Collective collapses and revivals in spontaneous emission of a partially inverted system of two-level atoms: Analytical solution

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(Received 10 May 1991; revised manuscript received 28 July 1992)

Applying a perturbation method, constructed in terms of SU(2) group representations, we study spontaneous emission of a system of N identical two-level atoms immersed in a single-mode ideal cavity with s atoms initially inverted. For the cases $s \geq 3$, characterized by nonequidistant spectra of the eigenvalues of the Hamiltonian, the phenomenon of collective collapses and revivals manifests itself. For $s=3$ the approximate solution is compared with the exact one and the limit of good applicability of our method is established.

PACS number(s): 32.80.−t, 42.50.Fx

I. INTRODUCTION

Since the fundamental paper of Dicke [1] on collective spontaneous emission, a considerable amount of increasing interest has been paid to an assembly of N two-level atoms located within a distance much smaller than the radiation wavelength. Such a system has commonly been referred to as the Dicke model. The recent experimental achievements [2,3] in realization of the Jaynes-Cummings model encourage further intensive investigations, among others of the Dicke model. The condition of a high-Q cavity requires the Rabi frequency to be much greater than the cavity-damping factor which is inversely proportional to the time that radiation is stored in the cavity. One of the ways of increasing the Rabi frequency is to increase the number of atoms interacting with the cavity field [4]. The Dicke model with a part of the atoms inverted is promising in this respect. Namely, for a given N the Rabi frequencies increase as the number of the initially excited atoms s decreases [5].

In many papers [6–9], the problem of the interactions of a system of N two-level atoms with a single field mode has been dealt with numerically. Tavis and Cummings [6] have found the eigenstates for such systems. Walls and Barakat [7] have shown that, if but a small number s of atoms from the system of N atoms is initially excited, the spectrum of the eigenvalues of the Hamiltonian is almost equidistant. Butler and Drummond [8] have studied the effect of the collectivity of the system on the magnitude of squeezing compared to the Jaynes-Cummings model. In turn, Li *et al.* [9] have discussed in the same way the problem of higher-order squeezing.

Bonifacio and Preparata [10] and Kumar and Mehta [11] have proposed approximate solutions to the problem of spontaneous emission from partially inverted systems in terms of elliptic functions. Cummings and Dorri [12] and Seke [13] have solved exactly the problem of spontaneous emission from a system of N two-level atoms in a cavity with only one atom initially inverted. Cummings and Dorri [12] have considered nonsymmetrical initial excitation of the atomic system while Seke [13] has performed calculations for symmetrical initial excitation. The effect of the symmetry properties of the initial state and of the spatial distribution of atoms on the collective emission rate has been discussed by Crubellier *et al.* [14] and Bužek [15].

Senitzky [16] analytically and Abate and Haken [17] numerically have revealed the effect of cooperative inhibition of radiation. According to them, total deexcitation of the initially inverted system is impossible. This problem was studied later by many authors, such as by Stroud *et al.* [18] and Cummings [19] among others.

With respect to the experimental realization of photon-number states [2,3,20], the problem of the radiation effects caused by the presence of such fields has already been examined as well. Seke and Rattay [21] studied numerically induced emission in the presence of a Fock-state field for different cavity dampings. They have shown, among others, that the initially inverted system of N two-level atoms coupled to the Fock field will exhibit quasiperiodic collapses and revivals. The influence of the photon-number state field on the radiatively decaying

atomic system has also been discussed by Hassan, Bullough, and Puri [22].

Here, we are interested in spontaneous emission by a system of N two-level atoms placed in a lossless cavity. In fact, the approach used in this paper describes very well the process when only a part s [$s \leq (N+4)/4$] of the N atoms radiates spontaneously in the presence of $N-s$ unexcited atoms. In our previous paper [5] we restricted the time evolution of the system to the purely harmonic terms; i.e., we neglected the anharmonic corrections to the eigenfrequencies for $s \geq 3$. Such an approximation is valid even for long times, albeit for s very small in comparison with N. The anharmonic corrections to the eigenfrequencies are responsible for the spread in the Rabi frequencies resulting in modulation of the oscillations. Here, it is our aim to present the time evolution of the system with these anharmonic corrections. A phenomenon of collective collapses and revivals, manifesting itself significantly for $s < N$ is found. Moreover, we present the exact solution for $s=3$ and compare it with the approximate one.

II. THEORY

We discuss a system of N identical two-level atoms coupled to a single-mode quantized radiation field in an ideal cavity and assume a small-sample approximation. Thus the linear dimensions of the atomic system are much smaller than the wavelength λ of the radiation and all the atoms are treated as occupying equivalent mode positions, i.e., we deal with a field invariant throughout the sample. However, the wave functions of the atoms are not supposed to overlap. The cooperative nature of the spontaneous emission from the system is then due to indirect atom-atom coupling via the field only. The above satisfies most of the requirements of the Dicke model. Such a model is realized in experiments with Rydberg atoms the transition wavelength of which is of the order of millimeters. It is then possible to prepare a sample of many atoms ("dilute gas") with negligible interatomic interactions in a volume small compared to λ [23].

