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Dispersion forces, self-reaction, and electromagnetic fluctuations

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It is demonstrated that semiclassically both the self-reaction and electromagnetic field fluctuations satisfy the condition that the field acting upon the electron is that due to its own charge and current fluctuations. The electron-field interaction energy can be found using the fluctuation-dissipation theorem, and equals $\frac{1}{2}\hbar$ multiplied by the sum of the frequency shifts as in the theory of dispersion forces. For an atom in an excited state, the self-consistently obtained frequencies have an imaginary part for $\hbar kc$ approximately equal to the atomic energy-level differences, giving spontaneous emission. Since the fields are real, the physically correct or Hermitian separation into self-reaction and electromagnetic contributions is obtained.

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I. INTRODUCTION

Dispersion forces between neutral atoms and macroscopic bodies are the result of correlated charge and current fluctuations and their associated electromagnetic fields and include the Casimir force. The dispersion energy can be found by obtaining the frequencies that self-consistently satisfy Maxwell's equations and the Schrödinger equation and evaluating $\frac{1}{2}\hbar$ multiplied by the sum of the frequency shifts [1]. This procedure has been justified by invoking an oscillator model of the atom, but equivalent results can be obtained using linear response theory and the fluctuation-dissipation theorem [2]. A perturbation at one point in a molecule causes a response that can be characterized by a spatially dependent dynamic susceptibility.

The Lamb-shift energy can be evaluated within this framework and gives, in lowest order, precisely the quantum electrodynamic (QED) result [3]. Both electromagnetic field fluctuations and the self-reaction contribute in the proportions obtained using Hermitian operators, and hence representing the physically meaningful separation of these effects [4]. In the absence of interactions the electromagnetic modes satisfy the dispersion relation $\omega=kc$, while the self-reaction takes place at the quantum-mechanical transition frequencies $\omega_{n'n}=(E_{n'}-E_n)/\hbar$, where $\{E_n\}$ are eigenvalues of the zero-order Schrödinger equation. When interaction between the particles and the self-consistently obtained fields is included, both these frequencies are shifted. The shifted quantum-mechanical frequencies $\omega_{n'n}$ are the result of the radiation reaction and the electron's Coulomb field, and the combination will be called the self-reaction following Ref. [4].

This natural separation into self-reaction and electromagnetic contributions might be considered a significant success of the dispersion energy formalism, since the equivalent result is not easily obtained using quantized fields. In the latter case the ordering of operators determines the physical interpretation [5]. When normal ordering is used, the radiative energy shift of an atom appears to be due to the radiation reaction. However, with symmetric ordering, the Lamb shift can be attributed to vacuum fluctuations.

The shifted frequencies will be obtained from the self-consistency condition that the field seen by the electron is the same as the field due to its own charge and current fluctuations. Thus both the radiation reaction and the electromagnetic parts can be attributed to fields produced by a fluctuating electron current. This implies that an electromagnetic field of the same magnitude as that normally identified with vacuum fluctuations can be attributed to the presence of matter. The question of whether the vacuum fluctuation energy is physically real or virtual is a fundamental one [6,7].

Using the dispersion energy formalism [1] the self-reaction and the electromagnetic modes can be treated symmetrically. The interaction energy comes from the frequency shifts, and any connection to the zero-point energy $\hbar\omega/2$ depends on the justification made for adding them to obtain $\Delta E=\frac{1}{2}\hbar\sum_{\mathbf{k}}[\sum_{n'}\Delta\omega_{\mathbf{k},n'n}+\Delta\omega_{kc}]$. A corresponding zero-point energy of $\frac{1}{2}\hbar\sum_{n'}\omega_{n'n}$ is not normally invoked, and there is no *a priori* reason to assume that the electromagnetic modes necessitate zero-point energy in the absence of matter.

