Photon polarization in radiative recombination of bare ions with low-energy free electrons

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The polarization of photons emitted from radiative recombination of bare ions with free electrons is discussed in the nonrelativistic dipole approximation in the low-relative-energy limit. Within this approach an analytical expression is derived for the degree of polarization of photons from radiative recombination of ions with fixed-energy electrons for an arbitrary (n,l) state. For a relative-velocity distribution of the electron beam, characterized by the longitudinal $kT_{\parallel}$ and transverse $kT_{\perp}$ electron-beam temperatures, we found also analytical forms for the polarization rates. For a flattened distribution $(kT_{\parallel} \ll kT_{\perp})$ the polarization rates for different (n,l) states peak at a direction, perpendicular to the ion-beam axis (in the moving frame), where the photons are completely linearly polarized for the s states. The results are discussed in the context of experiments aimed to observe the photons from radiative recombination in the electron coolers of storage rings.

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I. INTRODUCTION

In radiative recombination a free electron is captured into a bound state of an ion with the emission of a photon. The recombination into an (n,l) state of a bare ion with atomic number Z, where n and l are the main and orbital quantum numbers, respectively, may be written as follows:

$$A^{Z+}+e^{-}\rightarrow A^{(Z-1)+}(n,l)+E_{\gamma} ,$$

where the photon energy E_{γ} equals an initial electron kinetic energy E plus an electron binding energy in a final state E_{nl}, i.e., $E_{\gamma}=E+E_{nl}$. The time inverse of this process was studied theoretically already in the 1920s [1–4], when a quantum-mechanical description of photoionization was developed. The cross sections for both the radiative-recombination and photoionization processes are related via the principle of detailed balance [5]. A general result for the radiative-recombination cross section in a nonrelativistic dipole approximation was given in 1930 by Stobbe [6] for arbitrary (n,l) states. Later on, Bethe and Salpeter [7] derived an approximate formula for the radiative-recombination cross section for a fixed main quantum number n. It should be noticed that both the angular distributions and polarization of photons for arbitrary (n,l) states have not been quantitatively discussed in the papers mentioned above, with the exception of Stobbe's paper [6] where a short discussion of the photon polarization for recombination into the K shell was given.

Radiative recombination was recognized many years ago as an important energy loss process in plasma physics [8] and astrophysics [9]. In recent years, heavy-ion storage rings [10–13] equipped with electron coolers have offered new possibilities to study state-selective radiative recombination for bare and few-electron ions, e.g., via high-resolution spectroscopy of emitted photons [14,15] and laser-induced radiative recombination [16]. Recently, the first experimental results from measurements of total recombination rates for spontaneous recombination [17–22] and laser-induced state-selective recombination [23,24] have become available. Experiments for detecting the radiation from the spontaneous recombination process are in preparation. For the purpose of such experiments, it is worthwhile to know not only the radiative-recombination cross sections and rate coefficients, but also the angular distribution and polarization of the emitted radiation.

Numerical calculations of the angular distributions and polarization of photons from radiative recombination were published recently by Scofield [25]. In that work, the differential cross sections and polarization of photons in recombination of selected highly charged ions using electrons with a few kilo-electron-volts of energy were calculated. It was done in a relativistic treatment including higher multipoles and further on it was demonstrated there that a nonrelativistic dipole approximation describes very well the recombination process in a sub-electron-volt electron energy region, which is of interest for the present paper. A more detailed discussion of the applicability of a nonrelativistic dipole approximation to the description of recombination between up to highest-Z bare ions and low-energy electrons can be found elsewhere [26].

In a storage ring, the energy spread of cooling electrons in the ion frame is typically 0.2 eV or less, which is much smaller than the electron binding energy in low-(n,l) states, being on the order of some kilo-electron-volts for medium- and high-Z ions. Guided by this fact we have shown recently [26] that, within the so-called low-energy approximation $(E \ll E_{nl})$, most of the relations for the radiative-recombination process (recombination cross sections and rate coefficients) can be given in closed analytical forms for an arbitrary (n,l) state and an arbitrary electron-beam-velocity distribution, characterized by the longitudinal $(kT_{\parallel})$ and transverse $(kT_{\perp})$ beam temperatures. For typical cooling conditions of an ion beam [27], we found in Ref. [26] that the low-energy approximation is valid up to quite high-n states, namely, for $n \leq Z$. In this context, our previous paper [26] can be regarded as an introduction to the low-energy approximation for radiative recombination. Within this approximation we derive in the present paper analytical expressions for the degree of polarization of photons from radiative recombination.

Here, as well as in the previous paper [26], we discuss radiative recombination between *bare* ions and *low-energy* free electrons, however, we want to point out that the results can be applied to any system of structureless Coulomb-interacting particles, for instance, such as in the formation of antihydrogen ($e^+\bar{p}$) [16] or protonium ($p\bar{p}$) [28] via radiative recombination. Other references related to these interesting aspects can be found elsewhere [26].

The paper is organized as follows. In Sec. II we present the differential cross sections and the polarization of photons in radiative recombination for a fixed electron energy in the low-energy limit. In Sec. III we give the main results for the polarization averaged over the electron-beam-velocity distribution, which we call polarization rates. The polarization of photons observed in the laboratory system, in possible future experiments, is discussed in Sec. IV. Conclusions are made in Sec. V.

