Quantum theory of optical multistability in a two-photon three-level Λ-configuration medium

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An effective Hamiltonian for the two-photon two-level model is derived form the microscopic Hamiltonian of a three-level atom in a Λ configuration, interacting through two-photon transitions with a light field. The effective Hamiltonian includes a term describing the optical Stark shift. With this effective Hamiltonian, we study the steady-state equation and the squeezing spectrum for a cavity-field mode interacting with an ensemble of three-level "Λ-configuration" atoms by a two-photon transition. It is shown that significant changes in the transmission characteristics and the squeezing spectrum for the output field result from the inclusion of the Stark shift.

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I. INTRODUCTION

The three-level Λ medium displaying optical tristability has been the subject of theoretical [1–4] and experimental [5] studies for a decade and has attracted considerable interest for its possible utility as a squeezed-state generator [6,7] in recent years. Reid, Walls, and Dalton [6] pointed out that the three-level Λ medium is particularly promising for producing squeezed light. It has many advantages, such as negligible atomic saturation, substantial reduction in the light intensity, and the atomic density required. Savage and Walls [7] put forward a model of a single cavity-field mode interacting with an ensemble of three-level Λ atoms and employed an effective Hamiltonian to deal with the two-photon transitions therein. As the optical Stark shift is ignored, the effective Hamiltonian is exact only for special selections of the physical parameters [8–10].

From a semiclassical viewpoint, a complete theory for the two-photon process has been developed by Narducci *et al.* [11], which reduces the problem to an effective two-level model including the Stark shift. Holm and Sargent III [9] took into account the Stark shift arising from the interaction of the atoms with the classical pump field in their quantum theory of multiwave mixing. The effect of the Stark shift on squeezing was found to be approximately a simple translation of the atomic detuning [12].

In this paper we wish to present a fully quantum-statistical treatment of two-photon optical multistability in a Λ medium, including the optical Stark shift. In Sec. II, we present a quantum formulation of a modified treatment of Narducci *et al.* [11] to meet the Λ configuration, which leads to an effective Hamiltonian for the two-photon two-level model. In Sec. III, we use the method of Haken [13] and Drummond and Walls [14] to derive our basic equations. The effect of Stark shift on the steady-state deterministic equation and the multistable behavior of the transmission curve is discussed in Sec. IV. We eliminate the atomic variables adiabatically in the good-cavity limit in Sec. V and derived the linearized equation for the fluctuation in Sec. VI. Following the technique of Collett and Gardiner [15] and Collett and Walls [16], the squeezing spectrum in the output field is calculated in Sec. VII.

II. DERIVATION OF THE EFFECTIVE HAMILTONIAN

We consider a three-level atom in a Λ configuration interacting with a light field of frequency ω. The energy-level diagram of the atom is shown in Fig. 1. The two lower levels labeled $|2\rangle$ and $|1\rangle$, separated by energy $\hbar\omega_{21}$, are coupled to the upper level $|j\rangle$ by a direct dipole transition. The Hamiltonian of the atom–light-field system in the interaction representation is

$$\tilde{H}_I = i\hbar[g_2 e^{i\mathbf{k}\cdot\mathbf{x}}|2\rangle\langle j|e^{-i\omega_{j2}t}(be^{-i\omega t}+b^{\dagger}e^{i\omega t}) + g_1 e^{i\mathbf{k}\cdot\mathbf{x}}|j\rangle\langle 1|e^{-i\omega_{j1}t}(be^{-i\omega t}+b^{\dagger}e^{i\omega t})] + \text{H.c.}, \quad (1)$$

where $\omega_{ik}=\omega_i-\omega_k$. $b^{\dagger}$ and b are respectively the boson creation and annihilation operators of the light field. g_1 and g_2, assumed real without loss of generality, are coupling constants describing the interaction between the atom and the light field. We assume that ω_{21} is small and that the field frequency ω is far from resonance with ω_{j2} and ω_{j1}, hence an effective Hamiltonian coupling the two lower levels via two-photon transitions can be derived.

Taking the diagonal elements of (1) in the coherent-

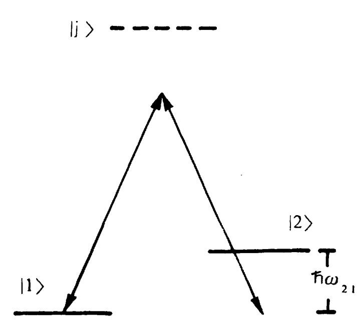

FIG. 1. Schematic energy-level diagram for a three-level atom in a Λ configuration.

state representation, we obtain the normally associated function $\tilde{H}_I^{(n)}$ corresponding to $\tilde{H}_I$ [17]. This amounts to replacing the operators $b^\dagger$ and b in (1) by c numbers α^* and α.