The electric-dipole Hamiltonian in the rotating-wave approximation for the system in question is ($\hbar=1$)

$$H=H_{\text{free}}+V_{\text{int}} ,$$
$$H_{\text{free}}=\omega a^{\dagger}a+\omega_f \sum_{j=1}^{N} S_j^3 , \tag{1}$$
$$V_{\text{int}}=gV , \quad V=\sum_{j=1}^{N} (a^{\dagger}S_j^- + aS_j^+) ,$$

where $a^{\dagger}$ (a) is the photon creation (annihilation) operator and S_j^-, S_j^+, and S_j^3 are pseudospin lowering, raising, and inversion operators of the jth atom, respectively. ω_f denotes the frequency of the field mode while ω is the atomic transition frequency. In what follows, we assume exact resonance and choose the scale in such a way that $\omega=\omega_f=1$. With respect to the small-sample approximation the coupling coefficient g is the same for all the atoms.

The excitation number operator $\mathcal{N}$

$$\mathcal{N}=a^{\dagger}a+\sum_{j=1}^{N} S_j^3+\frac{N}{2} \tag{2}$$

commutes with the Hamiltonian (1), i.e., is an integral of motion. We take its eigenstates $|s,m\rangle$ as the basis vectors:

$$|s,m\rangle=|s-m\rangle_a \otimes |m\rangle_f , \quad 0 \leq m \leq s , \tag{3}$$

where $|m\rangle_f$ denotes the Fock state of the field and $|s-m\rangle_a$ is the normalized symmetric Dicke state of the atomic subsystem with $s-m$ atoms excited:

$$|k\rangle_a=\left[\frac{N!}{k!(N-k)!}\right]^{-1/2} \sum |+\rangle_{j_1}\otimes|+\rangle_{j_2}\otimes\cdots\otimes|+\rangle_{j_k}\otimes \prod_{j\neq j_1,\ldots,j_k} |-\rangle_j ,$$
$$\text{for } k=s-m \ ; \ \text{also } {}_a\langle n|k\rangle_a=\delta_{nk} . \tag{4}$$

The summation is over all possible manners of choosing k indistinguishable atoms from the group of N atoms.

Consequently, the time evolution of the system is restricted to the $(s+1)$-dimensional subspace spanned by the basis vectors (3). In particular, the time evolution of the system in such a subspace is related with the initial condition $|s,0\rangle$ when s atoms are initially inverted and no photon is present. In this subspace there acts the representation of the group SU(2), the generators of which are determined as follows:

$$X^-|s,m\rangle=\sqrt{(m+1)(s-m)}\,|s,m+1\rangle ,$$
$$X^+|s,m\rangle=\sqrt{m(s-m+1)}\,|s,m-1\rangle ,$$
$$S^z|s,m\rangle=(s/2-m)|s,m\rangle , \tag{5}$$
$$S^X=(X^++X^-)/2 , \quad S^Y=(X^+-X^-)/2i .$$

They satisfy the commutations rules of su(2) Lie algebra.

Let us briefly recall the main steps of our method [5]. The free part H_{free} of the Hamiltonian (1) leads to an unimportant phase factor in the transition amplitudes. In turn, the interactions part V may be presented as follows:

$$V=V^++V^- ,$$
$$V^-=a^{\dagger}\sum_{j=1}^{N} S_j^- , \quad V^+=a\sum_{j=1}^{N} S_j^+ . \tag{6}$$

The nonvanishing matrix elements of these operators in the basis (3) are

$$\langle s,m+1|V^-|s,m\rangle=\sqrt{(m+1)(s-m)(N-s+m+1)}=\langle s,m|V^+|s,m+1\rangle,\quad 0\le m\le s\ . \tag{7}$$

We construct a perturbation theory by introducing a parameter ϵ

$$\epsilon=(N-s/2+\tfrac{1}{2})^{-1}\ . \tag{8}$$

In principle, the perturbation method we use requires not only ϵ to be small but $s\ll N$ as well. However, as it is also the case in other problems solved by perturbation methods our approximate formulas work satisfactorily well even for not small values of ϵ and s/N, at least for not very large s.