II. PHOTON ANGULAR DISTRIBUTION AND POLARIZATION IN THE LOW-ENERGY LIMIT

The differential radiative-recombination cross section into an (n,l) state, $[d\sigma_{nl}/d\Omega](\mathbf{v})$, can be expressed in a nonrelativistic dipole approximation in terms of the recombination cross section $\sigma_{nl}(E)$ and an anisotropy parameter $\beta_{nl}(E)$, describing the angular distribution of photoelectrons emitted in photoionization process, as follows [26,29,30]:

$$\frac{d\sigma_{nl}}{d\Omega}(\mathbf{v})=\frac{\sigma_{nl}(E)}{4\pi}\left[1+\frac{\beta_{nl}(E)}{2}[3(\mathbf{e}\cdot\mathbf{u}_p)^2-1]\right]. \tag{1}$$

Here E is the kinetic energy of an electron with velocity $\mathbf{v}$ in the moving frame; $\mathbf{e}$ and $\mathbf{u}_p$ are unit vectors along photon electric vector and electron momentum $\mathbf{p}$, respectively. In the low-energy limit ($E \ll E_{nl}$) the anisotropy parameter β_{nl} becomes *energy independent* and can be expressed as follows [26]:

$$\beta_{nl}=\frac{(l+2)(l+1)c_{l+1}^2(n,l)+l(l-1)c_{l-1}^2(n,l)-6l(l+1)c_{l+1}(n,l)c_{l-1}(n,l)}{(2l+1)[(l+1)c_{l+1}^2(n,l)+lc_{l-1}^2(n,l)]} \tag{2}$$

where the energy-independent reduced dipole matrix elements $c_{l\pm1}(n,l)$ are defined in Ref. [26], where also a further discussion of the anisotropy β_{nl} parameter can be found. In that article we also show that, within the nonrelativistic dipole approximation, β_{nl} can be determined in the *low-energy* regime quite accurately even for highest-Z *bare* ions.

Equation (1) shows that the angular distribution of photons depends only on an angle ϕ (see Fig. 1) between $\mathbf{e}$ and $\mathbf{p}$ (or $\mathbf{u}_p$), i.e., $\cos\phi=\mathbf{e}\cdot\mathbf{u}_p$. Taking into account that $\cos\phi=\sin\vartheta\cos\chi$, where χ is the angle between $\mathbf{e}$ and the plane defined by $\mathbf{p}$ and $\mathbf{k}$ as shown in Fig. 1, the differential radiative-recombination cross section can be written in the following form:

$$\frac{d\sigma_{nl}}{d\Omega}(\mathbf{v})=\frac{\sigma_{nl}(E)}{4\pi}\left[1+\frac{\beta_{nl}}{2}(3\sin^2\vartheta\cos^2\chi-1)\right]. \tag{3}$$

A flux of photons observed at an angle ϑ with a fixed polarization (i.e., $\mathbf{e}$ vector orientation) as described by an angle χ, is proportional to $[d\sigma_{nl}/d\Omega](\mathbf{v},\chi)$. Consequently, one can calculate the polarization of emitted radiation in terms of the differential recombination cross section given in Eq. (3). The degree of polarization $P_{nl}(\vartheta)$ of radiation from recombination into an (n,l) state observed at an angle ϑ is defined as follows:

$$P_{nl}(\vartheta)=\frac{I_{\parallel}(\vartheta)-I_{\perp}(\vartheta)}{I_{\parallel}(\vartheta)+I_{\perp}(\vartheta)}, \tag{4}$$

where $I_{\parallel,\perp}(\vartheta)$ means a photon flux at angle ϑ with an electric vector *in* and *perpendicular* to the $(\mathbf{p},\mathbf{k})$ plane, respectively. Since $I_{\parallel,\perp}(\vartheta)\sim d\sigma_{nl}(\vartheta,\chi)/d\Omega$ of Eq. (3) setting $\chi=0°$ and 90°, respectively, one obtains

$$P_{nl}(\vartheta)=\frac{3\beta_{nl}\sin^2\vartheta}{2(2-\beta_{nl})+3\beta_{nl}\sin^2\vartheta}. \tag{5}$$

In Fig. 2 the dependence of the photon polarization

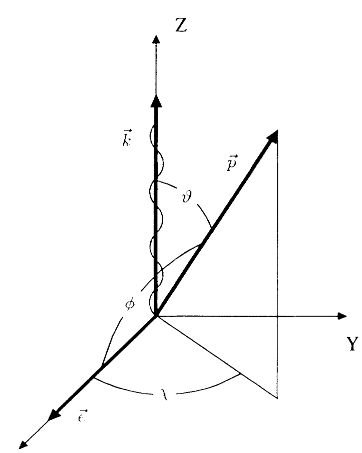

FIG. 1. Radiative-recombination process in the xyz coordinate system. The $\mathbf{p}$ vector denotes the electron momentum, the photon $\mathbf{k}$ vector points along the z axis, and a unit vector $\mathbf{e}$ along the photon electric vector is chosen along the x axis. In this coordinate system the following relation between ϕ, ϑ, and χ angles exists: $\cos\phi=\sin\vartheta\cos\chi$.

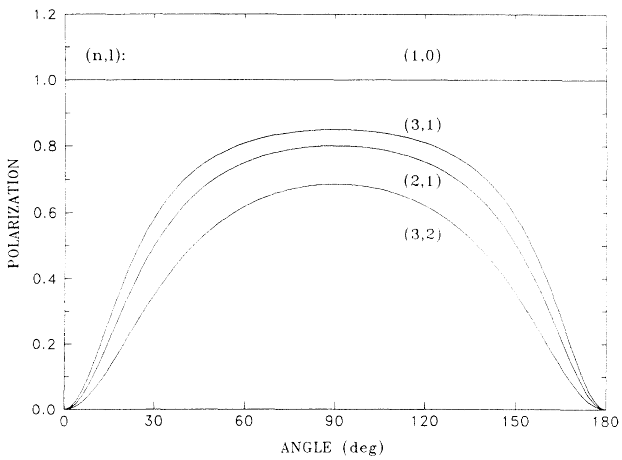

FIG. 2. Polarization $P_{nl}(\vartheta)$ of photons from radiative recombination into (n,l) states with $n \leq 3$, vs an angle ϑ between the photon wave vector $\mathbf{k}$ and electron momentum $\mathbf{p}$. Note that for s states the photons are completely linearly polarized at any angle ϑ.