The atomic-field system obeys the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle=\tilde{H}_I^{(n)}|\psi(t)\rangle \ . \tag{2}$$

For a three-level atom, the state vector $|\psi(t)\rangle$ can be expanded as

$$|\psi(t)\rangle=c_1|1\rangle+c_2|2\rangle+c_j|j\rangle \ . \tag{3}$$

Substituting (1) and (3) into (2), we obtain equations of motion for the atomic amplitudes c_1, c_2, and c_j. Following the lines of Narducci *et al.* [11], we eliminate c_j with use of the slowly-varying-amplitude approximation and the two-photon rotating-wave approximation. Retaining only the slowly varying terms, we obtain the following equations of motion for c_1 and c_2 in the Λ-configuration case:

$$\begin{aligned} i\dot{c}_1&=-g_1^2\frac{2\omega_{j1}|\alpha|^2}{\omega_{j1}^2-\omega^2}c_1+g_1g_2\frac{2\omega_{j2}e^{-i2\mathbf{k}\cdot\mathbf{x}}|\alpha|^2}{\omega_{j2}^2-\omega^2}c_2 \ ,\\ i\dot{c}_2&=g_1g_2\frac{2\omega_{j2}e^{i2\mathbf{k}\cdot\mathbf{x}}|\alpha|^2}{\omega_{j2}^2-\omega^2}c_1-g_2^2\frac{2\omega_{j2}|\alpha|^2}{\omega_{j2}^2-\omega^2}c_2 \ , \end{aligned} \tag{4}$$

which satisfy probability conservation requirements.

Thus the three-level model is reduced to an effective two-level model, with $H_I^{(n)}$ expressed in matrix form

$$H_I^{(n)}=\hbar\begin{bmatrix} -g_1^2\dfrac{2\omega_{j1}|\alpha|^2}{\omega_{j1}^2\omega^2} & g_1g_2\dfrac{2\omega_{j2}e^{-i2\mathbf{k}\cdot\mathbf{x}}|\alpha|^2}{\omega_{j2}^2-\omega^2}\\ g_1g_2\dfrac{2\omega_{j2}e^{i2\mathbf{k}\cdot\mathbf{x}}|\alpha|^2}{\omega_{j2}^2-\omega^2} & -g_2^2\dfrac{2\omega_{j2}|\alpha|^2}{\omega_{j2}^2-\omega^2}\end{bmatrix} . \tag{5}$$

With the normal ordering operator [17] operating on $H_I^{(n)}$ and introducing atomic spin operators s^-, s^+, and s_z to describe the two-level atom, we obtain the following effective Hamiltonian:

$$\begin{aligned} H_I&=\hbar g\bar{\delta}b^\dagger b s_z\\ &\quad+i\hbar g b^\dagger b[s^-e^{-i(2\mathbf{k}\cdot\mathbf{x}+\pi/2)}-s^+e^{i(2\mathbf{k}\cdot\mathbf{x}+\pi/2)}] \ , \end{aligned} \tag{6}$$

where

$$\begin{aligned} g&=g_1g_2\frac{\omega_{j2}}{\omega_{j2}^2-\omega^2} \ ,\\ g\bar{\delta}&=g_1^2\frac{2\omega_{j1}}{\omega_{j1}^2-\omega^2}-g_2^2\frac{2\omega_{j2}}{\omega_{j2}^2-\omega^2} \ , \end{aligned} \tag{7}$$

g is the effective two-photon coupling constant in [7], and the first term in (6) describes the optical Stark shift. It is seen that $g\bar{\delta}$ may have the same order of magnitude as g and must be included.

III. QUANTUM-MECHANICAL EQUATION

Now we consider the N three-level "Λ-configuration" atoms in a single-ported optical ring cavity interacting through two-photon transitions with a cavity mode. The two-photon transition is modeled by the effective two-photon Hamiltonian derived in Sec. II. With (6), our model Hamiltonian is

$$\begin{aligned} H&=H_F+H_E+H_I+H_D \ ,\\ H_F&=\hbar\omega_c b^\dagger b+\tfrac{1}{2}\hbar\omega_{21}\sum_{\mu=1}^{N}S_\mu^z \ ,\\ H_E&=i\hbar(\epsilon b^\dagger e^{-i\omega_L t}-\epsilon^* b e^{i\omega_L t}) \ ,\\ H_I&=\hbar g\bar{\delta}\sum_{\mu=1}^{N}b^\dagger b s_\mu^z\\ &\quad+i\hbar g b^\dagger b\sum_{\mu=1}^{N}[s_\mu^-e^{-i(2\mathbf{k}\cdot\mathbf{x}_\mu+\pi/2)}-s_\mu^+e^{i(2\mathbf{k}\cdot\mathbf{x}_\mu+\pi/2)}] \ ,\\ H_D&=\sum_{\mu=1}^{N}(\Gamma_P s_\mu^z+\Gamma_A s_\mu^+ +\Gamma_A^\dagger s_\mu^-)+\Gamma_F b^\dagger+\Gamma_F^\dagger b \ . \end{aligned} \tag{8}$$

H_F is the free Hamiltonian for the cavity-field mode of frequency ω_c, and for the lower two levels of the N atoms. H_E accounts for the driving of the cavity by an external coherent field of amplitude ϵ and frequency ω_L. H_I models the two-photon interaction of the cavity mode with the effective two-photon two-level medium, g and $g\bar{\delta}$ are given in (7), with ω replacing by ω_L, for the frequency of the intracavity field is equal to the frequency of the external driving field in steady states. Thus the Stark shift included comes from the interaction of the intracavity field with the atoms. H_D describes the coupling of atoms to reservoirs Γ_A, $\Gamma_A^\dagger$, representing incoherent pumping, and to the reservoir Γ_P, representing phase damping. The final terms couple the field to reservoirs Γ_F and $\Gamma_F^\dagger$ describing dissipation of the field through the cavity port.

