Two-dimensional optical lattices in a CO_2 laser

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(Received 24 July 1992)

Using a cavity with a large Fresnel number, a nontrivial large-scale order resulting from global multimode-interaction dynamics is found in the transverse structure of a CO_2 laser beam. Two-dimensional optical lattices displaying several hundred phase singularities each have been observed. The symmetry properties of these far-field patterns and their associate temporal spectra are studied as a function of two control parameters: the Fresnel number and the transverse intermode spacing.

PACS number(s): 42.55.Lt, 42.60.Jf

The problem of turbulence remains a major challenge in physics and has recently received extensive interest. The generalization of the nonlinear-dynamics approach has shown that this topic is not restricted to hydrodynamics but also concerns chemical reactions [1], optics [2], etc. In optical dynamical systems, interest has moved recently from purely temporal instabilities in which the total light intensity emitted by or transmitted through a system is measured, to spatio-temporal instabilities. This latter field covers pattern generation in optics and optical turbulence. The optical domain has some practical properties as short characteristic times that make it particularly attractive for the study of the onset of turbulence. Moreover, the question of optical turbulence in lasers is of general interest as shown, for instance, by the correspondence between the Maxwell-Bloch and Ginzburg-Landau equations, which was demonstrated by Coullet, Gil, and Rocca [3]. Optical turbulence has recently been investigated experimentally and completely disordered patterns were observed in photorefractive oscillators [2], in a liquid-crystal device with optical feedback [4], and in CO_2 lasers [5–7].

Increasing the number of degrees of freedom of a system does not necessarily lead to disorder. In this article, we show that it can also give rise to nontrivial large-scale ordered structures involving a large number of modes, contrary to previous observations in lasers [8] where only a small number of modes was involved. We stress that the patterns reported here are not the standard high-order modes of empty cavities but result from global multimode dynamics.

We have used a linear nearly degenerate CO_2-laser cavity [9] with an intracavity lens [5–7] (see Fig. 1). The cavity length is varied on both sides of the critical value of the purely degenerate case. The transverse pattern of the laser is monitored by a phosphorescent plate which can detect only slow time variations ($\sim$0.5 s) or by two Hg-Cd-Te detectors when a fast description of local intensities is required.

Two parameters play a dominant role in the onset of transverse patterns: (i) the Fresnel number N controls the transverse size of the dynamical system and consequently the number of degrees of freedom, and (ii) the transverse intermode spacing $\Delta\nu_T$ governs the coupling strength between the modes of the active cavity and thus the influence of the nonlinearities on the dynamical behavior of the system. The change of $\Delta\nu_T$, while keeping N constant, imposes a simultaneous variation of two main cavity parameters: its length and the position of the lens inside it. N is changed either by moving both the plane mirror and the lens or by means of an iris inserted inside the cavity close to the spherical mirror. Note that $\Delta\nu_T$ is not only a function of the lens position but also depends on the astigmatism of the optical elements, particularly on the symmetry breaking induced by the Brewster windows.

In a symmetric confocal resonator, the ratio N of the resonator surface area to the TEM_{00} mode area on one of the mirrors differs from the standard definition of the Fresnel number by a factor of π. N is a measure of the number of transverse modes allowed to oscillate without significant perturbation by the aperture diameter Φ. In each optical arrangement, N is calculated using the largest beam size inside the cavity, and will be called hereafter the "effective" Fresnel number.

When ϵ and η (with $|\epsilon| \ll R+f$ and $|\eta| \ll f+f^2/R$ as defined on Fig. 1) are chosen to obtain equal spot sizes on both the lens and the spherical mirror, N reaches a value N_d that may be approximated by

$$N_d \approx \frac{\pi\Phi^2}{4\lambda R}\sqrt{(R-f)/(R+f)} ,$$

where λ is the laser beam wavelength. It can easily be shown that the maximum N_d value is obtained when the ratio R/f is the golden mean.