$P_{nl}(\vartheta)$ on the angle ϑ is shown for selected low-(n,l) states. As the most interesting feature we point out that for recombination into s states, where $\beta_{n0}=2$ [26] the photons are completely linearly polarized with an electric vector *in* the $(\mathbf{p},\mathbf{k})$ plane. Such a result was given for the first time by Stobbe [6] for recombination into the K shell. With Eq. (5) we find that for higher-(n,l) states the photons are no longer completely linearly polarized and the polarization is strongest for $\vartheta=90°$. The maximum degree of polarization $P_{nl}=P_{nl}(\vartheta=90°)$ can be expressed in terms of β_{nl} as follows:

$$P_{nl}=\frac{3\beta_{nl}}{4+\beta_{nl}} \ . \tag{6}$$

In Fig. 3 the dependence of P_{nl} on the orbital quantum

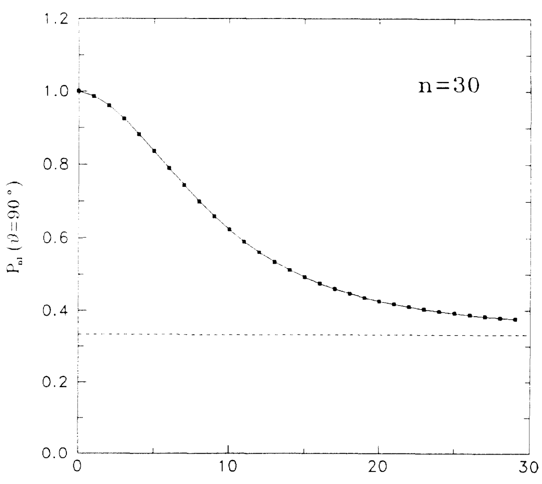

FIG. 3. Dependence of the maximum degree of polarization $P_{nl}(\vartheta=90°)$ on the orbital quantum number l for $n=30$. The polarization reaches the asymptotic value $\frac{1}{3}$ (dashed line) in a limit of high-l values.

number l is shown for $n=30$. We see that P_{nl} decreases from its maximum value $P_{n0}=1$ for s states to the lowest value $\frac{1}{3}$ in the limit of large l, where $\beta_{nl}\approx(l+2)/(2l+1)$ approaches $\frac{1}{2}$ [26].

III. POLARIZATION RATES

As it was discussed in the preceding section, the polarization of photons from radiative recombination (for a fixed momentum of the electron) can be expressed in terms of the differential recombination cross section $d\sigma_{nl}(\vartheta,\chi)/d\Omega$ given in Eq. (3). In the experimental condition of an electron beam merged with an ion beam, e.g., in the electron cooler of a storage ring, the relative electron-beam velocity $\mathbf{v}=(v_\parallel,v_\perp)$ has generally asymmetric, Maxwellian distribution [27,31]:

$$f(\mathbf{v})=\left[\frac{m}{2\pi}\right]^{3/2}\frac{1}{kT_\perp(kT_\parallel)^{1/2}}\exp\left[-\frac{mv_\perp^2}{2kT_\perp}-\frac{mv_\parallel^2}{2kT_\parallel}\right], \tag{7}$$

where m denotes the electron mass, k the Boltzmann constant, and $kT_\parallel$ and $kT_\perp$ are the effective longitudinal and transverse electron-beam temperatures (in eV), respectively. Under such conditions one has to introduce the radiative-recombination rate coefficient, being the radiative-recombination cross section folded with the electron-beam-velocity distribution. In a previous paper [26] we derived the analytical expressions for the double-differential $[d^2\alpha_{nl}/dE_\gamma d\Omega](E_\gamma,\theta)$, angle-differential $[d\alpha_{nl}/d\Omega](\theta)$, and total (α_{nl}) recombination rate coefficients in the low-energy approximation.

Since the flux of photons from radiative recombination in the electron cooler is proportional to the corresponding recombination rate coefficient, we can introduce so-called *polarization rates* using in the definition of the polarization [Eq. (4)] the rate coefficients $[d^2\alpha_{nl}^{\parallel,\perp}/dE_\gamma d\Omega](E_\gamma,\theta)$ or $[d^2\alpha_{nl}^{\parallel,\perp}/d\Omega](\theta)$ with a fixed orientation ($\parallel$ or $\perp$) of a photon electric vector with respect to the $(\mathbf{p},\mathbf{k})$ plane, similarly to the way it was defined for the photon fluxes $I_\parallel$ and $I_\perp$ in Eq. (4). We note that in terms of the κ angle in Fig. 4, this corresponds to set $\kappa=90°$ and 0° for the ($\parallel$) and ($\perp$) cases, respectively. The double-differential rate coefficients with a fixed orientation of the electric vector can be expressed as follows (see also Refs. [32,26]):

$$\frac{d^2\alpha_{nl}^{\parallel,\perp}}{dE_\gamma d\Omega}(E_\gamma,\theta)=\frac{2(E_\gamma-E_{nl})}{m^2}\int\frac{d\sigma_{nl}^{\parallel,\perp}(\mathbf{v})}{d\Omega_r}f(\mathbf{v})d\Omega_r \ , \tag{8}$$

where the angular integration should be performed over all possible orientations of the electron velocity $\mathbf{v}$ with the differential recombination cross section $d\sigma_{nl}^{\parallel,\perp}(\mathbf{v})/d\Omega$ defined as follows:

$$\frac{d\sigma_{nl}^{\parallel,\perp}}{d\Omega}(\mathbf{v})=\frac{\sigma_{nl}(E)}{4\pi}\left[1+\frac{\beta_{nl}}{2}(3\sin^2\vartheta\cos^2\chi_{\parallel,\perp}-1)\right]. \tag{9}$$

Here $\chi_{\parallel,\perp}$ denotes the value of the angle χ for which the

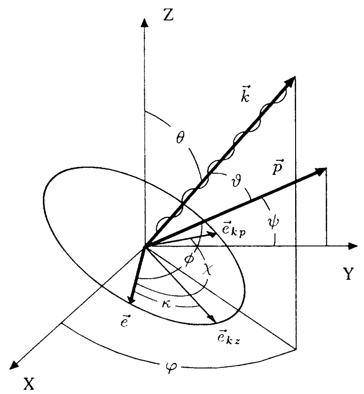

FIG. 4. The coordinate system used to calculate the rate coefficients when an orientation of the photon electric vector (shown by unit vector **e**) is fixed relative to the (**k**,**z**) plane. The ion-beam axis is set along the z axis and the electron momentum **p** is fixed in the yz plane. A photon wave vector **k** points into direction (θ,φ) and the angle between **k** and **p** vectors is denoted by ϑ. The vectors $\mathbf{e}_{kp}$ and $\mathbf{e}_{kz}$ are defined in the text.