Following the method of Haken [13] and Drummond and Walls [14], we may derive a master equation for the density operator ρ from the Hamiltonian (8), including various damping and incoherent pumping effects. Specifically, we included damping of the cavity mode at rate κ, phase damping of coherence between the lower levels at rate γ_p, population decay from level $|2\rangle$ to $|1\rangle$ at rate $\gamma_\downarrow$, and incoherent population pumping from level $|1\rangle$ to level $|2\rangle$ at rate $\gamma_\uparrow$.

The master equation for the density operator yields a Fokker-Planck (FP) equation for the distribution function P in positive P representation which is valid for a large number of atoms N. A correspondence between complex c numbers and system operators is established as follows:

$$\begin{aligned} &\alpha,\alpha^+\rightleftarrows b,b^\dagger \ ,\\ &V\rightleftarrows s^-=\sum_{\mu=1}^{N}s_\mu^-e^{-i(2\mathbf{k}\cdot\mathbf{x}_\mu+\pi/2)} \ ,\\ &V^+\rightleftarrows s^+=\sum_{\mu=1}^{N}s_\mu^+e^{+i(2\mathbf{k}\cdot\mathbf{x}_\mu+\pi/2)} \ ,\\ &D\rightleftarrows 2s_z=2\sum_{\mu=1}^{N}s_\mu^z \ . \end{aligned} \tag{9}$$

We transform the resulting FP equation with positive-definite diffusion into stochastic differential equations:

$$\dot{\alpha}=\epsilon-\kappa(1+i\varphi)\alpha-i\bar{\delta}g\frac{D}{2}\alpha-g(V^{+}-V)\alpha+\Gamma_{\alpha} ,$$
$$\dot{\alpha}^{+}=\epsilon^{*}-\kappa(1-i\varphi)\alpha^{+}+i\bar{\delta}g\frac{D}{2}\alpha^{+}+g(V^{+}-V)\alpha^{+}+\Gamma_{\alpha^{+}} ,$$
$$\dot{V}=-\gamma_{\perp}(1+i\Delta)V-i\bar{\delta}g\alpha^{+}\alpha V+g\alpha^{+}\alpha D+\Gamma_{V} , \qquad (10)$$
$$\dot{V}^{+}=-\gamma_{\perp}(1-i\Delta)V^{+}+i\bar{\delta}g\alpha^{+}\alpha V^{+}+g\alpha^{+}\alpha D+\Gamma_{V^{+}} ,$$
$$\dot{D}=-\gamma_{\parallel}(D-D_0)-2g\alpha^{+}\alpha(V+V^{+})+\Gamma_{D} ,$$

where

$$\varphi=\frac{\omega_c-\omega_L}{\kappa}, \quad \Delta=\frac{\omega_{21}}{\gamma_{\perp}} ,$$

and

$$\gamma_{\parallel}=\gamma_{\uparrow}+\gamma_{\downarrow}, \quad \gamma_{\perp}=\tfrac{1}{2}\gamma_{\parallel}+\gamma_{p}, \quad D_0=\gamma_{\parallel}^{-1}(\gamma_{\uparrow}-\gamma_{\downarrow})N . \qquad (11)$$

κ is the relaxation rate of the cavity, $\gamma_{\perp}$ and $\gamma_{\parallel}$ are the transverse and longitudinal relaxation rates of the two-level atoms, respectively, while γ_P is the rate of collision-induced phase decay of the atoms ($\gamma_{\perp}=\gamma_P+\gamma_{\parallel}/2$). In our paper we shall set $\gamma_{\perp}=0$ as in [7].

The $\Gamma(t)$s are δ-function-correlated noise functions with zero mean. The nonzero correlations of the quantum noise terms are

$$\langle\Gamma_{\alpha}(t)\Gamma_{V}(t')\rangle=(g\alpha D-i\bar{\delta}g\alpha V)\delta(t-t') ,$$
$$\langle\Gamma_{\alpha^{+}}(t)\Gamma_{V^{+}}(t')\rangle=(g\alpha^{+}D+i\bar{\delta}g\alpha^{+}V^{+})\delta(t-t') ,$$
$$\langle\Gamma_{V}(t)\Gamma_{V}(t')\rangle=2g\alpha^{+}\alpha V\delta(t-t') ,$$
$$\langle\Gamma_{V^{+}}(t)\Gamma_{V^{+}}(t')\rangle=2g\alpha^{+}\alpha V^{+}\delta(t-t') ,$$
$$\langle\Gamma_{V}(t)\Gamma_{V^{+}}(t')\rangle=[\gamma_P(D+N)+\gamma_{\parallel}N]\delta(t-t') , \qquad (12)$$
$$\langle\Gamma_{D}(t)\Gamma_{D}(t')\rangle=2[\gamma_{\parallel}(N-D)-2\gamma\alpha^{+}\alpha(V+V^{+})]\times\delta(t-t') ,$$
$$\langle\Gamma_{\alpha}(t)\Gamma_{D}(t')\rangle=-2g\alpha V^{+}\delta(t-t') ,$$
$$\langle\Gamma_{\alpha^{+}}(t)\Gamma_{D}(t')\rangle=2g\alpha^{+}V\delta(t-t') ,$$
$$\langle\Gamma_{V}(t)\Gamma_{D}(t')\rangle=-2\gamma_{\parallel}V\delta(t-t') ,$$
$$\langle\Gamma_{V^{+}}(t)\Gamma_{D}(t')\rangle=-2\gamma_{\parallel}V^{+}\delta(t-t') .$$

For later use, it is practical to define a ratio which gives the relative degree of radiative and collisional damping $f=\gamma_{\parallel}/2\gamma_{\perp}$.