For current fluctuations induced by an external potential, the total vector potential goes as $(\omega^2-k^2c^2)^{-1}$ in accordance with Maxwell's equations. The susceptibility χ that relates the total potential to the external potential has poles at the frequencies $\omega_{n'n}$ and kc. The current-potential correlation function, which can be obtained from the imaginary part of χ, describes fluctuations at both the self-reaction and the electromagnetic frequencies. There is a resonance, and hence dissipation, at $\omega=kc$ in addition to the quantum-mechanical resonance at $\omega_{n'n}$. Thus, according to the fluctuation-dissipation theorem, electromagnetic as well as self-reaction fluctuations occur as a consequence of the mathematical relationship between the correlation function and susceptibility.

The discussion here will be limited to a single electron in free space except possibly for a nucleus of infinite mass. Zero temperature will be assumed. In Sec. II the relationship between charge and current fluctuations and the electromagnetic vector and scalar potentials required by Maxwell's equations and the Schrödinger equation will first be obtained. Susceptibility will be defined and the self-consistency condition will be found. In Sec. III the interaction energy will be written in terms of the imaginary part of susceptibility using the fluctuation-dissipation theorem and shown to equal $\frac{1}{2}\hbar$ times the sum of the frequency shifts. Emission from excited atomic states which have complex frequency shifts will be examined. Finally, the results and their relationship to QED, especially the master equation approach [4], will be discussed.

II. DERIVATION OF SUSCEPTIBILITY AND THE SELF-CONSISTENCY CONDITION

The self-consistency condition will be derived by obtaining expressions for the ratio of current to potential from Maxwell's equations and the Schrödinger equation and requiring that they be the same. The current density can be calculated once the perturbed wave function is expressed as a function of the as yet unknown potentials. The Lorentz gauge and the four-vector current $J=(\rho c,\mathbf{j})$ and potential $A=(\varphi/c\ \mathbf{A})$ will be used where $\mathbf{j}$ is current density, ρ is charge density, and $\mathbf{A}$ and φ are the vector and scalar potentials. This is for convenience in writing Maxwell's equations. The potential and current will be written in Fourier expanded form which, for the mode of frequency ω, is

$$A(\mathbf{r},t)=\sum_{\mathbf{k}} A_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\cos(\omega t+\theta)\ , \tag{1}$$

with a similar expansion for the current four vector.

The interaction term in the Lagrangian is $\int(\mathbf{A}\cdot\mathbf{j}-\rho\varphi)d^3r$ and the source term in the electromagnetic field equation is the derivative of the Lagrangian density with respect to potential. Thus the quantum-mechanical current looks like a source in Maxwell's equations which become

$$(k^2-\omega^2/c^2)A_{\mathbf{k}}=\mu J_{\mathbf{k}}\ . \tag{2}$$

Treating $\mathbf{j}$ as a physical current density is not consistent with the probability density interpretation of $\psi^*\psi$. Since J is a probability density, A must be interpreted as the potential due to a moving point charge weighted by the probability that this charge has a particular position and velocity.

For the Schrödinger equation, interaction between the electron and the field results from the perturbation $\hat{H}'=\hat{H}'_1+\hat{H}'_2$ where

$$\hat{H}'_1\equiv-\frac{i\hbar e}{2m}(\nabla\cdot\mathbf{A}+\mathbf{A}\cdot\nabla)-e\phi\ , \tag{3}$$

$$\hat{H}'_2\equiv\frac{e^2}{2m}\mathbf{A}^2\ . \tag{4}$$

If a self-consistent solution is to be found it is essential that the nonlinear A^2 term can be neglected. In QED, A is proportional to $V^{-1/2}$ for the mode with wave vector $\mathbf{k}$ restricted to a volume V. While the source in Eq. (2) is the current weighted by the probability density, in the Schrödinger equation the coefficient of A is V independent. Thus $\hat{H}'_2$ can be expected to approach zero relative to $\hat{H}'_1$ as the volume goes to infinity. To compensate, the number of modes in a given volume of $\mathbf{k}$ space goes up proportional to V. The validity of neglecting $\hat{H}'_2$ will be discussed later after the frequency shifts have been obtained, allowing the calculation of A.

