Pair production in a strong wake field driven by an intense short laser pulse

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A preliminary investigation is presented of electron-positron pair production by means of plasma electrons accelerated by relativistic velocities in a strong wake field. The propagation distance of the plasma wake field, which is determined by the depletion of the short laser pulse due to wake-field generation, is much larger than the pulse length. For this case, the total number of electron-positron pairs produced is independent of the plasma concentration. For achievable parameters of the laser pulse, the total number of pairs may be quite significant ($\sim 10^6 Z^2$, where Z is the nuclear charge of the plasma ion).

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I. INTRODUCTION

Recent technological advances have made possible compact terawatt laser facilities with high intensities ($>10^{18}$ W/cm^2) and ultrashort pulses (<1 psec) [1,2]. The design of a neodymium laser with 10^3 TW power and a pulse duration of the order of 1 psec [3] is in the process of development. Using modern focusing devices, one can focus laser radiation at a very small spatial region with focal-spot dimensions only 2–3 times higher than the diffraction limit. The radiation intensity in such a region will be of the order of $I \approx 10^{21}$ W/cm^2, and the corresponding electric field would be $E \approx 10^{11}$ V/cm, which is two orders of magnitude larger than the internal field of the atom. In the field of such a strong radiation, the electron oscillation energy would be much higher than its rest energy. The character of the nonlinear response of the medium would radically change in the presence of these short laser pulses. In high-energy physics, the problems of nonlinear electron physics, such as Thomson and Compton scattering, multiphoton atomic ionization, etc., are of great importance. Furthermore, experimental verification of nonlinear quantum electrodynamics has also to be examined [4]. The plasma may be a convenient medium for this purpose.

The possibility of electron-positron pair production by means of powerful laser radiation has been discussed in the literature for a long time. In the focal region of laser radiation, the straight "pulling" of pairs from vacuum is possible [5,6]. The electric field E of the laser will form electron-positron pairs, if the work done by it upon virtual particles at a distance of the order of the Compton length ($\hbar/m_0c$) is of the order of the pair energy $2m_0c^2$, where $\hbar$ is Planck's constant, m_0 is the rest mass of the electron, and c is the speed of light. Then, for the characteristic electric field we find

$$E \approx 2m_0^2c^3/e\hbar \approx 10^{16}\ \text{V/cm} , \tag{1}$$

whereas the corresponding laser intensity in vacuum is $I \approx 10^{30}$ W/cm^2. Here, e is the magnitude of the electron charge. In Ref. [7] it has been shown that due to the ponderomotive potential the value of the electric field necessary to produce the pairs increases further. Accounting for this fact and the small probability of the process, one can conclude that pair production in vacuum is practically impossible in the field of laser radiation.

However, focused laser light creates hot plasmas, which can be used as a practical medium for pair production. In fact, laser-plasma interactions can produce high-energy electrons. When the electron kinetic energy exceeds the pair-production threshold $2m_0c^2$, the fast electrons can produce electron-positron pairs by scattering in the Coulomb potential of a nucleus. A number of authors [8,9] have presented a preliminary discussion of pair production by relativistic electrons accelerated by an intense laser focus. In this scheme, the radiation field during the time ($\approx 1/\omega_0$, where the laser frequency ω_0 is of the order of the electron plasma frequency ω_{pe}) transfers the energy,

$$\varepsilon = m_0c^2\gamma_{os} , \tag{2}$$

where

$$\gamma_{\cos} = (1+e^2E_0^2/m_0^2c^2\omega_0^2)^{1/2} , \tag{3}$$

to initially nonrelativistic electrons, where E_0 is the electric-field amplitude of the laser pulse. When the oscillatory electron energy ε exceeds $2m_0c^2$, then the accelerating electrons produce an electron-positron pair by scattering off heavy ions. Accordingly, the threshold value for the laser radiation intensity is

$$I_{th} \simeq \frac{2\times 10^{19}}{\lambda_0}\ \text{W/cm}^2 , \tag{4}$$

where $\lambda_0 = 2\pi c/\omega_0$ is the radiation wavelength, measured in micrometers. The volume, in which the electron-positron pairs are produced, is determined by the spatial size of the focused pulse. Because of the small volume, the number of produced pairs should also be small. On the other hand, the number of electron-positron pairs produced due to the electron bremsstrahlung [10] (electrons accelerating in the ion field emit high-energy photons, which are then transformed into electron-positron pairs) should be small, since this process consists of two stages.