If Stark shift is ignored ($\bar{\delta}=0$), Eqs. (10) and (12) are identical with the formulas derived by Savage and Walls [7]. It is seen that Stark shift brings about changes in both the drift coefficients and the correlations between the stochastic forces.

IV. THE STEADY-STATE DETERMINISTIC SOLUTION

Ignoring quantum fluctuations altogether in the first instance, we obtain the steady-state deterministic equations ($\alpha_s^{+}=\alpha_s^{*}$, $V_s^{+}=V_s^{*}$):

$$V_s=\frac{g|\alpha|^2N}{\gamma_{\perp}(1+i\Delta_d)\Pi}, \quad D_s=\frac{N}{\Pi} ,$$
$$Y=X\left[1+\left[\varphi+\frac{C\delta}{\Pi}+\frac{2CX\Delta_d}{\bar{X}}\right]^2\right] , \qquad (13)$$

where $C+(gN/2\kappa)(\gamma_{\parallel}/\gamma_{\perp})^{1/2}$ is the cavity cooperativity parameter, $n_0=(\gamma_{\parallel}\gamma_{\perp})^{1/2}/2g$ is the saturation intensity on resonance, and

$$\delta=\frac{\bar{\delta}}{\sqrt{2f}}, \quad X=\frac{\alpha_s\alpha_s^{+}}{n_0}, \quad Y=\frac{\epsilon\epsilon^{*}}{\kappa n_0} ,$$
$$\Delta_d=\Delta+f\delta X, \quad \Pi=1+\frac{X^2}{1+\Delta_d^2}, \quad \bar{X}=1+X^2+\Delta_d^2 .$$

Equation (13) clearly demonstrates that Stark shift not only changes the atomic detuning Δ into a dynamic detuning Δ_d, but also introduces a term $C\delta/\Pi$ to give the true cavity resonance. In other words, Stark shift provides another nonlinear mechanism which imposes on the mechanisms of saturation, dispersion and two-photon Kerr effect [18]. The change of transmission curve caused by inclusion of Stark shift is shown in Fig. 2. Owing to the competition of various mechanisms, multistable behavior of the transmission curve is displayed. The domain boundaries of the multistable behavior for different δ in the (φ, C) subspace are shown in Fig. 3.

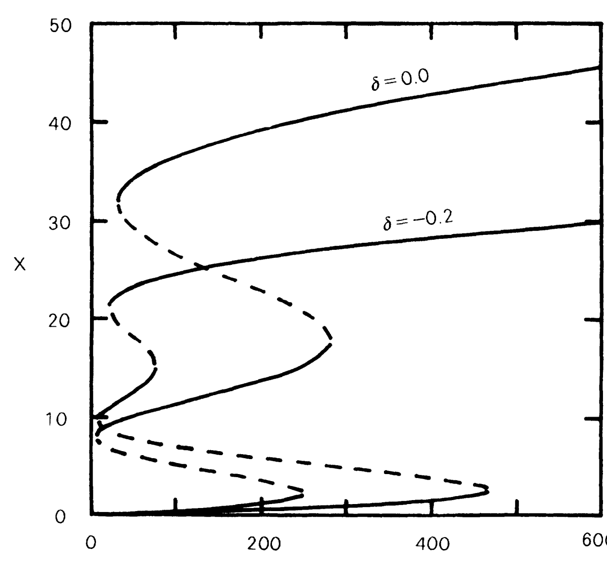

FIG. 2. Transmission curves. The dashed line indicates instability. Parameter values: $C=20$, $\Delta+16$, $\varphi=-16$, and $f=1$.