The main interest of this cavity geometry is to keep N as large as possible in a nearly degenerate configuration. This occurs because in such conditions, the transverse intermode spacing $\Delta\nu_T$ may be much narrower than the gain bandwidth $\gamma_\perp/\pi$ ($\sim$300 MHz) in which the field is amplified thus allowing for large-scale multimode operation ($\gamma_\perp$ is the transverse relaxation rate). Indeed, near the degenerate configuration, N typically varies from 1 to 36 and the corresponding transverse-mode spacing $\Delta\nu_T$ is in the range 0–3 MHz, depending on the cavity length

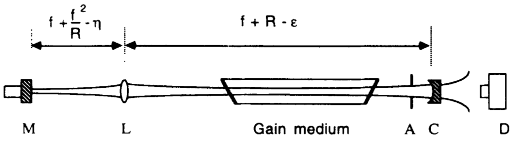

FIG. 1. Schematic representation of the CO_2 hemispherical Fabry-Pérot laser cavity. M, plane mirror; L, lens (focal $f=300$ mm); A, aperture ($1<\Phi<24$ mm); C, spherical mirror (radius of curvature: $R=1$ m); D, Hg-Cd-Te detector or phosphorescent plate filmed with a video camera. In the purely degenerate cavity, the cavity length $L_0\approx 1.69$ m and $\Delta\nu_L\approx 89$ MHz.

and on the intracavity lens position. Moving away from the degenerate case and using other focal lengths allows $\Delta\nu_T$ to be changed by up to a third of the free spectral range of the cavity ($\Delta\nu_T=36$ MHz).

The transverse structure of the output beam of the CO_2 laser evolves with N and $\Delta\nu_T$ as control parameters. We have observed a succession of spatially well-ordered transverse patterns whose complexity increases with N. These structures (see Fig. 2) are very different from the monomode structures associated with Hermite or Laguerre-Gauss functions in an empty cavity as they exhibit complex temporal spectra and are not stationary. When $\Delta\nu_T$ is reduced from a few MHz to zero, two different regions are encountered: a region ($\Delta\nu_T>1$ MHz) of weak to moderate coupling between modes in which optical lattices appear and a region ($\Delta\nu_T<1$ MHz) of strong-mode coupling in which the lattices collapse. Let us first focus our attention on the first region.

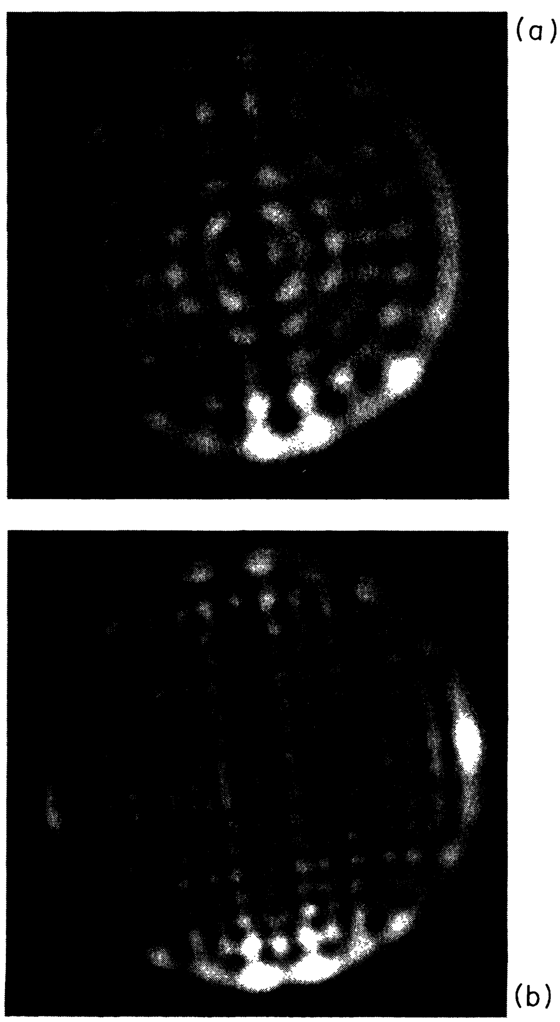

FIG. 2. Far-field transverse structure of the output beam monitored on a phosphorescent plate: (a) circular lattice at small effective Fresnel number ($N=20$, $\Delta\nu_T=5$ MHz), (b) square lattice at large effective Fresnel number ($N=30$, $\Delta\nu_T=3.7$ MHz).

The outer side of the pattern is always governed by the boundary conditions and appears as a succession of bright spots on a ring while the inner part strongly evolves with N. Depending on the fine adjustment of the angular and longitudinal positioning of one of the end mirrors, the transverse structure may appear sharp or blurred. At low N, the simplest sharp structures show a breaking of the cylindrical symmetry similarly to those observed and calculated by Brambilla *et al.* [8] in the Na_2 laser. When N is increased, more complex sharp patterns are regularly set out on successive concentric rings of bright and dark spots [Fig. 2(a)]. The dark spots which appear between the bright rings are a signature of the presence of a great number of phase singularities. Up to 10 concentric rings have been observed when N approaches 30. Each ring contains $2(2l+i+1)$ equidistant phase singularities, where l is the ring index and $i=0$ or 1 depending on the experimental conditions. We have checked by moving the lens along the axis of the cavity, that the number of rings grows proportionally to N and consequently that the number of phase singularities increases as N^2. At large N more than 220 phase singularities have been observed. When the iris placed close to the spherical mirror is progressively closed, in addition to the trivial reduction of the number of rings, we have observed, as is the case with single-mode structures, a noticeable increase of the distance Δr between the remaining rings ($\Delta r\sim N^{-0.5}$), and that the central part of the pattern acts as a well where the rings disappear after a continuouslike change in the structure.