In this paper, we present an efficient mechanism for producing electron-positron pairs in a transparent plasma. Specifically, we demonstrate the possibility of pair production by the accelerated electrons in a strong electric field of the electron plasma wave, which is driven by an intense short laser pulse.

The paper is organized as follows. In Sec. II we present the underlying physical principles of the wake-field generation and pair production. In Sec. III the theory is developed. Finally, Sec. IV contains a summary of our results.

II. PHYSICAL PRINCIPLE

At present, new plasma-based methods for the acceleration of electrons by strong electron plasma waves are being intensively developed [11–15]. A large-amplitude electron plasma wave can be excited by a single-frequency short laser pulse [11–14] as well as by relativistic electron beams [15]. In fact, the authors of Refs. [12–14] have proposed the use of ultrarelativistic laser pulses (viz., $eE_0/m_0\omega_0 c > 1$) for generating ultrastrong gradients of the wake field. It has been demonstrated that in a transparent plasma, $\omega_0 \gg \omega_{pe} = (4\pi n_0 e^2/m_0)^{1/2}$, where n_0 is the unperturbed electron number density, an ultrashort laser pulse (or the pulse with narrow fronts) of $\tau \approx \omega_{pe}^{-1}$ duration would generate a longitudinal wake field behind itself. The maximum potential difference in the wake field is reached when $\gamma_{os} \gg 1$, and has the value

$$\Delta\phi \simeq \frac{m_0 c^2}{e}\gamma_{os}^2 , \tag{5}$$

provided that the phase velocity of the plasma wave is close to c. In such strong potential fields, the electrons can gain the energy

$$\varepsilon_{\parallel} = m_0 c^2 \gamma_{\parallel} , \tag{6}$$

where the relativistic γ factor associated with the longitudinal waves is

$$\gamma_{\parallel} = (1 + p_{\parallel}^2/m_0^2 c^2)^{1/2} . \tag{7}$$

Here $p_{\parallel}$ is the electron longitudinal momentum. We note that for relativistically strong radiation $\gamma_{\parallel} \gg \gamma_{os}$.

In Ref. [14] it has been shown that some relativistic laser pulses ($\gamma_{os} \gg 1$) separated from each other by a certain time interval can generate an ultrarelativistic plasma wave with the following potential difference:

$$\Delta\phi \gg m_0 c^2/e , \tag{8}$$

while the energy of longitudinal oscillations of the electrons in the wake field can reach

$$\varepsilon_{\parallel} \simeq m_0 c^2 \gamma_{os}^{2n} , \tag{9}$$

where n is number of the pulse. Thus, for example, when two similar short laser pulses with $\gamma_{os} = 10$, a front duration $\tau_0 \le 0.1\omega_{pe}^{-1}$, and a time delay between the pulses $t_0 \approx 20\omega_{pe}^{-1}$ propagate into a transparent plasma, then a strong wake field is generated and in its field the maximum oscillation energy of the plasma electrons would be of the order of 200 $m_0 c^2$.

Thus, in the longitudinal wake field, the electrons can have ultra relativistic energy, which is, however, much higher than what they would directly gain in the region of laser pulse localization. If the electron energy in the wake field exceeds the threshold value $3m_0c^2$, one can then conclude that due to the electron scattering by heavy ions, an electron-positron pair will be created not only in the laser pulse region, but also behind it; namely, in the wake-field region. Because of the large extent of the wake field, the total number of pairs will be determined by the volume occupied by the wake field rather than by the laser pulse.

III. THEORETICAL CONSIDERATION

The pair concentration produced, as a result of the electron scattering on the ions, is found from the equation

$$\frac{dn_p}{dt} = \sigma_T n_e n_i v_e , \tag{10}$$

where n_e and n_i are the electron and ion concentrations, respectively, v_e is the electron velocity, and σ_T is the total cross section of the so-called trident electron-positron pair-production process [16]. The following approximate formula has been derived in Ref. [9] for the cross section

$$\sigma_T \cong 9.6\times 10^{-4}(Zr_0/137)^2(\gamma-3)^{3.6} , \tag{11}$$

where $r_0 = 2.8\times 10^{-13}$ cm is the classical electron radius, Z is the ion nuclear charge, and

$$\gamma = (1 + p^2/m_0^2 c^2)^{1/2} . \tag{12}$$

It should be noted that, according to the results of Ref. [16], the nuclear charge screening by the external ion and electrons seems to only slightly affect the scattering process. Equation (11) gives a good approximation at $\gamma \le 10$. For larger γ values, we shall use the well-known expression [16,17]

$$\sigma_T = (28/27\pi)(Zr_0/137)^2(\ln\gamma)^3 . \tag{13}$$

Figure 1 gives the dependence of the cross section of the trident process on $\gamma - 3$, which is in agreement with the matching of the expressions (11) and (13).