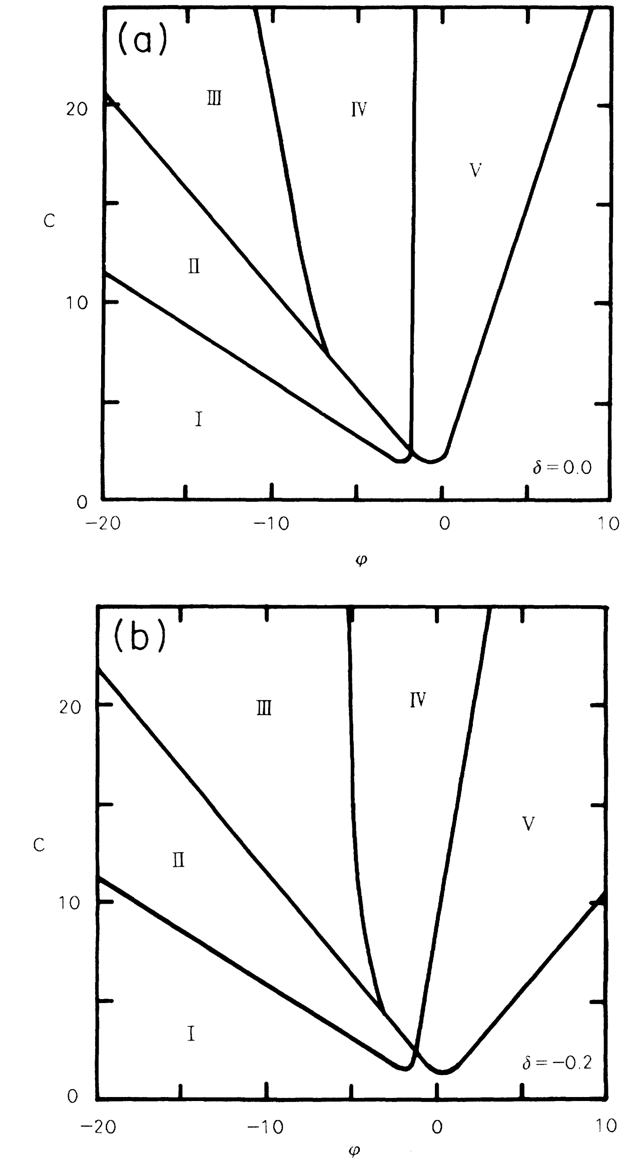

FIG. 3. Domain boundaries of the multistable behaviors in (C,φ) subspace for $\Delta=16$ and $f=1$. I, monostable; II, unsaturated bistable; III, tristable; IV, double bistable; V, saturated bistable. (a) $\delta=0.0$ and (b) $\delta=-0.2$.

V. ADIABATIC ELIMINATION OF ATOMIC VARIABLES

We eliminate the atomic variables under the assumption $\gamma_\perp,\gamma_\parallel \gg \kappa$, which allows us to set $\dot V=\dot D=0$. We find

$$\begin{aligned}\dot\alpha&=\epsilon-\kappa(1+i\varphi)\alpha-\frac{ig\bar\delta\alpha N}{2\Pi}-\frac{2ig^2\alpha^+\alpha^2N\Delta_d}{\gamma_\parallel\Pi(1+\Delta_d^2)}+F\ ,\\ \dot\alpha^+&=\epsilon^*-\kappa(1-i\varphi)\alpha^+\frac{+ig\bar\delta\alpha^+N}{2\Pi}+\frac{2ig^2\alpha\alpha^{+2}N\Delta_d}{\gamma_\perp\Pi(1+\Delta_d^2)}+F^+\ ,\end{aligned}\tag{14}$$

where

$$\begin{aligned}F=\Gamma_\alpha&-\frac{iX\Gamma_D}{2\Pi\alpha}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\\ &+\frac{g\alpha\Gamma_V}{\gamma_\perp(1+i\Delta_d)}\left[1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]\\ &-\frac{g\alpha\Gamma_{V^+}}{\gamma_\perp(1-i\Delta_d)}\left[1-\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]\ ,\\ F^+=\Gamma_{\alpha^+}&+\frac{iX\Gamma_D}{2\Pi\alpha^+}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\\ &+\frac{g\alpha^+\Gamma_{V^+}}{\gamma_\perp(1-i\Delta_d)}\left[1-\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]\\ &-\frac{g\alpha^+\Gamma_V}{\gamma_\perp(1+i\Delta_d)}\left[1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]\ .\end{aligned}\tag{15}$$

The correlations of the stochastic forces $F(t)$ and $F^+(t)$ are

$$\begin{aligned}&\langle F(t)F(t')\rangle=\kappa d\delta(t-t')\ ,\\ &\langle F^+(t)F^+(t')\rangle=\kappa d^*\delta(t-t')\ ,\\ &\langle F(t)F^+(t')\rangle=\kappa\Lambda\delta(t-t')\ ,\\ &\langle F^+(t)F(t')\rangle=\kappa\Lambda\delta(t-t')\ ,\end{aligned}\tag{16}$$

where

$$\begin{aligned}d=&\frac{2CX}{(1+i\Delta_d)\Pi}\left[1-\frac{if\delta X}{1+i\Delta_d}\right]\left[1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]+\frac{2iCX^2}{(1-i\Delta_d)\Pi^2}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\\ &-\frac{2CX}{1+\Delta_d^2}\left[(1-f)\left[1+\frac{1}{\Pi}\right]+2f\right]\left|1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right|^2\\ &-\frac{2CX}{1+\Delta_d^2}\left[(1-f)\left[1+\frac{1}{\Pi}\right]+2f\right]\left|1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right|^2\\ &+\mathrm{Re}\left\{\frac{8iCX^2f}{(1+i\Delta_d)^2\Pi^2}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\left[1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]\right\}\\ &\times\mathrm{Re}\left\{\frac{2CX^3f}{(1+i\Delta_d)^3\Pi}\left[1+\frac{iX}{\Pi}\left[\frac{X\Delta_d}{1+\Delta_d^2}+\frac{\delta}{2}\right]\right]^2\right\}\ ,\end{aligned}\tag{17a}$$

and

$$\Lambda = -\mathrm{Re}(d) \; . \tag{17b}$$

VI. LINEARIZED THEORY OF FLUCTUATIONS

The effect of fluctuations is estimated by linearizing (14) about the steady-state solutions α_s, V_s, and D_s of Eq. (13). Writing $\alpha = \alpha_s + \delta\alpha$, we obtain the following equation describing to first order the fluctuations:

$$\delta\dot{\alpha} = -\,\mathbf{A}\delta\alpha + \mathbf{D}^{1/2}\epsilon(t) \; , \tag{18}$$