angle κ is fixed, namely, $\chi_{\parallel}=\chi(\kappa=90^{\circ})$ and $\chi_{\perp}=\chi(\kappa=0^{\circ})$. To perform the angular integration of Eq. (8) we express $\cos^2\chi_{\parallel,\perp}$ in Eq. (9) by other angles, which define the directions of **p** and **k** vectors as shown in Fig. 4. This leads to the following expressions (see Appendix):

$$\cos^2\chi_{\parallel}=\frac{\cos^2\psi\cos^2\varphi}{\sin^2\vartheta}\ , \tag{10}$$

$$\cos^2\chi_{\perp}=\frac{(\sin\psi\sin\theta-\cos\psi\cos\theta\sin\varphi)^2}{\sin^2\vartheta}\ . \tag{11}$$

By inserting these in Eq. (9), the differential recombination cross section can be expressed by

$$\frac{d\sigma_{nl}^{\parallel,\perp}}{d\Omega}(\mathbf{v})=\frac{\sigma_{nl}(E)}{4\pi}\left[1-\frac{\beta_{nl}}{2}+\tfrac{3}{2}\beta_{nl}F_{\parallel,\perp}(\theta,\psi,\varphi)\right]\ , \tag{12}$$

where the $F_{\parallel,\perp}(\theta,\psi,\varphi)$ functions have the forms

$$F_{\parallel}(\theta,\psi,\varphi)=\cos^2\psi\cos^2\varphi\ , \tag{13}$$

$$F_{\perp}(\theta,\psi,\varphi)=(\sin\psi\sin\theta-\cos\psi\cos\theta\sin\varphi)^2\ . \tag{14}$$

With Eqs. (12–14) the integral in Eq. (8) is of the same type discussed previously [see Eq. (23) in Ref. [26], and Ref. [31]] calculating the double-differential rate coefficient $[d^2\alpha_{nl}/dE_\gamma d\Omega](E_\gamma,\theta)$ for a case when the polarization of radiation is not detected. After a rather lengthy, but straightforward angular integration in Eq. (8) (see Fig. 4 and also Ref. [31]) the double-differential rate coefficients $[d^2\alpha_{nl}^{\parallel,\perp}/dE_\gamma d\Omega](E_\gamma,\theta)$ can be expressed as follows:

$$\frac{d^2\alpha_{nl}^{\parallel}}{dE_\gamma d\Omega}(E_\gamma,\theta)=\frac{E_{nl}\sigma(n,l)}{\pi(2\pi m)^{1/2}}\frac{e^{-(E_\gamma-E_{nl})/kT_\perp}}{kT_\perp(kT_\parallel)^{1/2}}\times\left[\left[1-\frac{\beta_{nl}}{2}\right]f_0(a)+\tfrac{3}{4}\beta_{nl}[f_1(a)-2f_2(a)\sin^2\theta]\right] \tag{15}$$

and

$$\frac{d^2\alpha_{nl}^{\perp}}{dE_\gamma d\Omega}(E_\gamma,\theta)=\frac{E_{nl}\sigma(n,l)}{\pi(2\pi m)^{1/2}}\frac{e^{-(E_\gamma-E_{nl})/kT_\perp}}{kT_\perp(kT_\parallel)^{1/2}}\times\left[\left[1-\frac{\beta_{nl}}{2}\right]f_0(a)+\tfrac{3}{4}\beta_{nl}\right]\ , \tag{16}$$

where both the reduced radiative-recombination cross sections $\sigma(n,l)$ and the functions $f_i(a)$, with $a=(E_\gamma-E_{nl})(kT_\parallel-kT_\perp)/kT_\parallel kT_\perp$, are defined in our previous paper [26].

With Eqs. (15) and (16) and using the general definition of polarization [Eq. (4)], one can calculate the polarization rates $\pi_{nl}(E_\gamma,\theta)$ for recombination into (n,l) states for a fixed photon energy E_γ as follows:

$$\pi_{nl}(E_\gamma,\theta)=\frac{3\beta_{nl}f_2(a)\sin^2\theta}{2(2-\beta_{nl})f_0(a)+3\beta_{nl}[f_1(a)-f_2(a)\sin^2\theta]}\ . \tag{17}$$

One can notice here that due to the $\sin^2\theta$ factor in Eq. (17) the photons cannot be polarized in the ion-beam direction ($\theta=0^{\circ}$). For a Maxwellian electron-beam-velocity distribution ($kT_\parallel=kT_\perp$) or for a photon energy $E_\gamma\approx E_{nl}$ (i.e., when $a\approx 0$), keeping in mind that $f_0(0)=1$, $f_1(0)=\frac{2}{3}$, and $f_2(0)=0$ [26], it turns out from Eq. (17) that $\pi_{nl}(E_\gamma,\theta)=0$. This means that no polarization is expected for an isotropic Maxwellian electron velocity distribution or for a photon energy near the edge $E_\gamma=E_{nl}$. On the other hand, for a flattened electron-beam-velocity distribution ($kT_\parallel \ll kT_\perp$), when additionally $E_\gamma-E_{nl}\gg kT_\parallel$ ($a\gg 1$) which implies that $f_0(a)/f_1(a)\approx 1$ and $f_2(a)/f_1(a)\approx\frac{1}{2}$ [31,26], the photons are polarized. Under these conditions, the polarization rate of Eq. (17) becomes independent of the photon energy E_γ, and can be written as

$$\pi_{nl}^{f}(E_\gamma,\theta)\approx\frac{3\beta_{nl}\sin^2\theta}{2(4+\beta_{nl})-3\beta_{nl}\sin^2\theta}\ . \tag{18}$$