The A^2 term is a correction to the electron's rest mass and is a factor $\hbar k_M/mc$ smaller than the Coulomb self-energy [4] where k_M is the cutoff wave vector. Thus for nonrelativistic particle speeds and k small in comparison with the de Broglie wave vector, the lowest-order rest mass and kinetic-energy corrections are given by the Schrödinger equation with the perturbation $\hat{H}'_1$ only. This perturbation then describes an electron, whose speed is small compared with that of light, interacting with field modes of wave vector $k<mc/\hbar$ where m is the unrenormalized mass.

If the system is in state $|n\rangle$ to zero order, the wave function can be written as $\Psi_n(\mathbf{r},t)+\sum'_{n'}a_{n'}(t)\Psi_{n'}(\mathbf{r},t)$. The current component with frequency ω, $J\approx\sum_{n'}a^*_{n'}(t)J_{n'n}+\text{c.c.}$, can then be found using $\rho_{nn'}=-e\Psi^*_n\Psi_{n'}$ and

$$\mathbf{j}_{nn'}\equiv(i\hbar e/2m)(\Psi^*_n\nabla\Psi_{n'}-\Psi_{n'}\nabla\Psi^*_n)\ , \tag{5}$$

where $\rho_{n'n}$ and $\mathbf{j}_{n'n}$ satisfy a continuity equation, as do ρ and $\mathbf{j}$. Solution of the time-dependent Schrödinger equation in the presence of a potential A^a that is turned on infinitesimally slowly from $t=-\infty$ as $e^{\epsilon t}$ gives $\{a_{n'}(t)\}$ which can be used to calculate J. This can be substituted into Eq. (2) to give a matrix equation for the potential. $\tilde{A}=(\varphi/c,-\mathbf{A})$ and $\tilde{J}=(\rho c,-\mathbf{j})$ will be introduced to obtain the correct signs, since $\mathbf{j}\cdot\mathbf{A}$ and $\rho\varphi$ have opposite signs in the Hamiltonian. Using i to represent $(\mathbf{k},\alpha)$ where α indexes the components of J,

$$\tilde{A}_i=\sum_j\chi_{ij}(\omega)A^a_j\ . \tag{6}$$

A^a is the external potential in the definition of susceptibility [8], or the self-consistent potential in the description of fluctuations. For an input A^a proportional to $e^{(i\omega+\epsilon)t}$, the susceptibility tensor is

$$\chi_{ij}(\omega)\equiv-\frac{\mu c^2V}{\hbar[(\omega-i\epsilon)^2-k^2c^2]}\sum_{n'}\text{Re}\left[\frac{J_{i,nn'}\tilde{J}_{j,n'n}}{\omega_{n'n}+\omega-i\epsilon}\right]\ , \tag{7}$$

$$J_{i,n'n}\equiv V^{-1}\langle e^{-i\mathbf{k}\cdot\mathbf{r}}J_{\alpha,n'n}\rangle\ , \tag{8}$$

and ϵ is to be allowed to go to zero. In the notation used here J is the self-consistent current, $J_{n'n}$ is defined in Eq. (5), and the subscript i or $\mathbf{k}$ denotes the corresponding Fourier components. The angular brackets stand for the integral over volume. Re symbolizes the real part of everything to its right. The last factor is due to the Schrödinger equation and gives the real current resulting from a complex potential. The first factor then converts this source current to a potential according to Eq. (2). For $\omega\neq kc$, $\omega_{n'n}$, and a real A^a where both ω and $-\omega$ contribute, the susceptibility becomes

$$\chi_{ij}(\omega)\equiv-\frac{\mu c^2V}{\hbar(\omega^2-k^2c^2)}\sum_{n'}\frac{2\omega_{n'n}J_{i,nn'}\tilde{J}_{j,n'n}}{\omega^2-\omega^2_{n'n}}\ . \tag{9}$$

This can be used to find the in-phase response. The 90°

out of phase or imaginary part must be obtained from the full Eq. (7).