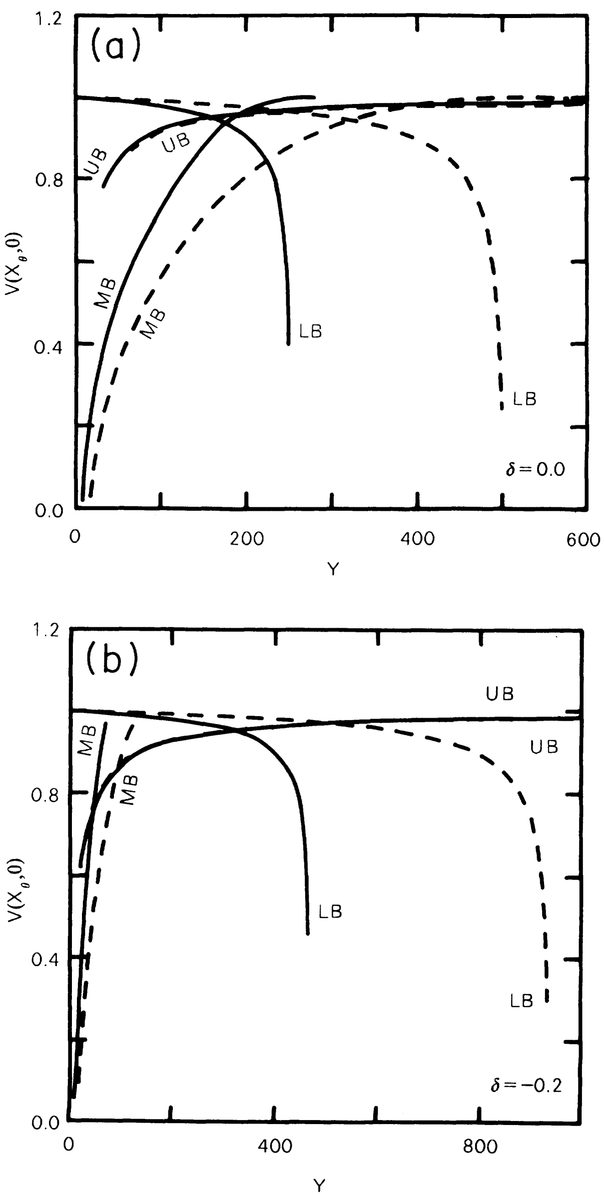

FIG. 4. $V(X_\theta,0)$ vs driving strength Y. $\Delta = 16$ (solid line), $\Delta = 32$ (dashed line). Other parameters as for Fig. 2. (a) $\delta = 0.0$ and (b) $\delta = -0.2$. LB, MB, and UB denote lowest branch, middle branch, and upmost branch, respectively.

where

$$\delta\alpha = \begin{bmatrix} \delta\alpha \\ \delta\alpha^{+} \end{bmatrix}, \quad \epsilon(t) = \begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \end{bmatrix},$$

$$\langle \epsilon_i(t)\epsilon_j(t') \rangle = \delta_{ij}\delta(t-t') \; , \tag{19}$$

$$\mathbf{A} = \kappa \begin{bmatrix} a & b \\ b^{*} & a^{*} \end{bmatrix}, \quad \mathbf{D} = \kappa \begin{bmatrix} d & \Lambda \\ \Lambda & d^{*} \end{bmatrix}$$

and

$$a = 1 + i\left[\varphi + \frac{C\delta}{\Pi} + \frac{2CX\Delta_d}{\bar{X}}\right] + b \; ,$$
$$b = \frac{2iCX}{\bar{X}^2}[(\Delta_d - \delta X)(1 + \Delta\Delta_d) + X(\delta f - X\Delta)] \; . \tag{20}$$