We will discuss the angular dependence of the polarization rates $\pi_{nl}^{f}(E_\gamma,\theta)$ for different (n,l) states later on after we show that Eq. (18) corresponds exactly to the photon-energy-integrated polarization rate $\pi_{nl}(\theta)$ derived from the angle-differential $[d\alpha_{nl}^{\parallel,\perp}/d\Omega](\theta)$ rate coefficients, as obtained from $[d^2\alpha_{nl}^{\parallel,\perp}/dE_\gamma d\Omega](E_\gamma,\theta)$ [Eqs. (15) and (16)] after integration over the photon energy E_γ. In the same way we calculated the angle-differential recombination rate coefficient $[d\alpha_{nl}/d\Omega](\theta)$ for a polarization-insensitive experiment [see Eq. (30) in Ref. [26]] we write the expressions for the longitudinal and transverse angle-differential rate coefficients as

$$\frac{d\alpha_{nl}^{\parallel,\perp}}{d\Omega}(\theta)=\int_{E_{nl}}^{\infty}dE_\gamma\,\frac{d^2\alpha_{nl}^{\parallel,\perp}}{dE_\gamma d\Omega}(E_\gamma,\theta)\ . \tag{19}$$

With the help of Eqs. (15) and (16), using the definition of the functions $f_i(a)$ from Ref. [26], these integrals can be expressed as follows:

$$\frac{d\alpha_{nl}^{\parallel,\perp}}{d\Omega}(\theta)=\frac{E_{nl}\sigma(n,l)}{\pi(2\pi m)^{1/2}}\frac{(t+1)^{1/2}}{(kT_\perp)^{1/2}}\times\int_0^1 dx\,[w_1^{\parallel,\perp}(\theta)-w_2^{\parallel,\perp}(\theta)x^2]\times\int_0^\infty dw\,\exp[-(1+tx^2)w]\ , \tag{20}$$

where $t=(kT_\perp-kT_\parallel)/kT_\parallel$ denotes the electron-beam asymmetry parameter and the functions $w_i^{\parallel,\perp}(\theta)$ are defined as follows:

$$w_1^\perp(\theta)=1+\tfrac{1}{4}\beta_{nl}-\tfrac{3}{4}\beta_{nl}\sin^2\theta\ , \tag{21}$$

$$w_2^\perp(\theta)=\tfrac{3}{4}\beta_{nl}(1-3\sin^2\theta)\ , \tag{22}$$

$$w_1^\parallel(\theta)=1+\tfrac{1}{4}\beta_{nl}\ , \tag{23}$$

$$w_2^\parallel(\theta)=\tfrac{3}{4}\beta_{nl}\ . \tag{24}$$

The double integration in Eq. (20) can be performed in a closed analytical form (see Ref. [26]), so, finally, we have

$$\frac{d\alpha_{nl}^{\parallel,\perp}}{d\Omega}(\theta)=\frac{E_{nl}\sigma(n,l)}{\pi(2\pi m)^{1/2}}\frac{1}{(kT_\perp)^{1/2}}(t+1)^{1/2}t^{-3/2}\times\{[w_1^{\parallel,\perp}(\theta)t+w_2^{\parallel,\perp}(\theta)]\arctan(\sqrt{t}\,)-w_2^{\parallel,\perp}(\theta)\sqrt{t}\,\}\ . \tag{25}$$

Using this in the definition of the polarization [Eq. (4)], the photon-energy-integrated polarization rate $\pi_{nl}(\theta)$ reads as follows:

$$\pi_{nl}(\theta)=\frac{[(w_1^\parallel-w_1^\perp)t+(w_2^\parallel-w_2^\perp)]\arctan(\sqrt{t}\,)-(w_2^\parallel-w_2^\perp)\sqrt{t}}{[(w_1^\parallel+w_1^\perp)t+(w_2^\parallel+w_2^\perp)]\arctan(\sqrt{t}\,)-(w_2^\parallel+w_2^\perp)\sqrt{t}}\ . \tag{26}$$

This is a general result for the total polarization rate for recombination into an (n,l) state and for an arbitrary electron-beam asymmetry, characterized by the parameter $t=(kT_\perp-kT_\parallel)/kT_\parallel$. As before, for a Maxwellian distribution $(t=0)$ the photons are not polarized $[\pi_{nl}^M(\theta)\equiv 0]$. For a flattened electron-beam-velocity distribution $(t\to\infty)$ we find that $\pi_{nl}(\theta)\to\pi_{nl}^f(\theta)=(w_1^\parallel-w_1^\perp)/(w_1^\parallel+w_1^\perp)$ which with Eqs. (21) and (23) gives the result found already above [Eq. (18)]:

$$\pi_{nl}^f(\theta)=\frac{3\beta_{nl}\sin^2\theta}{2(4+\beta_{nl})-3\beta_{nl}\sin^2\theta}\ . \tag{27}$$

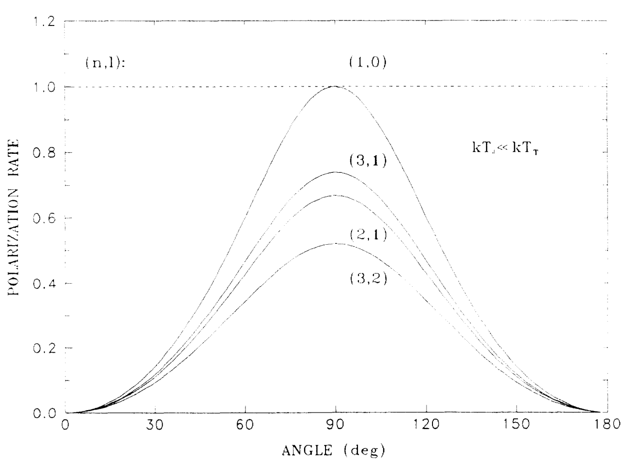

FIG. 5. Dependence of the polarization rate $\pi_{nl}^f(\theta)$ [Eq. (27)] on the observation angle θ (in a moving frame) with respect to a beam direction for selected $n\leq 3$ states for a flattened electron-beam-velocity distribution $(kT_\parallel \ll kT_\perp)$. Note that for $\theta=90°$ the photons from recombination into s states are completely linearly polarized.