Expanding the matrix elements (7) in a power series in ϵ one gets

$$\langle s,m+1|V^-|s,m\rangle=\left[\frac{(m+1)(s-m)}{\epsilon}\right]^{1/2}\left[1+\frac{\epsilon}{2}\left[\tfrac{1}{2}-\frac{s}{2}+m\right]-\frac{\epsilon^2}{8}\left[\tfrac{1}{2}-\frac{s}{2}+m\right]^2+\cdots\right]\ . \tag{9}$$

Comparing the appropriate matrix elements obtainable from Eqs. (5) and those given by (9) it is easily seen that the operator V may be expressed in terms of the generators of the group SU(2) as follows:

$$V=\frac{1}{\sqrt{\epsilon}}\sum_{n=0}^{\infty}\epsilon^n V_n\ ,$$
$$V_n=V_n^+ +V_n^-=K_n[X^-(\tfrac{1}{2}-S^z)^n+(\tfrac{1}{2}-S^z)^nX^+]\ , \tag{10}$$

where the coefficients K_n read as

$$K_n=\begin{cases}1 & \text{for } n=0\\ \tfrac{1}{2} & \text{for } n=1\\ (-1)^{n+1}\dfrac{(2n-3)!!}{2^n n!} & \text{for } n\ge 2\ .\end{cases} \tag{11}$$

In particular,

$$V_0=X^-+X^+=2S^x\ . \tag{12}$$

So, the zeroth-order interaction Hamiltonian V_0 coincides with the generator $2S^X$ of the $(s+1)$-dimensional irreducible representation of the SU(2) group. The spectrum of the eigenvalues of the generators of the SU(2) group representation is well known. For the Hamiltonian (12) we obtain

$$V_0|s,p\rangle^{(0)}=\lambda_{p,s}^{(0)}|s,p\rangle^{(0)}\ , \tag{13}$$

where its eigenvalues $\lambda_{p,s}^{(0)}$ are

$$\lambda_{p,s}^{(0)}=s-2p\ ,\quad 0\le p\le s\ . \tag{14}$$

Their spectrum is equidistant, irrespective of the value of s/N.

In turn, the eigenvectors $|s,p\rangle^{(0)}$ read [5]

$$|s,p\rangle^{(0)}=(\alpha_{0p}^s,\alpha_{1p}^s,\ldots,\alpha_{sp}^s)^T\ ,$$
$${}^{(0)}\langle s,k|s,p\rangle^{(0)}=\delta_{kp}\ , \tag{15}$$

where

$$\alpha_{mp}^s=\langle s,m|s,p\rangle^{(0)}=\left[\frac{m!p!}{2^s(s-m)!(s-p)!}\right]^{1/2}\times\sum_{j=0}^{\min(m,p)}\frac{(-2)^j(s-j)!}{j!(m-j)!(p-j)!} \tag{16}$$

are the matrix elements of the transition between the vector basis (3) and the basis of the eigenvectors of the Hamiltonian V_0.

Basing on the expansion (10) of the Hamiltonian V one can construct a perturbation theory with small parameter ϵ and with the operator V_0 treated as the zeroth-order unperturbed term.

Let us denote by $\Lambda_{p,s}$ the eigenvalues, by $|\Psi_{s,p}\rangle$ the eigenvectors of the interaction Hamiltonian V,

$$V|\Psi_{s,p}\rangle=\Lambda_{p,s}|\Psi_{s,p}\rangle\ , \tag{17}$$

and by A_{mp} the matrix elements of the transition from the vector basis $|s,m\rangle$ to the vector basis $|\Psi_{s,p}\rangle$,

$$A_{mp}=\langle s,m|\Psi_{s,p}\rangle\ . \tag{18}$$

Expanding the eigenvalues $\Lambda_{p,s}$ and the eigenvectors $|\Psi_{s,p}\rangle$ in a power series in ϵ, by standard perturbation methods in the first-order approximation for the eigenvectors we find that

$$A_{mp}=\alpha_{mp}^s+\tfrac{1}{8}\epsilon[(s-2p+1)\sqrt{p(s-p+1)}\alpha_{mp-1}^s-(s-2p-1)\sqrt{(p+1)(s-p)}\alpha_{mp+1}^s]\ , \tag{19}$$

while within an accuracy of ϵ^2 for the eigenvalues one obtains

$$\Lambda_{p,s}^{(2)}=\frac{\lambda_{p,s}^{(0)}}{\sqrt{\epsilon}}\left\{1-\frac{\epsilon^2}{16}\left[5p(s-p)-\frac{(s-1)(s-2)}{2}\right]\right\},\quad 0\le p\le s\ . \tag{20}$$

Because of the choice of the form of the parameter ϵ, the first-order correction to the eigenvalues vanishes. In fact, all odd-order corrections to the eigenvalues are equal to zero. For simplicity, we will omit the second subscript s following Λ.

