Fluctuations and squeezing in resonance fluorescence emitted near a phase conjugator

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Atomic resonance fluorescence, emitted near the surface of a four-wave-mixing phase conjugator, exhibits squeezing for certain values of the optical parameters of the incident laser field and for a certain range of the phase-conjugate reflectivity. A condition for the occurrence of squeezing is derived, and it is shown that the presence of the phase-conjugating medium tends to increase the quantum fluctuations, thereby diminishing the squeezing.

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Electromagnetic radiation with reduced quantum fluctuations has been the subject of many recent studies. In particular, the generation of squeezed states, for which the fluctuations are below the limit suggested by the uncertainty relation, has attracted much attention because of possible applications in high-precision optical measurements. Two-photon coherent states which are produced in degenerate four-wave mixing exhibit squeezing in certain field quadrature components [1–3]. Such squeezed states can be obtained, for instance, when the four-wave mixer operates as a phase-conjugating mirror, and the incident field on the nonlinear medium is combined with its phase-conjugate replica. Another source of squeezed radiation is single-atom resonance fluorescence [4–7], provided that the optical parameters are chosen carefully. Also, many atoms in a cavity [8–11], a linear array of atoms [12], or a single atom in a cavity [13–18] are predicted to radiate squeezed light under certain conditions. In this Brief Report we consider a combination of the two mechanisms. A nonlinear medium is pumped by two strong counterpropagating lasers with frequency $\bar{\omega}$. Then the medium will operate as a four-wave-mixing phase conjugator (PC) for weak incident radiation. A two-state atom is near the surface of the medium, and is irradiated by a laser beam of frequency ω_L which propagates parallel to the surface. The frequency ω_L is in close resonance with the transition frequency ω_0 between the atomic ground state $|g\rangle$ and excited state $|e\rangle$, and $\Delta=\omega_L-\omega_0$ indicates the detuning from resonance. Part of the emitted resonance fluorescence by the atom is incident upon the PC, which then generates the phase-conjugate image of this field. The radiation which travels into the direction of a detector far away from the surface is therefore the sum of ordinary fluorescence, emitted directly towards the detector, and a phase-conjugate image which is produced in the four-wave mixer. It can be shown [19] that the positive-frequency part of the total field attains the form

$$E(t)^{(+)}=\gamma^*[d^\dagger(t)-P^*e^{-2i\bar{\omega}t}d(t)]\ , \tag{1}$$

where $d=|e\rangle\langle g|$ is the atomic raising operator, P is the Fresnel reflection coefficient for a plane wave (the efficiency of the phase conjugator), and γ^* is an overall constant which includes the projection of the atomic dipole moment onto the polarization direction of the detector. The time dependence of the operators signifies the Heisenberg picture. Whether the field, as given by Eq. (1), exhibits any squeezing will depend on the details of the time evolution of the lowering and raising operators.

At the position of the atom, the driving laser has the form

$$\mathbf{E}_L(t)=E_0\mathrm{Re}\boldsymbol{\epsilon}_L e^{-i[\omega_L t+\phi(t)]}\ . \tag{2}$$

The phase $\phi(t)$ is a random process which accounts for the laser linewidth, and we shall take $\phi(t)$ to be the independent-increment process. This leads to a Lorentzian laser line shape, and it includes the phase-diffusion model as a special case. Following Collett, Walls, and Zoller [7], we define the slowly varying amplitude of the fluorescence field, with respect to the incident field, by

$$E_\theta(t)=E(t)^{(+)}e^{i[\omega_L t+\phi(t)-\theta]}+\mathrm{H.c.} \tag{3}$$

In an experiment, the phase θ can be varied in order to obtain a maximum reduction of the fluctuations [20]. For $\theta=0$ and $\pi/2$ this field reduces to the familiar in-phase and out-of-phase quadrature components of the field, respectively. As it turns out, however, these values of θ do not give the maximum possible reduction of phase fluctuations. The uncertainty relation for the variances of two quadrature components with different θ is

$$\Delta E_\theta(t)\Delta E_{\theta'}(t)\geq\tfrac{1}{2}|\langle[E_\theta(t),E_{\theta'}(t)]\rangle|\ , \tag{4}$$

and with Eq. (3) this becomes

$$[\Delta E_\theta(t)]^2[\Delta E_{\theta'}(t)]^2\geq\langle[E(t)^{(+)},E(t)^{(-)}]\rangle^2\sin^2(\theta-\theta')\ . \tag{5}$$