To describe the wake-field generation by a single-frequency short laser pulse, we follow Refs. [12] and [14]. In order to carry out the numerical analysis, we shall confine ourselves to considering a one-dimensional case. Besides, we will neglect the change of the pulse profile. Then the equation, describing the wake-field generation by a circularly polarized laser radiation, takes the form

$$\frac{d^2\phi}{d\xi^2}=(\omega_{\rm pe}^2/c^2)\gamma_g^2\{(v_g/c)(1+\phi)[(1+\phi)^2-\gamma_{\rm os}^2(\xi)/\gamma_g^2]^{-1/2}-1\}\ , \quad (14)$$

where

$$\gamma_g=(1-v_g^2/c^2)^{-1/2}\ , \quad (15)$$

and

$$\xi=z-v_g t\ . \quad (16)$$

Here, $\gamma_{\rm os}$ is determined from (3), ϕ is normalized by m_0c^2/e, v_g is the group velocity of the laser pulse, which is close to the speed of light, and γ_g is estimated by the following expression [18]:

$$\gamma_g\simeq(\omega_0/\omega_{\rm pe})\gamma_{\rm os}^{3/4}\ . \quad (17)$$

In order to determine the electron-positron pair concentrations, we make use of (12), (15), and (16) into (10) to obtain

$$\frac{dn_p}{d\xi}=\left[1-\frac{1}{\gamma^2}\right]^{1/2}\left[1-\frac{1}{\gamma_g^2}\right]^{-1/2}n_0 n_e \sigma_T\ . \quad (18)$$

While deriving (14) and (18), the characteristic process time was considered to be $t \gg \omega_{pi}^{-1}$, where ω_{pi} is the ion plasma frequency, so that the ions are at rest. That means $n_i=n_0$. Using the Maxwell and relativistic hydrodynamic equations for the electron motion written in the ξ-frame of reference [e.g., Eq. (16)], we have

$$\gamma=[(1+\phi)^2+(v_g/c)^2\gamma_{\rm os}^2(\xi)]/\{1+\phi+(v_g/c)[(1+\phi)^2-\gamma_{\rm os}^2(\xi)/\gamma_g^2]^{1/2}\} \quad (19)$$

and

$$\frac{n_e}{n_0}=1+\frac{c^2}{\omega_{\rm pe}^2}\frac{d^2\phi}{d\xi^2}\ . \quad (20)$$

We consider a transparent plasma in which $\omega_0>\omega_{\rm pe}$. Assuming $\gamma_g \gg \gamma_{\rm os}$, we obtain from (14) the equation

$$\frac{d^2\phi}{d\xi^2}=(\omega_{\rm pe}^2/2c^2)[\gamma_{\rm os}^2(\xi)(1+\phi)^{-2}-1]\ , \quad (21)$$

which has been investigated in Refs. [12–14]. According to the results of these papers, it follows that for the laser pulse with $eE_0/m_0\omega_0 c>1$ and the characteristic width (or the front width) $\Delta l \le c/\omega_{\rm pe}$ the longitudinal wake field excited behind the pulse has the parameter

$$\phi_{\rm max}=\gamma_{\rm os\ max}^2-1,\quad \phi_{\rm min}=\gamma_{\rm os\ max}^{-2}-1\ , \quad (22)$$

and the extent of the wake field is

$$\lambda=4\gamma_{\rm os\ max}(c/\omega_{\rm pe})E(s)\ , \quad (23)$$

where $s=(1-1/\gamma_{\rm os\ max}^4)^{1/2}$, and $E(s)$ is the elliptic integral of the second type. For $\gamma_{\rm os} \gg 1$, we have

$$\lambda\approx 4\gamma_{\rm os\ max}(c/\omega_{\rm pe})\ . \quad (24)$$

The maximum values of the density perturbation and the electron longitudinal energy in the wake field are, respectively,

$$\delta n_e/n_0=(\tfrac{1}{2})\gamma_{\rm os,max}^4\ , \quad (25)$$

and

$$\varepsilon_{\parallel\,\rm max}/m_0c^2=(\tfrac{1}{2})\gamma_{\rm os\ max}^2\ . \quad (26)$$

We note that a previous simulation work [19] has already found Eq. (26).