For $s=1$, p may take the value 0 and 1. For these values of p the terms in square brackets are zero. In turn, for $s=2$, p may be 0, 1, and 2. Again, for $p=0$ and 2 the terms in square brackets vanish while for $p=1$ the zeroth-order eigenvalue $\lambda_p^{(0)}$ is equal to zero. In consequence, the spectrum of the eigenvalues is equidistant for $s=1$ and 2 and the time evolution of the systems with such s is truly periodic. For $s\ge 3$ the spectra of the ei-

genvalues become unequidistant due to the nonlinear dependence of Eq. (20) on p. However, if s is very small in comparison with N, these spectra may be approximately treated as equidistant. Agreement of the eigenvalues given by Eq. (20) with those numerically obtained by Walls and Barakat [7] is amazingly good even if the condition $s \ll N$ is not satisfied [5].

In general, the problem in question is exactly solvable for $s \leq 8$. The eigenvalues predicted by our method are commensurate quantities, irrespective of s. The eigenvalues given by the exact solutions are commensurate for $s=1$ and 2. For the remaining s ($s \geq 3$) the exact eigenvalues would be noncommensurate [4]. Hence, in principle, the perturbation method we propose can yield even the exact results for $s=1$ and 2 but is unable to give them for $s \geq 3$. In fact, for the cases $s=1$ and 2 one gets the exact solutions [24] in the zeroth- and second-order approximations for the eigenvectors, respectively [5].

III. TIME EVOLUTION

The expectation value of the collective operator $S^3=\sum_{j=1}^{N} S_j^3$, representing the inversion of the atomic energy for the whole system of N atoms, in the eigenstate $|s,m\rangle$ is

$$\langle s,m|S^3|s,m\rangle = -N/2+s-m \ . \tag{21}$$

In turn, from Eqs. (5) we have

$$\langle s,m|S^z|s,m\rangle = s/2-m \ . \tag{22}$$

Comparing both these quantities we see that the latter represents inversion of the atomic energy for the group of s atoms. In what follows, we will calculate the quantity (22).

The time evolution of the expectation value of the atomic inversion is obtained with the standard formula

$$E_{\rm at}^s(t)=\langle s,0|e^{iHt}S^z e^{-iHt}|s,0\rangle \ , \tag{23}$$

where $|s,0\rangle$ is the initial state of the system with s atoms excited and with the number of photons equal to zero.

Making use of the transformation (19) and its orthogonality we find quite generally that

$$\begin{aligned} E_{\rm at}^s(t) &= \sum_{p,q=0}^{s}\sum_{m=0}^{s}\left[\frac{s}{2}-m\right]A_{0p}A_{0q}A_{mp}A_{mq}e^{igt(\Lambda_q-\Lambda_p)} \\ &= \frac{s}{2}-\sum_{p=0}^{s}A_{0p}^2\sum_{m=0}^{s}mA_{mp}^2-2\sum_{p=0}^{s}\sum_{q=p+1}^{s}A_{0p}A_{0q}\cos[gt(\Lambda_q-\Lambda_p)]\sum_{m=0}^{s}mA_{mp}A_{mq} \ . \end{aligned} \tag{24}$$

The expectation value of the photon number is readily obtained from Eqs. (2) and (22) with the help of Eqs. (23) and (24) at the eigenvalue s for the excitation number operator.

From Eq. (24), after some lengthy algebra, in the first-order approximation for the eigenvectors and in the second-order approximation for the eigenvalues we arrive at

$$\begin{aligned} {}^{12}E_{\rm at}^s(t)=\frac{1}{2^s}\sum_{p=0}^{s-1}\frac{s!}{p!(s-p-1)!}\{(1-\tfrac{1}{8}\epsilon[(s-2p-2)^2-2p-1]) \\ \times\cos[gt(\Lambda_{p+1}^{(2)}-\Lambda_p^{(2)})]-\tfrac{1}{4}\epsilon p\cos[gt(\Lambda_{p+1}^{(2)}-\Lambda_{p-1}^{(2)})]\}+\frac{\epsilon s(s-1)}{16} \ . \end{aligned} \tag{25}$$

The above expression contains the factor which does not vanish under the operation of time averaging:

$$(\overline{{}^{12}E_{\rm at}^s})^t=\frac{s(s-1)}{8(2N-s+1)} \ , \tag{26}$$

which simply means that some portion of the energy is trapped in the atomic subsystem. For fixed s this quasistationary value of the energy decreases as N increases. This situation is the opposite of that for nonsymmetrical initial excitation [1,12] when the effect of energy trapping grows with increasing N. In turn, for a given N this energy diminishes with decreasing s.