For fluorescence, the quantity $[E(t)^{(+)},E(t)^{(-)}]$ is not a

c number, and its expectation value is not necessarily positive. Therefore, the usual definition of squeezing, which requires the normally ordered variance to be negative [21], has to be generalized. As suggested by Eq. (5), we define $E_\theta(t)$ to be squeezed if

$$[\Delta E_\theta(t)]^2 < |\langle [E(t)^{(+)}, E(t)^{(-)}]\rangle| \ . \tag{6}$$

Under this condition, the fluctuations in $E_\theta(t)$ are minimal, but, according to Eq. (5), at the expense of enhanced fluctuations in the component of the field which is 90° out of phase with this $E_\theta(t)$. Note that the right-hand side of Eq. (6) is independent of θ, as it should be.

As a measure for the amount of fluctuation in the field, or the randomness of the phase of the field, we introduce the parameter r, defined by

$$r = \frac{(\Delta E_\theta)^2}{\langle E_\theta^2\rangle} = 1 - \frac{\langle E_\theta\rangle^2}{\langle E_\theta^2\rangle} \ . \tag{7}$$

Obviously, the value of r is restricted by $0 \le r \le 1$. For $r=0$ there are no fluctuations at all, and for $r=1$ we have $\langle E_\theta\rangle = 0$, which corresponds to a purely random phase. Similarly, the amount of squeezing can be expressed in terms of the normalized quantity

$$s = \frac{(\Delta E_\theta)^2 - |\langle [E(t)^{(+)}, E(t)^{(-)}]\rangle|}{\langle E_\theta^2\rangle} \ . \tag{8}$$

Then the condition for squeezing in $E_\theta(t)$ becomes $s<0$.

The various quantities which determine the parameters r and s can be expressed in terms of matrix elements of the atomic density operator σ (in the rotating frame). After averaging over the fluctuating laser phase and taking the steady-state limit, we obtain

$$\langle E_\theta\rangle = \gamma^* e^{-i\theta}\sigma_{eg} + \text{c.c.} \ , \tag{9}$$

$$\langle E_\theta^2\rangle = \gamma_0^2(1+P_0^2) \ , \tag{10}$$

$$\langle [E(t)^{(+)}, E(t)^{(-)}]\rangle = \gamma_0^2(1-P_0^2)(\sigma_{gg}-\sigma_{ee}) \ , \tag{11}$$

where $P_0 = |P|$. In the derivation of Eq. (9) from Eq. (3) we have used the following identity. If we indicate by $\tilde{\sigma}(t)$ the stochastically fluctuating density matrix, and by $\langle\ \rangle$ an average over the laser phase, then it holds that [22]

$$\lim_{t\to\infty} \langle \tilde{\sigma}(t)\exp[in\phi(t)]\rangle = 0 \ , \tag{12}$$

for n integer, $n \ne 0$. Notice that the average of the commutator in Eq. (11) vanishes for unit reflectivity ($P_0^2 = 1$). In that case, $s = r \ge 0$, and squeezing can never occur. The density operator matrix elements are [22]

$$\sigma_{ee} = \frac{1}{2}\frac{\Omega_0^2\eta + AP_0^2(\Delta^2+\eta^2)}{\Omega_0^2\eta + A(1+P_0^2)(\Delta^2+\eta^2)} \ , \tag{13}$$

$$\sigma_{gg} = 1 - \sigma_{ee} \ , \tag{14}$$

$$\sigma_{eg} = -\tfrac{1}{2}\Omega\frac{A(\Delta - i\eta)}{\Omega_0^2\eta + A(1+P_0^2)(\Delta^2+\eta^2)} \ . \tag{15}$$