We suggest that the distance of the laser pulse displacement without substantially changing its form is determined by the depletion of the pulse energy due to the loss by the excitation of the wake field. It is easy to show that the order of magnitude of this distance is [18]

$$L_D\approx(\omega_0/\omega_{\rm pe})^2 c\tau\ , \quad (27)$$

where τ is the pulse duration.

Let us now estimate the maximum concentration of the electron-positron pairs produced. For $\gamma_{\rm os}\ge 3$, the pairs will be produced in the regions occupied both by the laser pulse and by the wake field. The maximum concentration value should be expected at a distance of the order of L_D from the laser pulse. For $\gamma_g, \gamma_{\rm os\ max} \gg 1$, one can show that

$$n_{p\ \rm max}({\rm cm}^{-3})\approx 10^{-22}n_0 n_c \tau(\gamma_m-3)^{3.6}Z^2\ , \quad (28)$$

for $\gamma_m \le 10$, and

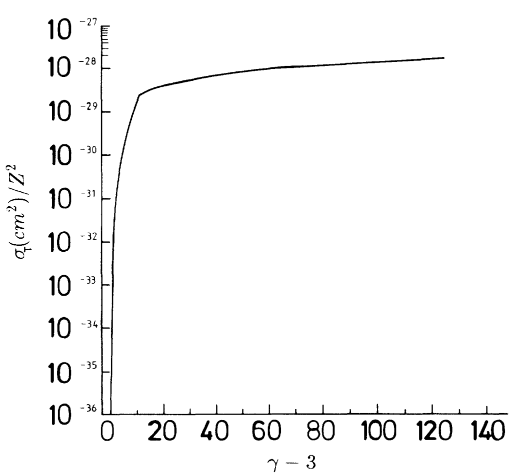

FIG. 1. Total cross section σ_T of the trident process plotted vs the dimensionless electron-energy excess above the threshold. Here $\gamma=E/m_0c^2$.

$$n_{p\,\max}(\mathrm{cm}^{-3}) \approx 4\times 10^{-20} n_0 n_c \tau (\ln\gamma_m)^3 Z^2 , \qquad (29)$$

for $\gamma_m > 10$. Here, γ_m ($>\gamma_{os\,\max}$) is the maximum value of the relativistic factor for electrons in the wake field, and $n_c = \omega_0^2 m_0/4\pi e^2$ is the plasma critical density. In (28) and (29), the number densities are measured in cm^{-3} and τ in seconds.

Since in a transparent plasma, $\omega_0 \gg \omega_{pe}$, it follows from (27) that the length L_D of the stationary wake field is much larger than the laser pulse length. Therefore, the total number of electron-positron pairs will be mainly determined by the volume occupied by the wake field. If the laser and wake fields are limited by the area S in the perpendicular plane, then for the total number of pairs formed at the length L_D, we have

$$N_{p\,\max} \simeq 3\times 10^{-12} n_c^2 \tau^2 S(\gamma_m - 3)^{3.6} Z^2 , \qquad (30)$$

for $\gamma_m \le 10$, and

$$N_{p\,\max} \simeq 1.3\times 10^{-9} n_c^2 \tau^2 S(\ln\gamma_m)^3 Z^2 , \qquad (31)$$

for $\gamma_m > 10$. Consequently, the total number of pairs produced by the ultrarelativistic wake field in the plasma is independent of the plasma number density and is determined by the laser pulse duration and the electron energy in the wake field.

Figure 2 shows the potential distribution in the stationary wake field, calculated from (14). The pulse at the beginning of the wake field corresponds to the chosen form of the transverse field localization of the laser pulse. For the oscillating relativistic factor [Eq. (3)], the maximum value of $\gamma_{os\,\max} = 10$ is given in the laser pulse. As is seen from Fig. 2, the maximum value of the normalized potential in the generated wake field is strongly relativistic, and is $\Phi_{\max} = e\phi_{\max}/m_0c^2 = 33$. Figure 3 shows the distribution of the electron-positron pair concentration in the wake field, corresponding to the solution of (18) in which the pair-production cross section , given in Fig. 1, is used.

As Fig. 2 exhibits, the periodic distribution of the potential in the wake field has peaklike minima. Therefore, at the latter, according to (19) and (20), the electron concentration and energy should have sharp maxima [at the points of the potential minimum, the denominator of (19) takes the least value]. The jumps in the electron-positron pair concentration in Fig. 3 are explained by a rapid increase of the electron concentration and energy at the points of the potential minimum in the wake field.

