle = \frac{\varepsilon_k^2}{\hbar^2(\omega_0+\omega_k)^2} + \frac{\varepsilon_k^2 x}{\hbar^2}\left[\frac{n}{(\omega_0-\omega+\omega_k)^2} + \frac{n+1}{(\omega_0+\omega+\omega_k)^2} - \frac{2n+1}{(\omega_0+\omega_k)^2}\right] \quad \left[\frac{\omega_0}{\omega} > n \gg 1, \ \ \varepsilon\sqrt{n} \ll \hbar\omega\right] , \quad (22)$$

where the first term yields the virtual cloud in the absence of the driving field (as expected) and the second, which is proportional to $\varepsilon^2 n$ for large $n$, is the correction due to the driving field. On the other hand, for the strong driving field case $\varepsilon\sqrt{n} \sim \hbar\omega$ it is easy to guess that the dominant contribution to the sum over $n'$ in (21) comes from the low-order Bessel functions for which $n' \sim n$. In this case one can obtain a rough representation of the behavior of $\langle a_k^\dagger a_k \rangle$ as

$$\langle a_k^\dagger a_k \rangle = \frac{\varepsilon_k^2}{\hbar^2}\frac{J_0^2(2\varepsilon\sqrt{n}/h\omega)}{(\omega_0+\omega)^2} \quad \left[\frac{\omega_0}{\omega} > n \gg 1, \ \ \varepsilon\sqrt{n} \sim \hbar\omega\right] . \quad (23)$$

We shall refrain here from discussing the approximation of (16) valid in the oscillatory region of the Laguerre polynomial [11], which is in the range of $x$ including point $x = 4n+2$, since this implies $\varepsilon > \hbar\omega$, which we consider nonphysical at optical frequencies and out of the scope of this paper.

In conclusion, we summarize our results as follows. We have shown that for a two-level atom in the presence of an external driving field described by the model Hamiltonian (1) it is possible to obtain a range of eigenstates of the atom–driving-field system, which are capable of supporting a cloud of virtual photons created around the atom by the vacuum fluctuations. This cloud of virtual photons depends on the intensity of the external field.

We have calculated the dependence on this intensity exactly and analytically at order $\varepsilon_k$, the atom vacuum fluctuation coupling constant. The resulting expression has been considered in various ranges of the physical parameters, and simplified expressions have been obtained for weak ($\varepsilon\sqrt{n} \ll \hbar\omega$) and strong ($\varepsilon\sqrt{n} \sim \hbar\omega$) driving field.

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