Here, Ω is the (complex) Rabi frequency, $\Omega_0 = |\Omega|$, A is the Einstein coefficient for spontaneous decay, and $\eta = \lambda + A(1+P_0^2)/2$, with λ the half width at half maximum of the laser profile. The combination of Eqs. (9)–(15) then yields for the parameter r

$$r = 1 - \frac{\Omega_0^2 A^2}{(1+P_0^2)[\Omega_0^2\eta + A(1+P_0^2)(\Delta^2+\eta^2)]^2} \times (\Delta\cos\beta - \eta\sin\beta)^2 \ . \tag{16}$$

Angle β is given by $\beta = \theta + \arg(\gamma) - \arg(\Omega)$, which depends on the phase of the atomic dipole through both γ and Ω. By varying the mixing angle θ we can vary angle β, and thereby the parameter r. It follows from Eq. (16) that r is minimal if β is chosen as the solution of

$$\tan(\beta) = -\eta/\Delta \ . \tag{17}$$

Then $(\Delta\cos\beta - \eta\sin\beta)^2 = \Delta^2 + \eta^2$, and the value of r follows from Eq. (16). For this value of θ, the fluctuations in the phase of the fluorescence are minimal, given the optical parameters. The quadrature component of the field which is 90° out of phase, compared to the solution of Eq. (17), has a mixing angle θ for which β is the solution of $\tan(\beta) = \Delta/\eta$. Then $(\Delta\cos\beta - \eta\sin\beta)^2 = 0$, which yields $r=1$. This gives $\langle E_\theta\rangle = 0$, which reflects that this out-of-phase component has a completely random phase.

The solution for θ which follows from Eq. (17) also minimizes the squeezing function s, because the commutator in Eq. (8) does not depend on θ. For this value of θ the parameter s is found to be

$$s = 1 - \frac{A(\Delta^2+\eta^2)}{(1+P_0^2)[\Omega_0^2\eta + A(1+P_0^2)(\Delta^2+\eta^2)]^2} \times [\Omega_0^2(A + |1-P_0^2|\eta) + A|1-P_0^4|(\Delta^2+\eta^2)] \ . \tag{18}$$

Therefore, squeezing occurs under the condition

$$(1+P_0^2)[\Omega_0^2\eta + A(1+P_0^2)(\Delta^2+\eta^2)]^2 < A(\Delta^2+\eta^2)[\Omega_0^2(A + |1-P_0^2|\eta) + A|1-P_0^4|(\Delta^2+\eta^2)] \ . \tag{19}$$

In the absence of the phase conjugator we have effectively $P_0 = 0$, and Eq. (19) reduces to

$$A(A-\eta)\Delta^2 > \eta^2[\Omega_0^2 + A(\eta - A)] \ , \tag{20}$$

which is the familiar result for a free atom [5]. Another interesting limit is the case $\Omega_0^2 = 0$, which represents the atom near the PC, but without the driving laser field. This spontaneously emitted phase-conjugated fluorescence is squeezed under the condition

$$(1+P_0^2)^2 < |1-P_0^2| \ , \tag{21}$$

and it is easy to see that this condition cannot be satisfied for any value of the reflectively P_0^2. Apparently, in order for phase-conjugated atomic radiation to be squeezed, we have to drive the atomic transition with an external field. Figure 1 illustrates the region where squeezing occurs as a function of P_0^2 and Ω_0^2/A^2 (proportional to the laser power) for the case of resonance ($\Delta=0$) and zero laser linewidth ($\lambda=0$). As shown in the figure, the light will only be squeezed for small values of the reflectivity and