On substitution of the eigenvalues (20) into Eq. (25) one gets

$${}^{12}E_{\rm at}^s(t)=\frac{1}{2^s}\sum_{p=0}^{s-1}\frac{s!}{p!(s-p-1)!}\{(1-\tfrac{1}{8}\epsilon[(s-2p-2)^2-2p-1])\cos(2\Omega_{p1}t)-\tfrac{1}{4}\epsilon p\cos(4\Omega_{p2}t)\}+\frac{\epsilon s(s-1)}{16} \ , \tag{27}$$

where the Rabi frequencies Ω_{p1} and Ω_{p2} read

$$\Omega_{p1}=\Omega_0\{1+\tfrac{1}{16}3\epsilon^2[5p(p-s+1)+(s-1)(s-2)]\}\ ,$$
$$\Omega_{p2}=\Omega_0\{1+\tfrac{1}{32}3\epsilon^2[10p(p-s)+2s^2-s+4]\}\ , \tag{28}$$

and

$$\Omega_0=g\left[N-\frac{s-1}{2}\right]^{1/2}. \tag{29}$$

At $t=0$ the system is prepared in a definite state. Hence all terms in Eq. (27) are correlated. As time goes on, these terms begin to oscillate with different frequencies and become dephased. Later on, they rephase again leading to modulation of the oscillations (except in the cases $s=1$ and $s=2$), in particular to their collapses and revivals. This collective mechanism of modulation of the oscillations in spontaneous emission of the Dicke model has its source in the unequidistancy of the eigenvalues spectra.

Seke and Rattay [21] considering emission of the initially inverted atomic system stimulated by a Fock field, have revealed collective collapses and revivals of the Rabi oscillations. Here we find the possibility of such effects for spontaneous emission from partially excited atomic systems. In the case of spontaneous emission of the totally inverted systems this phenomenon does not manifest itself so distinctly [4].

Putting $\epsilon=0$ in the frequencies (28), from Eq. (27) we get our earlier result [5] obtained in the first-order approximation for the eigenvectors and in the zeroth-order approximation for the eigenvalues:

$$^{10}E^s_{\rm at}(t)=\frac{s}{2}\left[\cos[2gt(N-s/2+\tfrac{1}{2})^{1/2}]+\frac{s-1}{4(2N-s+1)}\{1-\cos[4gt(N-s/2+\tfrac{1}{2})^{1/2}]\}\right]. \tag{30}$$

From Eq. (30) it is more easily seen that the first-order correction to the oscillation amplitudes is of the order $x=\epsilon(s-1)/8$. Therefore, for a given fraction $\rho=s/N$ this correction decreases as s and N decrease (contrary to ϵ which then becomes greater). As for the eigenvalues, the correction is of the order x^2. In consequence, for a given ρ Eq. (27) approximates reality better for small s and N than for large s and N. Moreover, since the corrections to the eigenvalues are of the order x^2 our method for given s and N describes the eigenvalues with a smaller error than the oscillation amplitudes although, as mentioned, it gives them as commensurate quantities. On the other hand, however, even subtle differences in the eigenvalues may lead in long times to a phase shift of the oscillations predicted by Eq. (27), relative to the exact ones.

For $s=1$ the second term in Eq. (30) vanishes while the first term represents the exact solution for spontaneous emission of the system with one atom initially inverted. For $s=2$ the exact solution [5] is obtained in the second approximation for the eigenvectors. For $s\geq 3$ the above formula is still able to describe the real situation for relatively short times if N is not significantly greater than s, or for long times if s is extremely small compared to N. Then, in fact, even the pure zeroth-order approximation [the first term in Eq. (30)] is sufficient, at least for realistic times.

IV. EXACT SOLUTION FOR $s=3$

The case $s=3$ is the first in the hierarchy of those characterized by the unequidistant eigenvalue spectra. The eigenvectors $|s,m\rangle$ for $s=3$ are $|3,3\rangle$, $|3,2\rangle$, $|3,1\rangle$, and $|3,0\rangle$. The total wave function of the system in the interaction picture reads as

$$|\Phi(t)\rangle=C_0(t)|3,3\rangle+C_1(t)|3,2\rangle+C_2(t)|3,1\rangle+C_3(t)|3,0\rangle\ , \tag{31}$$

where $C_k(t)$ are the probability amplitudes.

The time-dependent Schrödinger equation leads to the four coupled equations of motion

$$\begin{aligned}
\dot C_0&=-ig\sqrt{3N}\,C_1\ ,\\
\dot C_1&=-ig\sqrt{3N}\,C_0-2ig\sqrt{N-1}\,C_2\ ,\\
\dot C_2&=-2ig\sqrt{N-1}\,C_1-ig\sqrt{3(N-2)}\,C_3\ ,\\
\dot C_3&=-ig\sqrt{3(N-2)}\,C_2\ ,
\end{aligned} \tag{32}$$

with the initial condition $C_3(t=0)=1$.