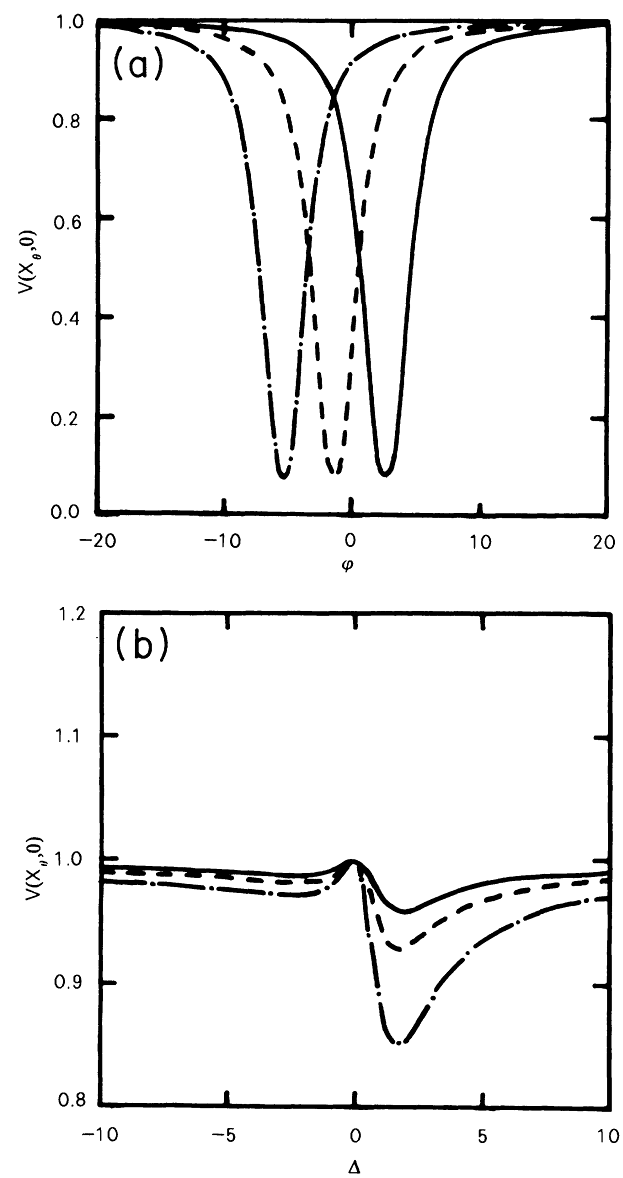

FIG. 5. (a) $V(X_\theta,0)$ vs φ and (b) $V(X_\theta,0)$ vs Δ. Solid, dashed, and dash-dotted line correspond to $\delta = -0.2$, 0.0, and 0.2, respectively. Other parameters as for Fig. 2 except $X = 0.3$.

Λ and d are given in (17).

This procedure of assuming small fluctuation about a steady-state deterministic solution is only consistent if the deterministic solution α_s is stable, i.e., if the eigenvalues of the matrix $\mathbf{A}$ have positive real parts. The stability criteria are:

$$\mathrm{Tr}(\mathbf{A})=2\geq 0 , \tag{21a}$$

$$\det(\mathbf{A})>0 . \tag{21b}$$

One can easily verify that

$$\det(\mathbf{A})=\frac{dY}{dX} . \tag{22}$$

Criterion (21a) is always satisfied, so the stability is only determined by the slope of transmission curve, as shown in Fig. 2.

VII. SQUEEZING SPECTRUM IN THE OUTPUT FIELD

Of particular interest to us is the field transmitted through the cavity port. Using the techniques developed by Collett and Gardiner [15] and Collett and Walls [16], we calculate the squeezing spectrum for the output field from the linearized drift and diffusion coefficients.

The squeezing spectrum is defined as

$$V(X_\theta,\omega)=\int_{-\infty}^{+\infty} e^{i\omega\tau}\langle X_\theta(t+\tau),X_\theta(t)\rangle d\tau , \tag{23}$$

where $X_\theta(t)=a_{\mathrm{out}}(t)e^{-i\theta}+a_{\mathrm{out}}^\dagger(t)e^{i\theta}$ is the quadrature phase amplitude of the output field in which we seek squeezing and the angle θ specifies the particular quadrature of interest.

The squeezing spectrum can be calculated from the co-

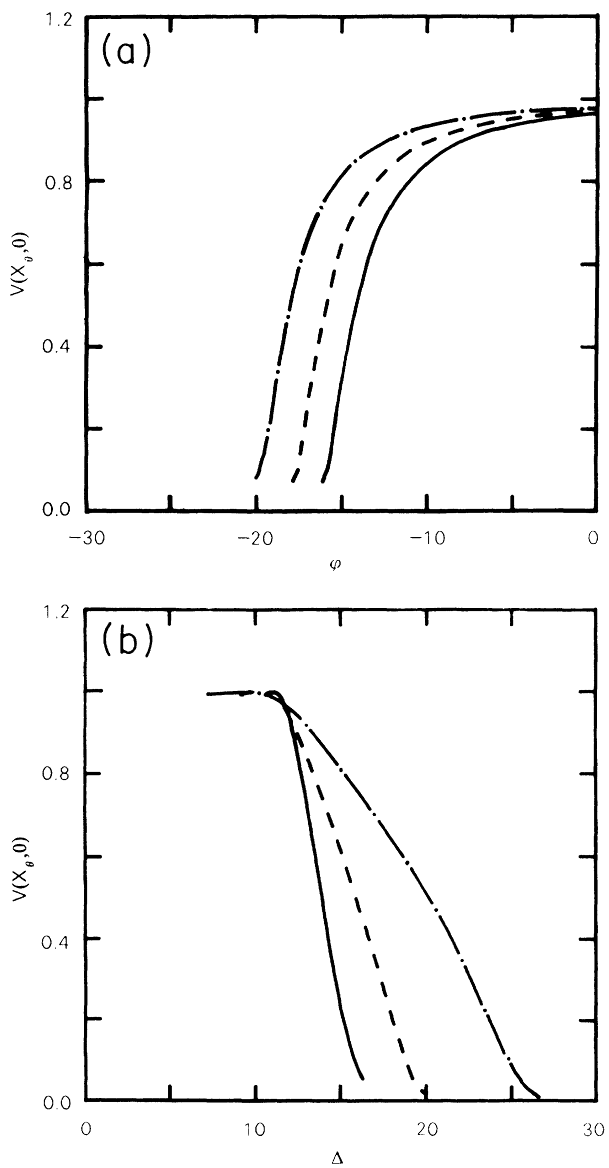

FIG. 6. (a) $V(X_\theta,0)$ vs φ and (b) $V(X_\theta,0)$ vs Δ. Parameters as for Fig. 5 except $X=10$.