These results are plotted in Fig. 5 versus the photon observation angle θ in an ionic frame for different low-(n,l) states $(n\leq 3)$ for the flattened electron-beam-velocity distribution. The polarization rates peak at $\theta=90°$ and the degree of polarization reaches unity for recombination into s states. It means that photons are completely linearly polarized in a plane perpendicular to the ion-beam direction. A dependence of the polarization rate for s states on the electron-beam asymmetry parameter t, given in Eq. (26), is shown in Fig. 6. From this figure one sees that for a typical electron-beam temperature $kT_\perp=0.2$ eV and $kT_\parallel=0.002$ eV corresponding to $t\approx 100$, the degree of polarization is about 0.8 for s states. For recombination into final states with $l\neq 0$ the polarization rates no longer reach unity and the maximum de-

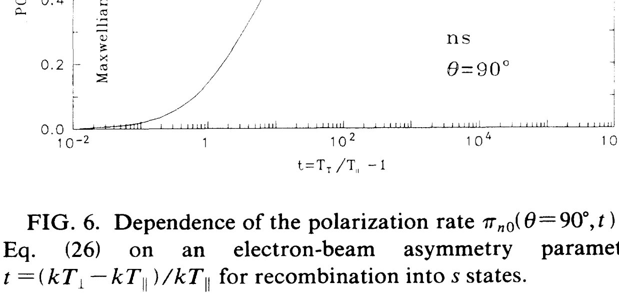

FIG. 6. Dependence of the polarization rate $\pi_{n0}(\theta=90°,t)$ of Eq. (26) on an electron-beam asymmetry parameter $t=(kT_\perp-kT_\parallel)/kT_\parallel$ for recombination into s states.

TABLE I. Anisotropy parameters β_{nl} of Eq. (2), the maximum polarization $P_{nl}=P_{nl}(\vartheta=90°)$ of Eq. (6), and the maximum polarization rates $\pi^f_{nl}=\pi^f_{nl}(\theta=90°)$ of Eq. (28) for a flattened electron-beam-velocity distribution ($kT_\parallel \ll kT_\perp$) as derived in the low-energy approximation for different (n,l) states with $n \leq 5$.

(n,l)	β_{nl}	P_{nl}	π^f_{nl}
1 0	2.000	1.000	1.000
2 0	2.000	1.000	1.000
2 1	1.455	0.800	0.667
3 0	2.000	1.000	1.000
3 1	1.579	0.849	0.734
3 2	1.182	0.684	0.520
4 0	2.000	1.000	1.000
4 1	1.662	0.881	0.787
4 2	1.290	0.732	0.577
4 3	1.027	0.613	0.442
5 0	2.000	1.000	1.000
5 1	1.719	0.902	0.821
5 2	1.378	0.769	0.624
5 3	1.110	0.652	0.483
5 4	0.929	0.565	0.394

gree of polarization π^f_{nl} as observed for $\theta=90°$ is

$$\pi^f_{nl}=\frac{3\beta_{nl}}{8-\beta_{nl}} . \tag{28}$$

For extreme values of the anisotropy parameter ($\frac{1}{2}\leq\beta_{nl}\leq 2$) we find that π^f_{nl} varies between unity (for s states for which $\beta_{nl}=2$) and $\frac{1}{5}$ for the lowest value of $\beta_{nl}=\frac{1}{2}$ for high-l states [26]. Numerical values of the anisotropy parameter β_{nl} and the maximal polarizations, both P_{nl} of Eq. (6) and π^f_{nl} of Eq. (28), of the final states with $n \leq 5$ are summarized in Table I.

IV. DISCUSSION

In the previous section we derived, in the low-energy approximation, analytical expressions for the degree of polarization of photons in radiative recombination, both for a fixed electron energy and for typical electron-beam-velocity distributions existing in the electron cooler of a storage ring. As the most interesting result of our study we find that the photons emitted from recombination into s states are completely linearly polarized at any observation angle relative to the incoming electron velocity. Moreover, in a realistic experimental condition, where one has to average the intensity of the observed photon flux with respect to the electron velocity distribution in the electron cooler, the photons from recombination into s states are still completely linearly polarized at angle $\theta=90°$ (in a moving frame) for a flattened electron-beam-velocity distribution ($kT_\parallel \ll kT_\perp$). For higher-$l$ states the degree of polarization generally decreases with increasing l values.

The results above are valid in the moving frame of an ion. The transformation of degree of polarization from this frame to the laboratory system can be done using a relation between the angles in both systems [31]:

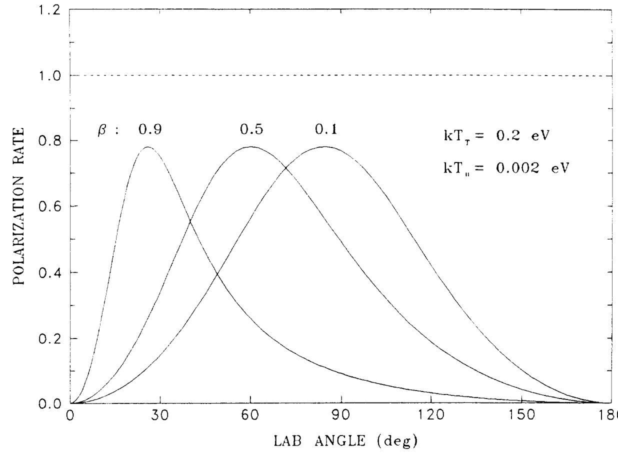

FIG. 7. Polarization rates $\pi_{n0}(\theta_{\rm lab})$, tranformed into the laboratory system, for different values of $\beta=v_{\rm ion}/c$ vs the laboratory observation angle $\theta_{\rm lab}$ for typical electron-beam temperatures $kT_\perp=0.2$ eV and $kT_\parallel=0.002$ eV.