The solutions for the probability amplitudes are

$$\begin{aligned}
C_0(t)&=-\frac{6ig^3\sqrt{N(N-1)(N-2)}}{\Omega_1^2-\Omega_2^2}\left[\frac{\sin\Omega_1 t}{\Omega_1}-\frac{\sin\Omega_2 t}{\Omega_2}\right],\\
C_1(t)&=\frac{2g^2\sqrt{3(N-1)(N-2)}}{\Omega_1^2-\Omega_2^2}(\cos\Omega_1 t-\cos\Omega_2 t)\ ,\\
C_2(t)&=-\frac{ig\sqrt{3(N-2)}}{\Omega_1^2-\Omega_2^2}\\
&\quad\times\left[\frac{\Omega_1^2-3Ng^2}{\Omega_1}\sin\Omega_1 t-\frac{\Omega_2^2-3Ng^2}{\Omega_2}\sin\Omega_2 t\right],\\
C_3(t)&=\frac{1}{\Omega_1^2-\Omega_2^2}\{[\Omega_1^2-g^2(7N-4)]\cos\Omega_1 t\\
&\quad-[\Omega_2^2-g^2(7N-4)]\cos\Omega_2 t\}\ ,
\end{aligned} \tag{33}$$

where the Rabi frequencies Ω_1 and Ω_2 have the form

$$\Omega_1 = g[5N-5+(16N^2-32N+25)^{1/2}]^{1/2} ,$$
$$\Omega_2 = g[5N-5-(16N^2-32N+25)^{1/2}]^{1/2} . \quad (34)$$

They are really noncommensurate quantities contrary to those given by Eq. (28). A sketch of the solution (33) has already been proposed by Senitzky [21] though not solved to the end.

The energy inversion referred to the group of three atoms is

$$^{ex}E_{at}(t) = \tfrac{3}{2} - 3|C_0(t)|^2 - 2C_1(t)^2 - |C_2(t)|^2 . \quad (35)$$

The final analytical expression for the energy inversion obtained on substitution of the probability amplitudes (33) would be rather complicated, so we leave Eq. (35) in the above general form.

V. DISCUSSION

Let us now compare graphically the time evolution of the systems with $s=3$ described by the exact solution (35) and by the approximate one (27). In all figures the dashed line corresponds to the exact solution while the solid line corresponds to Eq. (27).

The graph in Fig. 1 represents the case $s=N=3$. As it could be suspected, this case is beyond the scope of our approximate result. However, as is evident from Fig. 2, representing the case $s=3$ and $N=6$, agreement between these two curves is already satisfactory. The fundamental difference is in the minimal values of the energy inversion. The approximate solution predicts them too small. Moreover, a noticeable difference is also seen during the time interval corresponding to collapse. The exact solution then gives a slightly higher upper limit of the oscillations. The oscillations are of irregular nature during the collapses, especially the oscillations described by the approximate Eq. (27). For clarity in the upper graph in Fig. 2 (also of the remaining figures) we simply cut out the local maxima and minima and hence the envelope looks strange in this time interval. The form of the oscillations in the collapse region and in its vicinity is presented in the lower graph in Fig. 2. The oscillations described by Eqs. (27) and (35) are in phase before and after the first collapse region. During collapse they differ not only in their amplitudes but become slightly phase shifted as well. In longer times, both these oscillations may become more and more shifted in phase generally, not only in the collapse intervals.

Figure 3 gives the envelope of the oscillations for $s=3$ and $N=8$. In this case one is already justified in claiming good agreement between these two solutions, and thus good applicability of our approach.

In the case of $s=3$ and $N=6$ the quantity x takes the value $\frac{1}{20}$, while for $s=3$ and $N=8$ x amounts to $\frac{1}{28}$. Accepting such values of x as the limits of satisfactory and good applicability of our approach for $s>3$ as well, the following relations between s and N arise: $N=3(s-1)$ for the former and $N=4(s-1)$ for the latter value of x. The same relations are obtained from a comparison of the exact result [5,24] and that (30) for $s=2$.

In the upper graph in Fig. 4 the envelope of the oscillations of the atomic inversion is plotted for $s=3$ and $N=9$, while, as previously, the lower graph represents their time behavior during collapse. Analysis of Figs.

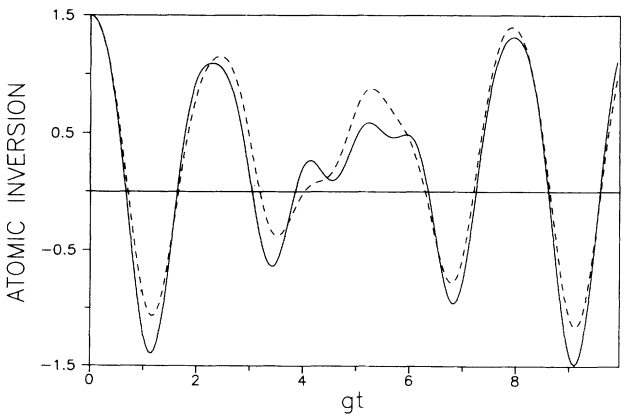

FIG. 1. Time evolution of the atomic inversion for $s=N=3$. Dashed line corresponds to the exact solution, solid line to Eq. (27). In the following figures the same notation is preserved.

