Strong squeezing by repeated frequency jumps

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The consequences of sudden frequency changes of a quantum oscillator from ω_1 to ω_2 and back is considered. A strong dependence of squeezing on the time interval between the frequency jumps is found. It is shown that a high degree of squeezing can be achieved by a series of sudden frequency changes.

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In recent years the problem of the generation of squeezed states by a change in the frequency of the harmonic oscillator was widely discussed [1–9] (see also, for other time-dependent oscillators, [10–12]). It was found that any nonadiabatic frequency change leads to squeezing and as shown by Agarwal and Kumar the squeezing is especially pronounced in the case of a sudden frequency jump [9]. In the present paper we shall show that a series of well-timed frequency jumps leads to an even stronger squeezing.

The behavior of a quantum oscillator with time-dependent frequency and unit mass is described by the Hamiltonian

$$H(t)=\tfrac{1}{2}P^2+\tfrac{1}{2}\omega^2(t)Q^2 \ . \tag{1}$$

Here Q and P are the coordinate and momentum operators.

In [1] we considered a harmonic oscillator having frequencies ω_1 at $t<0$ and ω_2 at $t>0$. Introducing the creation and annihilation operators ($\hbar=1$)

$$\begin{aligned} a_j^\dagger&=\frac{1}{\sqrt{2}}\left[\sqrt{\omega_j}Q-\frac{i}{\sqrt{\omega_j}}P\right],\\ a_j&=\frac{1}{\sqrt{2}}\left[\sqrt{\omega_j}Q+\frac{i}{\sqrt{\omega_j}}P\right],\quad j=1,2\ , \end{aligned} \tag{2}$$

we found the solution for the operator $a_2(t)$ at $t>0$ in the Heisenberg picture [1,5]

$$\begin{aligned} a_2(t)&=\left[\sqrt{\omega_2}Q(0)+\frac{i}{\sqrt{\omega_2}}P(0)\right]e^{-i\omega_2 t}\\ &=[ua_0+va_0^\dagger]e^{-\omega_2 t}\ , \end{aligned} \tag{3}$$

$$\begin{aligned} &u=\frac{\omega_2+\omega_1}{2\sqrt{\omega_2\omega_1}},\quad v=\frac{\omega_2-\omega_1}{2\sqrt{\omega_2\omega_1}}\ ,\\ &u^2-v^2=1,\quad a_0\equiv a_1(-0)\ . \end{aligned} \tag{4}$$

Considering the simplest case when the oscillator at $t=-0$ was in its ground state, defined by $a_1|0\rangle_{\omega_1}=0$, we find for the variances of the operators $X_+^{(2)}=a_2+a_2^\dagger$ and $X_-^{(2)}=-i(a_2-a_2^\dagger)$ just after the frequency jump,

$$\Delta X_+^{(2)}=(\omega_2/\omega_1)^{1/2},\quad X_-^{(2)}=(\omega_1/\omega_2)^{1/2}\ , \tag{5}$$

and also the variances of the coordinate and the momentum at $t>0$,

$$\Delta Q=\frac{1}{2\sqrt{\omega_2}}\left[\frac{\omega_2}{\omega_1}(1+\cos2\omega_2 t)+\frac{\omega_1}{\omega_2}(1-\cos2\omega_2 t)\right]^{1/2}\ , \tag{6}$$

$$\Delta P=\frac{\sqrt{\omega_2}}{2}\left[\frac{\omega_1}{\omega_2}(1+\cos2\omega_2 t)+\frac{\omega_2}{\omega_1}(1-\cos2\omega_2 t)\right]^{1/2}\ . \tag{7}$$

while for $t<0$, $\Delta Q=1/\sqrt{2\omega_1}$ and $\Delta P=\sqrt{\omega_1/2}$.

It should be noted that though the state is squeezed for the new potential with frequency ω_2, an immediate frequency jump back to the original frequency would destroy any squeezing and the state would be in its ground state again.