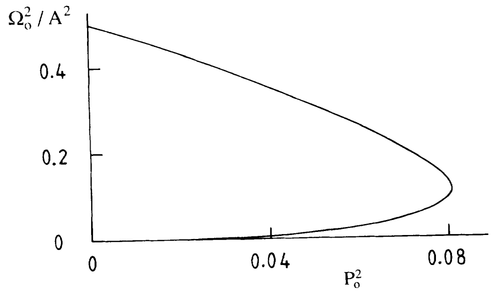

FIG. 1. For values of Ω_0^2/A^2 and P_0^2 corresponding to a point inside the lobe, squeezing occurs. Here we have assumed $\Delta=\lambda=0$.

moderate laser power.

The fluctuation parameter r and the squeezing parameter s can be minimized by a proper choice of optical parameters. For a laser intensity of

$$\Omega_0^2=\frac{A}{\eta}(1+P_0^2)(\Delta^2+\eta^2)\ , \tag{22}$$

the parameter r has a local minimum, which is equal to

$$r=1-\frac{A}{\eta}\frac{1}{4(1+P_0^2)^2}\ . \tag{23}$$

On the other hand, the minimum value of s is given by

$$s=1-\frac{(A+\eta|1-P_0^2|)^2}{4\eta A(1+P_0^2)^2}\ , \tag{24}$$

which is attained for

$$\Omega_0^2=\frac{A}{\eta}(1+P_0^2)(\Delta^2+\eta^2)\frac{A-\eta|1-P_0^2|}{A+\eta|1-P_0^2|}\ , \tag{25}$$

provided that $\eta|1-P_0^2|\le A$. Notice that the extreme values of r and s are independent of the detuning Δ.

Given the optimum value for Ω_0^2, the parameter r can be further minimized by setting $\lambda=P_0^2=0$, for which we find $r=\frac{1}{2}$. This corresponds to an intensity of $\Omega_0^2=2\Delta^2+A^2/2$, which can be obtained for any value of the detuning. For the optimum Ω_0^2, parameter s is also minimal for linewidth $\lambda=0$. The parameter s is then found to be

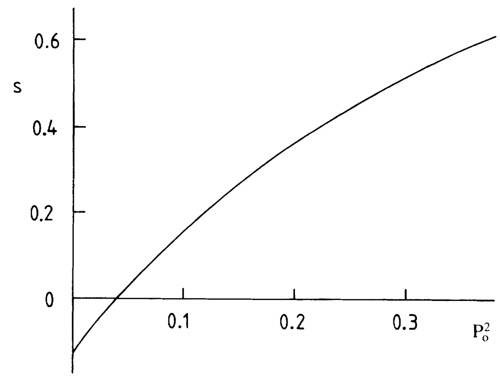

FIG. 2. Value of the squeeze parameter s as a function of the phase-conjugate reflectivity P_0^2 for $\lambda=0$ and the optimum value of Ω_0^2.

$$s=1-\frac{(2+|1-P_0^4|)^2}{8(1+P_0^2)^3} \tag{26}$$

for all Δ, and provided that $P_0^2\le\sqrt{3}$ (a larger value of P_0^2 would give a negative value for Ω_0^2). The behavior of s as a function of P_0^2 is shown in Fig. 2. The minimum value of s is equal to $-\frac{1}{8}$, which is reached for $P_0^2=0$, and the corresponding value of the intensity is $\Omega_0^2=(4\Delta^2+A^2)/6$. Furthermore, we have $s=0$ for $P_0^2\approx 0.04$.

It has been shown that the superposition of atomic resonance fluorescence and its phase-conjugate replica exhibits squeezing for certain values of the optical parameters, and the condition under which this occurs is given by Eq. (19). A necessary condition turned out to be that the atom has to be irradiated by a laser field, e.g., spontaneously emitted fluorescence near a phase conjugator is never squeezed.

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