$$\cos\theta=\frac{\cos\theta_{\rm lab}-\beta}{1-\beta\cos\theta_{\rm lab}} , \tag{29}$$

where index lab denotes angle in the laboratory system and β measures the ion velocity $v_{\rm ion}$ relative to the speed of light c, i.e., $\beta=v_{\rm ion}/c$. The polarization rate in the laboratory system versus observation angle $\theta_{\rm lab}$ for recombination into s states for typical electron-beam temperatures $kT_\perp=0.2$ eV and $kT_\parallel=0.002$ eV are shown in Fig. 7 for different values of the ion velocity parameter β. In this figure we find that the highest degree of polarization in the laboratory system moves towards forward angles with increasing values of the β parameter. For high-energy storage rings, such as, for instance, the Experimental Storage Ring (ESR) at GSI, Darmstadt [11], where ions will have $\beta\approx 0.8$, the radiation emitted in forward direction ($\theta \leq 30°$) can be strongly polarized. That can be particularly important when a crystal spectrometer is used to detect the photons. This result seems to be even more important in the context of planned relativistic ($\beta\approx 1$) electron-cooling storage rings [33] where for the case of recombination into s states the photon flux could be strongly linearly polarized for very small angles relative to the ion-beam direction. Another interesting aspect of polarization of photons in radiative recombination appears in the study of state-selective laser-induced recombination [16]. Such experiments are being performed [23] and are planned at different storage-ring facilities [32]. A measurement of a degree of polarization of photons from the electron cooler, due to its dependence on the electron-beam asymmetry parameter t [see Eq. (26)], can also be used as a diagnostic tool probing the electron-beam-velocity distribution.

V. CONCLUSIONS

We found that photons emitted from radiative recombination of bare ions with free electrons, as happens in the electron cooler of a storage ring, are generally *polarized.* Under a condition of the low-energy approximation

($E \ll E_{nl}$) the polarization of photons is completely described by an *energy-independent* anisotropy parameter β_{nl} [Eq. (2)]. For recombination into s states the photons are linearly polarized in the $(\mathbf{p},\mathbf{k})$ plane.

The polarization rates, introduced to describe the photon polarization after averaging over, generally *asymmetric* ($kT_{\parallel} \le kT_{\perp}$), Maxwellian electron-beam-velocity distribution in the cooler, can be expressed by a closed analytical formula in terms of an electron-beam asymmetry parameter $t=(kT_{\perp}-kT_{\parallel})/kT_{\parallel}$. For a symmetric distribution ($kT_{\parallel}=kT_{\perp}$) the polarization rate averages out [$\pi_{nl}^{M}(\theta)=0$], but for a typical in the electron-cooler flattened distribution ($kT_{\parallel} \ll kT_{\perp}$) the strongest polarization is observed at $\theta=90°$ (in the moving frame). The polarization rate, transformed to the laboratory system, peaks increasingly in the forward direction with increasing ion velocity. The present results on polarization of photons from radiative recombination have to be taken into account in experiments using crystal spectrometers to perform high-resolution spectroscopy. On the other hand, a measurement of polarization can be used as a diagnostic method of the electron beam in the cooler.

The present paper discussing the polarization of photons in radiative recombination, together with our previous paper [26] presenting the results for polarization-insensitive experiment, summarize our results on description of the radiative-recombination process between bare ions and free electrons in the low-energy limit $E \ll E_{nl}$, i.e., in an approximation which is perfectly fulfilled for low-n states ($n \le Z$) in the electron cooler of a heavy-ion storage ring.

APPENDIX

In this appendix we find the relations between $\cos^2\chi_{\parallel,\perp}$ and the angles ϑ, ψ, and φ, for a fixed angle θ between a photon observation direction $\mathbf{k}$ and ion-beam axis $\mathbf{z}$, where $\chi_{\parallel,\perp}$ are the angle between electric unit vector $\mathbf{e}$ and a plane $(\mathbf{k},\mathbf{p})$ (see Fig. 4), when $\mathbf{e}$ is perpendicular or lies in $(\mathbf{k},\mathbf{z})$ plane.

Introducing the unit vectors $\mathbf{e}_{kp}$ and $\mathbf{e}_{kz}$, both being perpendicular to the photon wave vector $\mathbf{k}$, which lie in $(\mathbf{k},\mathbf{p})$ and $(\mathbf{k},\mathbf{z})$ planes, respectively, the angle χ is just an angle between $\mathbf{e}_{kp}$ and $\mathbf{e}_{kz}$ vectors. Similarly, the angle κ (see Fig. 4) is an angle between $\mathbf{e}$ and $\mathbf{e}_{kz}$. With these the angles $\chi_{\parallel,\perp}$ are defined as follows: $\chi_{\parallel}=\chi(\kappa=90°)$ and $\chi_{\perp}=\chi(\kappa=0°)$. With the Cartesian xyz coordinate system chosen in such a way that $\mathbf{p}$ lies in the yz plane (see Fig. 4), the unit vectors along $\mathbf{k}$ and $\mathbf{p}$ vectors, $\mathbf{u}_k$ and $\mathbf{u}_p$, respectively, have the following coordinates: $\mathbf{u}_k=(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)$ and $\mathbf{u}_p=(0,\cos\psi,\sin\psi)$. With this one can write the unit vectors $\mathbf{e}_{kp}$ and $\mathbf{e}_{kz}$ as follows:

$$\mathbf{e}_{kp}=(1/\sin\vartheta)\mathbf{u}_p-\cot\vartheta\,\mathbf{u}_k \ , \tag{A1}$$

$$\mathbf{e}_{kz}=(1/\sin\theta)\mathbf{z}-\cot\theta\,\mathbf{u}_k \ , \tag{A2}$$

where $\mathbf{z}$ is the unit vector along the z axis. These expressions were obtained from the conditions that $\mathbf{e}_{kp}$ and $\mathbf{e}_{kz}$ vectors can be expressed as the normalized linear combinations of $\mathbf{u}_k$ and $\mathbf{u}_p$, and, respectively, $\mathbf{z}$ and $\mathbf{u}_k$ unit vectors. From this, since $\mathbf{e}_{kp}\cdot\mathbf{e}_{kz}=\cos(\chi-\kappa)$ (see Fig. 4) we find

$$\cos(\chi-\kappa)=\frac{\sin\psi-\cos\vartheta\cos\theta}{\sin\vartheta\sin\theta} \ . \tag{A3}$$