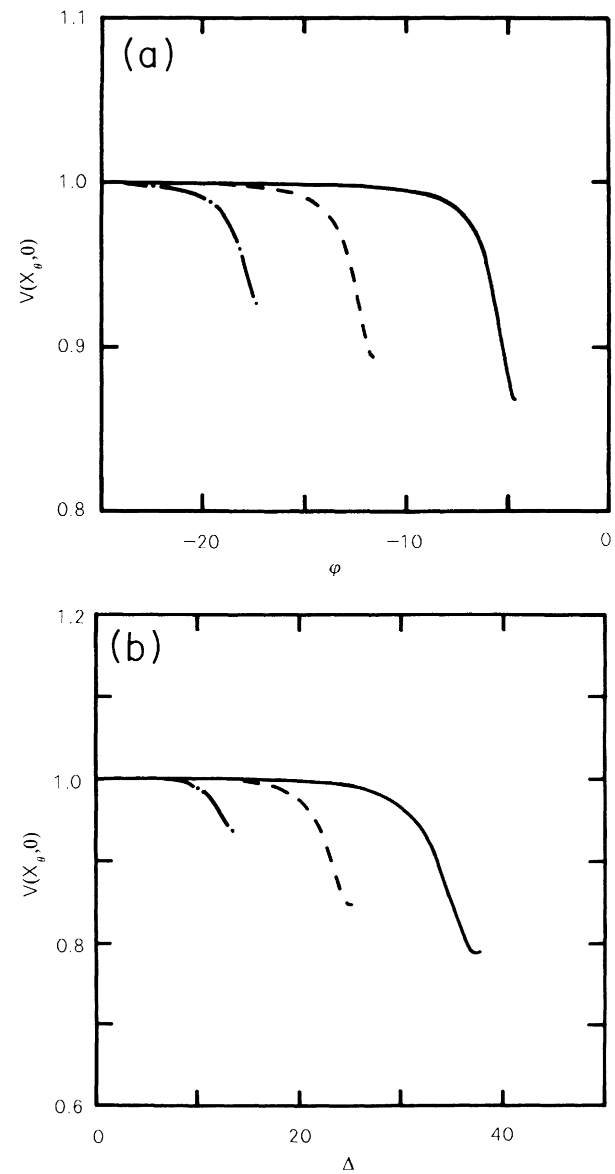

FIG. 7. (a) $V(X_\theta,0)$ vs φ and (b) $V(X_\theta,0)$ vs Δ. Parameters as for Fig. 5 except $X=50$.

variance spectrum $\mathbf{S}(\omega)$ in the stationary state:

$$V(X_\theta,\omega)=1+2\kappa[S_{12}(\omega)+S_{21}(\omega)+e^{-2i\theta}S_{11}(\omega)+e^{2i\theta}S_{22}(\omega)] \ . \quad (24)$$

Choosing θ to maximize the squeezing for given ω, we find

$$V(X_\theta,\omega)=1+2\kappa[S_{12}(\omega)+S_{21}(\omega)-2|S_{11}(\omega)|] \ , \quad (25)$$

where S_{ij} is the element of the covariance spectrum matrix $\mathbf{S}(\omega)$ which can be calculated from drift and diffusion matrix, as follows:

$$\mathbf{S}(\omega)=(\mathbf{A}+i\omega\mathbf{I})^{-1}\mathbf{D}(\mathbf{A}^T+i\omega\mathbf{I})^{-1} \ . \quad (26)$$

It is a straightforward numerical task to calculate the squeezing using the above equations. Figures 4(a) and 4(b) are the zero-frequency component of squeezing $V(X_\theta,0)$ plotted against the strength Y of the driving field, for the lowest branch (LB), middle branch (MB), and the upmost branch (UB) in the tristability. In contrast with a medium with "Ξ-configuration" atoms, squeezing is also found in the upmost branch, due to the absence of spontaneous emission. Good squeezing is found at the turning point of the lowest branch, while perfect squeezing is found at the turning point of the middle branch. But the regimes where good squeezing can be found in both cases are rather narrow, only limited at the turning points. Optical Stark shift changes both the turning points and the squeezing depth significantly.

Figures 5(a) and 5(b) exhibit the squeezing versus cavity detuning φ and atomic detuning Δ in the limit of low intensities. We see from Fig. 5(a) that the apparent effect of Stark shift is simply to translate the curve. This can be explained by noting that in this limit $\Delta_d \approx \Delta$ and the only significant change in the drift and diffusion coefficient is a change from φ to $\varphi + C\delta$. On the contrary, the curve of $V(X_\theta,0)$ vs Δ is by no means a simple translation with δ.

Figures 6(a), 6(b), 7(a), and 7(b) are plots of $V(X_\theta,0)$ vs φ and Δ in the middle- and high-intensity regimes, which show more complicated variations.

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