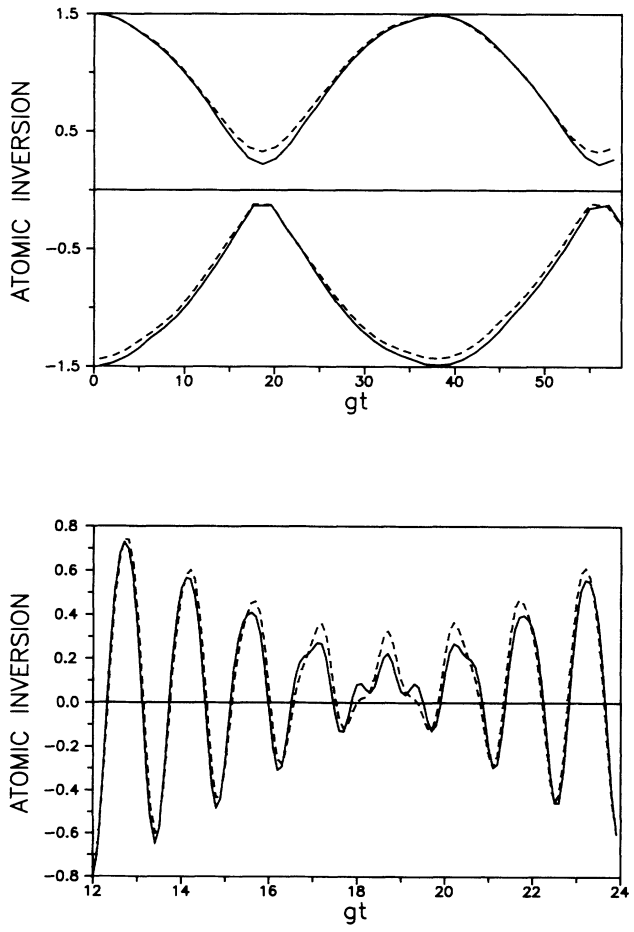

FIG. 2. The upper panel shows the envelope of the atomic inversion for $s=3$ and $N=6$. The lower panel shows the oscillations during the first collapse.

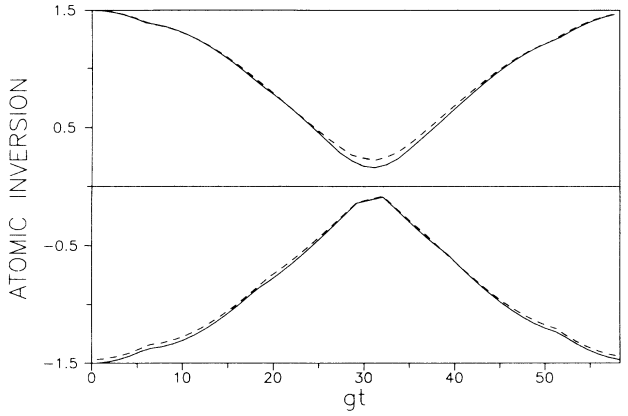

FIG. 3. Envelope of the atomic inversion for $s=3$ and $N=8$.

2–4 shows that an increasing number of atoms N (at a given s) gives longer revival and collapse periods.

In Fig. 5 the envelopes of the oscillations are plotted for $s=7$ and $N=15$, 18, and 24. The numbers $N=18$ and 24, in accord with the above statement, fulfill the condition of satisfactory and good applicability of the approximate result. In fact, comparison of these curves with the exact computer ones confirms this to some extent. For $N=18$ one observes in long times the above-mentioned phase shift of the oscillations. As could be expected, an increasing number of initially excited atoms (of the Rabi frequencies) renders the shapes of the envelopes more interesting.