III. REAL AND COMPLEX FREQUENCY SHIFTS

The self-consistency condition that determines the frequencies is obtained if A^a is set equal to A in Eq. (2) giving $|\delta_{ij}-\chi_{ij}|=0$. The matrix χ given by Eq. (9) is diagonal in $\mathbf{k}$ in the case of a free electron for which $\mathbf{k}_{n'}=\mathbf{k}_n+\mathbf{k}$ and $\omega_{n'n}=\hbar(k^2+2\mathbf{k}\cdot\mathbf{k}_n)/2m$. For discussion of emission by an atom one self-reaction frequency $\omega_{n'n}$ will dominate. Neglecting off-diagonal terms which are of order $[(\omega-kc)(\omega-\omega_{n'n})\chi]^2$ and higher, the self-consistency condition implies that

$$\omega_{\mathbf{k}}^2 \approx \frac{1}{2}\left[k^2c^2+\omega_{n'n}^2 \pm \left[(k^2c^2-\omega_{n'n}^2)^2-\frac{8\mu c^2\omega_{n'n}}{\hbar V}\sum_{\alpha}\langle e^{i\mathbf{k}\cdot\mathbf{r}}J_{\alpha,nn'}\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{r}}\tilde{J}_{\alpha,n'n}\rangle\right]^{1/2}\right] . \tag{10}$$

In Ref. [3] the frequency shifts were obtained from the trace of $\sum_r \chi^r/r$. The lowest-order shifts are approximately, for the modes of wave-vector $\mathbf{k}$,

$$\Delta\omega_{kc} = -\frac{\mu c^2}{V\hbar}\sum_{\alpha,n'}\langle e^{i\mathbf{k}\cdot\mathbf{r}}J_{\alpha,nn'}\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{r}}\tilde{J}_{\alpha,n'n}\rangle \times[(kc)^2-\omega_{n'n}^2]^{-1}\frac{\omega_{n'n}}{kc} , \tag{11a}$$

$$\Delta\omega_{\mathbf{k},n'n} = \frac{\mu c^2}{V\hbar}\sum_{\alpha}\langle e^{i\mathbf{k}\cdot\mathbf{r}}J_{\alpha,nn'}\rangle\langle e^{i-i\mathbf{k}\cdot\mathbf{r}}\tilde{J}_{\alpha,n'n}\rangle \times[(kc)^2-\omega_{n'n}^2]^{-1} . \tag{11b}$$

k is the magnitude of the mode wave vector but $\omega_{n'n}$ can be positive or negative. The longitudinal current can be eliminated using the charge-current conservation equation which implies that $c^2\rho^2-j_1^2=[1-(\omega/kc)^2]c^2\rho^2$, removing the pole at $\omega=kc$. The factor $[(kc)^2-\omega_{n'n}^2]^{-1}$ in Eqs. (11) is then replaced by $(kc)^{-2}$ as in Ref. [3].

For longitudinal fields alone the energy in the electric field $\mathbf{E}$ is $\frac{1}{2}\epsilon_0\int d^3r\,\mathbf{E}\cdot\mathbf{E}=\frac{1}{2}\int d^3r\,\rho\varphi$, or half the expectation value of the perturbation. Thus half the expectation value of $-e\varphi$ is just the energy in the electron's Coulomb field. In the presence of current fluctuations, $-e\varphi$ is replaced by $\hat{H}'_1$ with an additional contribution from the electromagnetic field. The factor $\frac{1}{2}$ compensates for double counting. The imaginary part of χ_{ij} is $\pi/\hbar$ times the correlation function of the response A in Eq. (6) and the coefficient of the external driving force A^a in $\hat{H}'_1$. Thus it is the correlation function of the potential A and the current operator. The fluctuation-dissipation theorem (at zero temperature) then takes the form

$$\Delta E=\langle\tfrac{1}{2}\hat{H}'_1\rangle=\frac{\hbar}{2\pi}\int_0^\infty d\omega\,\mathrm{Im}\left[\sum_i\chi_{ii}(\omega)\right] . \tag{12}$$

The longitudinal part is just Callen and Welton's original result [9] in the form $\frac{1}{2}\epsilon_0\int d^3r\,\mathbf{E}\cdot\mathbf{E}=(\epsilon_0/\pi)\int d\omega\frac{1}{2}\hbar\omega\sigma^{-1}(\omega)$ with $\epsilon_0\sigma^{-1}\omega=\mathrm{Im}(\epsilon_0/\epsilon)=\mathrm{Im}(\sum_i\chi_{ii})$, σ being conductivity. The interaction energy is thus given by the trace of the imaginary part of the susceptibility matrix.