Let us now consider a second frequency jump from ω_2 back to ω_1 after some time delay τ. Analogously with Eq. (3) we find

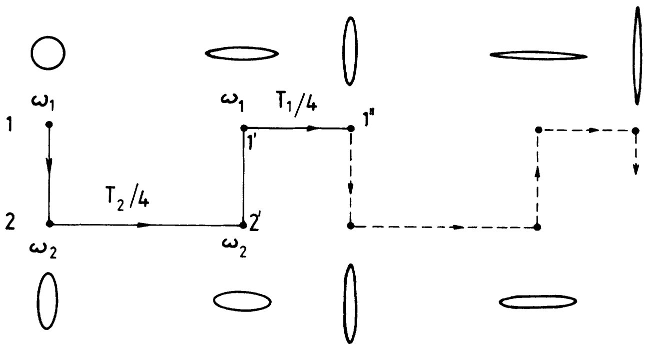

FIG. 1. Sudden change of frequency from (1) to (2) leads to squeezing for a state originally in the ground state. The corresponding uncertainties are shown as a circle and standing oval. An immediate jump back would cancel squeezing and restore the ground state. After waiting a quarter of a period of the new frequency the squeezing changes to its opposite value (from standing to lying oval at 2′). A sudden frequency jump back to its old value enhances the squeezing (more elongated oval lying at 1′). Waiting a quarter of the old period completes the cycle from (1) to (1″), which can be repeated many times (dashed line) generating very strong squeezing.

$$a_1(t,\tau)=(u'a_0+v'a_0^\dagger)e^{-i\omega_1(t-\tau)},$$
$$u'=\cos\omega_2\tau-1(u^2+v^2)\sin\omega_2\tau, \quad (8)$$
$$v'=-2iuv\sin\omega_2\tau,\quad t\geq\tau.$$

According to (8) no squeezing is left, if the time interval between the frequency jumps was $\tau=mT_2/2$, where $T_2=2\pi/\omega_2$ is the period with ω_2 and $m=0,1,2,\ldots$. On the contrary, the squeezing increases if

$$|v'|^2>v^2,\quad \text{i.e., } \sin^2\omega_2\tau>1/4u^2. \quad (9)$$

The most enhanced squeezing occurs at $\tau=T_2/4$ or at any $\tau=(m+\frac{1}{2})T_2/2$. At maximal squeezing $(\tau=T_2/4)$ the uncertainties are $\Delta X_+^{(1)}=(\omega_1/\omega_2)$ and $X_-^{(1)}=(\omega_2/\omega_1)$.

We notice that after waiting a quarter of a period of frequency ω_1 (i.e., $T_1/4$) we complete a cycle which can be repeated again and again (Fig. 1). Just after n cycles we have for the annihilation operator

$$a_1(nT)=u_na_0+v_na_0^\dagger,\quad T=(T_1+T_2)/4,$$
$$u_n=(-1)^n\frac{\omega_2^{2n}+\omega_1^{2n}}{2\omega_2^n\omega_1^n}, v_n(-1)^n\frac{\omega_2^{2n}-\omega_1^{2n}}{2\omega_2^n\omega_1^n}, \quad (10)$$

and the corresponding uncertainties are

$$\Delta X_\pm^{(1)}=(\omega_2/\omega_1)^{\pm n}. \quad (11)$$

This way we showed that by well-timed jumps to a new frequency and back one can achieve a very high degree of squeezing.

The excitation of an oscillator by a series of sudden frequency jumps is a purely quantum-mechanical process; as in the case of a classical oscillator originally in its ground state the frequency jumps would leave the system unchanged, while the same jumps excite the quantum oscillator rather considerably due to the amplification of the zero-phonon energy.

It should be noticed that the above-described method of squeezing can be used for practical applications, for example, in squeezing trapped ions by sudden changes of their confinement.

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