For the two particular cases we are interested in, namely, for $\chi_{\parallel}=\chi(\kappa=90°)$ and $\chi_{\perp}=\chi(\kappa=0°)$, we find

$$\cos^2\chi_{\parallel}=\frac{\cos^2\psi-\cos^2\vartheta-\cos^2\theta+2\sin\psi\cos\vartheta\cos\theta}{\sin^2\vartheta\sin^2\theta} \ , \tag{A4}$$

$$\cos^2\chi_{\perp}=\frac{(\sin\psi-\cos\vartheta\cos\theta)^2}{\sin^2\vartheta\sin^2\theta} \ . \tag{A5}$$

These expressions can be further simplified taking into account that

$$\cos\vartheta=\cos\psi\sin\theta\sin\varphi+\cos\theta\sin\psi \tag{A6}$$

which can be easily obtained from $\mathbf{u}_p\cdot\mathbf{u}_k=\cos\vartheta$. With this we get the final expressions for $\cos^2\chi_{\parallel,\perp}$ as

$$\cos^2\chi_{\parallel}=\frac{\cos^2\psi\cos^2\varphi}{\sin^2\vartheta} \ , \tag{A7}$$

$$\cos^2\chi_{\perp}=\frac{(\sin\psi\sin\theta-\cos\psi\cos\theta\sin\vartheta)^2}{\sin^2\vartheta} \ . \tag{A8}$$

These are used in Eq. (9) in Sec. III to calculate the double-differential recombination rate coefficient $d^2\alpha_{nl}^{\parallel,\perp}(E_\gamma,\theta)/dE_\gamma d\Omega$ for a fixed orientation of the photon electric vector with respect to the $(\mathbf{k},\mathbf{z})$ plane.

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[1] H. A. Kramers, Philos. Mag. **46**, 836 (1923).
[2] J. R. Oppenheimer, Phys. Rev. **31**, 349 (1928).
[3] W. Wessel, Ann. Phys. (Leipzig) **5**, 611 (1930).
[4] E. G. Stueckelberg and P. M. Morse, Phys. Rev. **36**, 16 (1930).
[5] I. I. Sobelman, *Atomic Spectra and Radiative Transitions* (Springer, Berlin, 1979).
[6] M. Stobbe, Ann. Phys. (Leipzig) **7**, 661 (1930).
[7] H. Bethe and E. Salpeter, *Quantum Mechanics of One- and Two-Electron Systems*, Handbuch der Physik Vol. 35 (Springer, Berlin, 1957).
[8] *Atomic and Molecular Processes*, edited by D. R. Bates (Academic, New York, 1962).
[9] W. H. Tucker, *Radiation Processes in Astrophysics* (MIT University Press, Cambridge, MA, 1975).
[10] D. Habs *et al.*, Nucl. Instrum. Methods B **43**, 390 (1989).
[11] F. Bosch, Nucl. Instrum. Methods B **23**, 190 (1987).
[12] S. P. Møller, in *Proceedings of the European Particle Conference, Rome 1988*, edited by S. Tazzari (World Scientific, Singapore, 1989), p. 112.
[13] K.-G. Rensfelt, in *Proceedings of the IEEE-91 Particle Accelerator Conference, San Francisco, May 1991*, edited by M. Allen (IEEE, in press).
[14] R. Schuch, Nucl. Instrum. Methods B **24-25**, 11 (1987).

[15] F. Bosch, Phys. Scr. **36**, 730 (1987).

[16] R. Neumann, H. Poth, A. Winnacker, and A. Wolf, Z. Phys. A **313**, 253 (1983).

[17] L. H. Andersen, J. Bolko, and P. Kvistgaard, Phys. Rev. Lett. **64**, 729 (1990).

[18] L. H. Andersen and J. Bolko, Phys. Rev. A **42**, 1184 (1990).

[19] L. H. Andersen and J. Bolko, J. Phys. B **23**, 3167 (1990).

[20] L. H. Andersen, G. Y. Pan, and H. T. Schmidt, J. Phys. B **25**, 277 (1992).

[21] A. Müller *et al.*, Phys. Scr. **T37**, 62 (1991).

[22] A. Wolf *et al.*, Z. Phys. D **21**, S69 (1991).

[23] U. Schramm, J. Berger, M. Grieser, D. Habs, E. Jaeschke, G. Kilgus, D. Schwalm, A. Wolf, R. Neumann, and R. Schuch, Phys. Rev. Lett. **67**, 22 (1991).

[24] F. B. Yousif, P. Van der Donk, Z. Kucherovsky, J. Reis, E. Brannen, and J. B. A. Mitchell, Phys. Rev. Lett. **67**, 26 (1991).

[25] J. H. Scofield, Phys. Rev. A **40**, 3054 (1989).

[26] M. Pajek and R. Schuch, Phys. Rev. A **45**, 7894 (1992).

[27] H. Poth, Phys. Rep. **196**, 135 (1990).

[28] L. Bracci, G. Fiorentini, and O. Pitzurra, Phys. Lett. **85B**, 280 (1979).

[29] J. Cooper and R. N. Zare, in *Lectures in Theoretical Physics,* edited by S. Geltman, K. T. Mahanthappa, and W. E. Britten (Gordon and Breach, New York, 1968), Vol. XI-C, p. 317.

[30] M. Ya. Amusia, *Atomic Photoeffect* (Plenum, New York, 1990).

[31] D. Liesen and H. F Beyer, GSI Report (Darmstadt) No. GSI-ESR/86-04, 1986 (unpublished).

[32] R. Schuch, A. Bárány, H. Danared, N. Elander, and S. Mannervik, Nucl. Instrum. Methods B **43**, 411 (1989).

[33] T. J. P. Ellison, in *Proceedings of the Workshop on Electron Cooling and New Cooling Techniques, Legnaro, 1990,* edited by R. Calabrese and L. Tecchio (World Scientific, Singapore, 1991), p. 29.