A general mathematical relationship between frequency shifts, the imaginary part of susceptibility, and the correlation function can be found using the solvability condition for Eqs. (6) and (9). The frequencies can be obtained approximately by setting ω equal to its zero-order value except where it occurs as a frequency shift. After solving for this shift $\Delta\omega_i$ and using $\lim_{\xi\to 0}[1/(x-i\xi)]=P(1/x)+i\pi\delta(x)$, it can be seen from Eq. (7) for ϵ approaching zero that

$$\mathrm{Im}\chi_{ii}=\pi\Delta\omega_i[\delta(\omega-\omega_i)-\delta(\omega+\omega_i)] . \tag{13}$$

The difference of δ functions implies a 90° phase shift between the external and the total potential and the use of imaginary numbers is merely a mathematical convenience here as in circuit theory. From the fluctuation-dissipation theorem, Eq. (12), the interaction energy is then $\Delta E=\frac{1}{2}\hbar\sum_i\Delta\omega_i$. Thus the dispersion energy expression for interaction energy in terms of frequency shifts for self-consistently determined frequencies is equivalent to the fluctuation-dissipation theorem. For the electron-fluctuating field interaction considered here, ω_i is ω_{kc} and $\omega_{\mathbf{k},n'n}$. The transverse contribution to this energy is then

$$\Delta E_t=\frac{\mu c}{2V}\sum_{\mathbf{k},\alpha}\frac{\langle e^{i\mathbf{k}\cdot\mathbf{r}}J_{\alpha,nn'}\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{r}}\tilde{J}_{\alpha,n'n}\rangle}{k(kc+\omega_{n'n})} . \tag{14}$$

The subscript t indicates that transverse currents only should be included.

The Fourier expanded interaction energy is $\Delta E=-\frac{1}{2}V\sum_i A_i\tilde{J}_i$ where i represents $\mathbf{k}$ and α as in Eq. (8) and A_i and J_i are defined by Eq. (1). Thus J_i is the $\mathbf{k}$th term in the Fourier expansion of a particular component of the current four-vector matrix element. The self-reaction modes are associated with virtual transitions to a state $|n'\rangle$ and the corresponding current fluctuations and J_i and hence A_i are proportional to V^{-1}. For the electromagnetic modes, the Schrödinger equation requires that $\mathbf{j}_{\mathbf{k}}=-(\hbar V)^{-1}2\omega_{n'n}(k^2c^2-\omega_{n'n}^2)^{-1}\mathbf{A}_{\mathbf{k}}$, and Eq. (11a) implies that the transverse potential fluctuations satisfy $\mathbf{A}_{\mathbf{k}}^2=\hbar c\mu/2kV$ as in QED [10]. These electromagnetic field fluctuations induce virtual transitions to excited states and the coherent current due to these transitions sustains the field. $\langle\hat{H}'_1\rangle$ is the same as in the conventional QED treatment, but here only half this expectation value is taken. The other half of the photon emission contribution comes from the radiation reaction. The use of $\langle\hat{H}'_1\rangle$ is consistent with A being external, while the use of $\frac{1}{2}$ is appropriate if the field is the result of the electron current itself. Since $A_{\mathbf{k}}$ is proportional to $V^{-1/2}$ for the electromagnetic modes and V^{-1} for the self-reaction modes, the neglect of the $A_{\mathbf{k}}^2$ term in H' was justified when deriving the self-consistency condition for the mode with wave vector **k**.