To conclude, our perturbation approach seems astonishingly accurate in the description of the eigenvalue spectra [5] and sufficiently good to describe the time evolution of the Dicke model in an ideal cavity with some part of the atoms initially inverted for fractions ρ up to

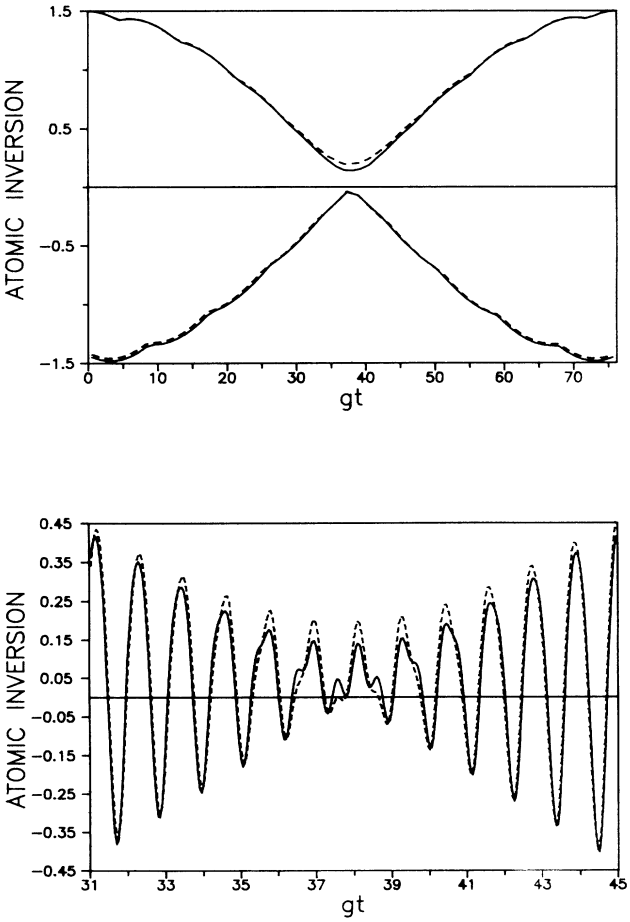

FIG. 4. The same as in Fig. 2 but for $s=3$ and $N=9$.

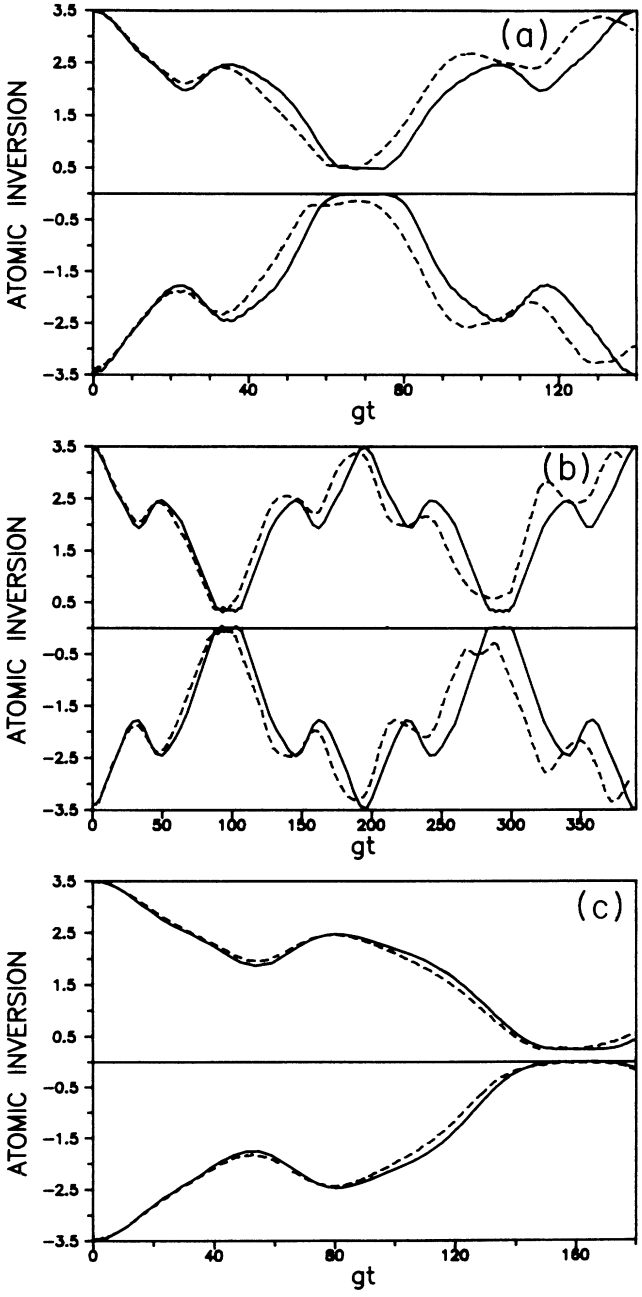

FIG. 5. Envelopes of the atomic inversion for $s=7$: (a) $N=15$, (b) $N=18$, (c) $N=24$.

the value $(N+4)/4N$. The accuracy of Eq. (27) becomes better as ρ decreases. But, certainly, we considered it more interesting to present the graphs for the limiting cases of applicability of our approach.

ACKNOWLEDGMENTS

This work was carried out within the program 2 0150 9101 of the Polish Committee for Scientific Research.

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