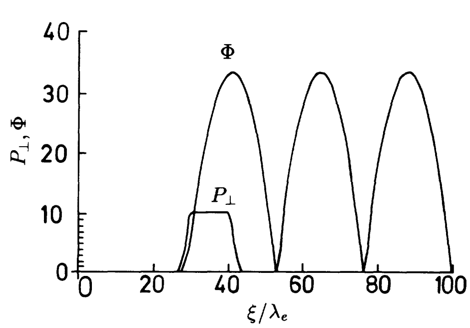

FIG. 2. Spatial distributions of the laser pulse (shown as a rectangular shape) and the dimensionless wake-field potential (shown as a chain of three strong pulses) vs ξ/λ_e. Here $P_\perp = p_\perp/m_0c \equiv eE_0/m_0c\omega_0$, $\Phi = e\phi/m_0c^2$, $\xi = z - v_g t$, and $\lambda_e = c/\omega_{pe}$.

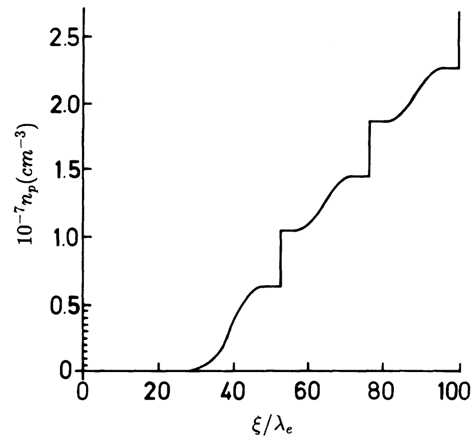

FIG. 3. Spatial distribution of the pair concentration vs ξ/λ_e.

Let us estimate the number of pairs which can be formed in the wake field while using the laser proposed to be built at Livermore [3]. The parameters of this laser are as follows: the pulse duration $\tau \approx 1$ psec., the maximum oscillatory relativistic factors of the electrons, $\gamma_{os\,\max} \approx 27$, and the focal spot diameter $r_s \approx 10\ \mu$m. According to (30) and (31), in a plasma with concentration $n_0 \approx 9.1\times 10^{17}\ \mathrm{cm}^{-3}$ at length $L_D \approx 30$ cm, the maximum number of electron-positron pairs will be of the order of $10^6 Z^2$, while for the maximum value of the electron-positron pair concentration we obtain $n_{p\,\max} \approx 3\times 10^{10} Z^2$ cm^{-3}.

The above estimates are based on the one-dimensional model of wake-field generation. This is justified for the case in which the transverse scale of the localization of the wake field is much larger than its wavelength. In estimating the total number of electron-positron pairs, the transverse dimension of the wake field is limited by the size of the laser focal spot. Although the transverse effect in conjunction with relativistic nonlinearity may provide the possibility of self-focusing and the guiding of intense laser beams, the present scenario of one-dimensional wake-field generation ought to be reexamined for a multidimensional situation. In the latter case, if the strength of plasma waves becomes stronger, one could expect the total number of pairs to be increased even further.

Emission of hard x-rays must also take place due to the bremsstrahlung of the electrons in the wake field. To distinguish the wake-field x-rays from those of electron-positron pair annihilation, a more detailed analysis is required, taking into account the distribution of pairs with respect to their energies.

IV. SUMMARY AND DISCUSSION

To summarize, we have presented a mechanism for creating electron-positron pairs in a transparent plasma.

Specifically, it has been demonstrated that the ultrastrong electric field of the plasma wave, which is driven by an intense short circularly laser pulse, can accelerate an electron to relativistic velocities. The fast electron can be scattered off in the Coulomb potential of the stationary positive ions, thereby producing an electron-positron pair via the trident process.

For a given intense-laser-pulse distribution, we have calculated the wake-field profile and have used the corresponding result to evaluate the number of electron-positron pairs. It has been found that the latter are independent of the plasma number density, but they are solely determined by the laser pulse duration and the electron energy in the wake field.

In this paper, we have not discussed annihilation of electrons and positrons, which is the analog of recombination in plasmas composed of electrons and ions. However, this process is relatively less important even at an electron density of 10^{12} cm^{-3} and a temperature as low as 1 eV, because the positron annihilation time is greater than 1 sec [20].

In conclusion, we emphasize that the future perspective of an efficient electron-positron pair-production scheme, as discussed here, may rest on the development and success of plasma-based laser-wake-field electron accelerators. If they can be made to work, then an experimental endeavor for creating electron-positron pairs in a controlled fashion should be made.

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