If the electron is initially in an excited state, that is if $\omega_{n'n} < 0$, then the frequencies that satisfy Eq. (10) are complex for $kc \approx \omega_{n'n}$. Since $\Delta E = \frac{1}{2}\hbar\sum_i \Delta\omega_i$ the decay rate Γ is proportional to the sum over wave vectors in the range where ω given by Eq. (10) has an imaginary part. The real frequency, approximately kc, is greater than Γ and the solution is effectively self-consistent over time periods short in comparison with Γ^{-1}. If the sum is converted to an integral, Γ is proportional to the area under the frequency shift versus electromagnetic field frequency curve,

$$\int_{-2\sqrt{A}}^{2\sqrt{A}} [4A - (kc - \omega_{n'n})^2]^{1/2} dkc = 2\pi A \ , \tag{15}$$

where

$$A \equiv -\frac{\mu c |\omega_{n'n}|}{2\hbar V k} \sum_{\alpha} \langle e^{i\mathbf{k}\cdot\mathbf{r}} J_{\alpha,nn'} \rangle \langle e^{-i\mathbf{k}\cdot\mathbf{r}} \tilde{J}_{\alpha,n'n} \rangle \ . \tag{16}$$

For the range of **k** values for which ω is complex, $\omega = \omega_0 \pm i\Gamma/2$ where ω_0 and Γ are real. There is a decaying and a growing solution, but the latter is not physically acceptable since it would make $|a_n|^2 > 1$. The width of the imaginary frequency shift peak goes to zero as $V^{-1/2}$ and the height of the peak goes to infinity as $V^{1/2}$ after multiplication by the density of states $\rho(k) = V(2\pi)^{-3} 4\pi k^2$. Thus the integrand is a δ function multiplied by the area given by Eq. (15) and $\rho(k)$. Since $|a_n|^2$ is proportional to $|\exp(i\omega t)|^2$ integration over wave vector gives

$$\Gamma = 2\pi A \rho(k) c \ . \tag{17}$$

This is numerically equivalent to using $\mathrm{Im}(kc + \omega_{n'n})^{-1} = \pi\delta(kc + \omega_{n'n})$ in Eq (14). However, Γ here comes directly from the imaginary frequency shifts and there is no need to introduce this procedure.

IV. DISCUSSION

The results obtained here in the dispersion energy framework can be compared with the QED-based master-equation formulation of Cohen-Tannoudji and co-workers [4]. These authors define a velocity-velocity correlation function C_S for the system, a reservoir potential-potential correlation function C_R, and corresponding susceptibilities that link particle velocity to the vector potential and vice versa. The radiation reaction is obtained from the frequency integral of the product $\chi_R C_S$ and the energy due to vacuum fluctuations from the integral of $\chi_S C_R$. The former is essentially the same as the contribution to ΔE of the $\omega_{n'n}$ modes here, and the latter is equivalent to the contribution of the ω_{kc} modes. Other than the use of the long wavelength approximation in Ref. [4], the main difference between their results and the present ones is that the factor $[(kc)^2 - \omega_{n'n}^2]^{-1}$ in Eq. (11b) is replaced by $(kc)^{-2}$ in Ref. [4]. This allows separation of system and reservoir fluctuations. Here it is the product of vector potential and current that is averaged, where the factor $[(kc)^2 - \omega_{n'n}^2]^{-1}$ takes into account retardation for the modes with frequency $\omega_{n'n}$.

The rate of energy loss is the real part of the scalar product of current and electric field, that is $\mathrm{Re}\langle \mathbf{E}\cdot\mathbf{j} \rangle = \mathrm{Re}(i\omega\hat{H}'_1\rangle$ per unit volume. Only the transverse modes can contribute divergent frequency shifts due to the factor $(kc \pm \omega_{n'n})^{-1}$. If $\mathrm{Im}(kc \pm \omega_{n'n})^{-1} = \pi\delta(kc \pm \omega_{n'n})$ is used, the radiation reaction Eq. (11b) looks like it generates loss at a rate of $-(6\pi\epsilon_0 c^3)^{-1} |j_{n'n}|^2 \omega_{n'n}^2$. This is the energy radiated by an accelerated charge classically. For an atom in its ground state the shifts in the electromagnetic frequencies [Eq. (11a)] contribute an absorption term of equal magnitude as in Ref. [4]. However, from the perspective of the present self-consistent theory, these frequencies are shifted away from each other and there is only virtual emission and absorption. The approximation used to derive Eqs. (11) is invalid for $kc = \omega_{n'n}$ and only for an atom in an excited state is there an imaginary frequency shift that represents emission.

The present calculation gives identical energy shifts [Eq. (14)] to conventional QED in the nonrelativistic limit [10]. This is discussed in Ref. [3] in the case of electron mass renormalization and the Lamb shift. It is based on self-fields, and is complimentary to the formulation of Barut and Dowling [7]. On the other hand, the separation into radiation reaction and electromagnetic parts is obtained in a unique way here. It results from the fact that the current and potential are real. This aspect is consistent with what is obtained in Ref. [4] by requiring that the self-reaction and vacuum fluctuation contributions to the rate of variation of any atomic observable be Hermitian. There is thus no indetermination in the separation of self-reaction and electromagnetic effects. From Eqs. (11) it can be seen that the radiation reaction contribution is approximately state independent and hence represents renormalization of the electron mass. The state dependent or Lamb-shift part of ΔE comes from electromagnetic fluctuations to leading order in $\omega_{n'n}/kc$ as with symmetric ordering in QED.

It can also be seen from Eqs. (11) that the electromagnetic and radiation reaction contributions combine so as to eliminate the factor $(kc - \omega_{n'n})^{-1}$ that would result in divergence. This cancellation is essential to avoid predicting the absorption of photons at zero temperature. Only terms proportional to $(kc + \omega_{n'n})^{-1}$ that imply emission of virtual photons remain. It is remarkable that this result falls naturally out of the dispersion energy formalism. Imaginary frequencies satisfy Eq. (10) only for a particle in an excited state for which $\omega_{n'n} < 0$. In this regard the present calculation is qualitatively different from the formulation in Ref. [7] which is based on the self-reaction only and where both emission and absorption terms occur. It is the balance between the radiation reaction and electromagnetic effects which removes the absorption term at zero temperature that is most characteristic of the present formulation. This is perhaps best illustrated by interplay between the radiation reaction and electromagnetic frequencies required to obtain complex frequency shifts in Eq. (10).

In summary, the interaction of an electron with fluctuating fields has been analyzed in the dispersion energy framework where the electron is treated quantum mechanically and the field is real. The field induces virtual transitions to excited states described by the time-dependent Schrödinger equation and hence produces a fluctuating current that is the source term in Maxwell's equations. The frequencies that satisfy these equations self-consistently are the self-reaction modes at frequencies $\omega_{n'n}+\Delta\omega_{\mathbf{k},n'n}$, and the electromagnetic modes at $kc+\Delta\omega_{kc}$. The interaction energy is $\Delta E = \frac{1}{2}\hbar\sum_{\mathbf{k}}[\sum_{n'}\Delta\omega_{\mathbf{k},n'n}+\Delta\omega_{kc}]$ as in the theory of dispersion forces, and agrees with the electron-fluctuating field energy obtained using conventional QED arguments. It is equivalent to using the fluctuation-dissipation theorem to find the expectation value of the perturbation. Since the fields and current are real, the separation of ΔE into self-reaction and electromagnetic contributions is the same as is obtained by defining Hermitian operators corresponding to these two components. Spontaneous emission is obtained naturally in a range of frequencies for which the self-consistent solutions are complex. The decaying solutions are obtained only for an atom in an excited state as the shifted frequencies are always real for an atom in its ground state. In the present theoretical framework, both the radiation reaction and electromagnetic field fluctuations of the magnitude of vacuum fluctuations in QED result from current fluctuations that they themselves produce.

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