The symmetry properties of the large-scale structures are spontaneously modified at large N. The central part of these structures is then organized in a square lattice [Fig. 2(b)] whose axes are determined by the orientation of the Brewster windows. The patterns distort progressively near the edges of the cavity where the circular symmetry is restored. This shows that Brewster windows play a key role in the appearance of square lattices. The symmetry properties of the patterns evolve as one Brewster window is rotated with respect to the other one, and it becomes more difficult to obtain rings by adjusting the cavity mirror when the angle between the windows exceeds 20°. When the spherical mirror is slightly tilted, a strain is applied on the beam whose transverse pattern is hence modified. Some patterns display fixed defects (Fig. 3) appearing by pair and having a hexagonal or pentagonal shape. They are often symmetrically located with respect to an axis of the structure. Due probably to mechanical instabilities, these defects may fluctuate be-

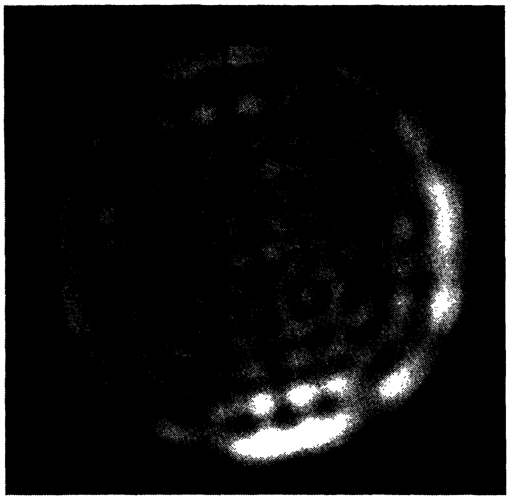

FIG. 3. Defects in the transverse pattern of a CO_2-laser beam.

tween two different positions.

Thermal-plate observation gives only access to the average intensity pattern. To compare more carefully sharp and blurred patterns, we have studied the local signals at two different points of the transverse structure. They depend strongly on the point where they are taken from and show irregular pulsations superimposed on a large-scale cw component. The power spectrum of blurred patterns does not show any continuous background, but exhibits a great number (~ 100) of moving spectral components in the range 0–15 MHz. At large N, even if a few families of Gauss-Laguerre modes could oscillate simultaneously, the homodyne radio-frequency spectrum would display an enormous amount of spectral components owing to the degeneracy lift induced by the symmetry breaking appearing in the system. Here, it is clear that the modal expansion is no longer relevant, and that nonlinear interactions strongly couple modes to each other and lead to frequency locking and pulling. For instance, when the transverse pattern appears blurred, no more than 100 separate spectral components between 0 and 15 MHz can be resolved with a spectrum analyzer whose selectivity is 12.5 kHz even though the bandwidth of the system is much larger [Fig. 4(a)]. We have observed that blurring is often associated with multiline operation. The laser emits randomly between 10P16 and 10P24 (P branch of the 10-μm band of the CO_2 molecule), but single-line operation, which is strongly correlated with sharp patterns, can be restored through cavity-length tuning. As soon as a sharp pattern appears, these spectral components are subjected to cluster formation [Fig. 4(b)], with a power spectrum which does not depend on the measurement point. Depending on the N value, between 5 to 10 spectral clusters, regularly separated by a distance close to 500 kHz, are spontaneously formed. The width of the clusters is found to be 50–150 kHz, i.e., much larger than the resolution of the spectrum analyzer [see Fig. 4(b)]. The clusters are likely to contain spectral components separated by a low frequency ($f_c < 50$ kHz). A strong analogy exists between these clusters and the phenomenon of mode frequency locking described by Lugiato, Oldano, and Narducci [10]. In the latter case, nearly frequency degenerate modes lock onto a common frequency. In our case, a very large amount of modes with different frequencies gathers in a small number of equidistant clusters. Although there is no clear interpretation for the appearance of this ~ 500-kHz frequency, it should be noted that it is of the order of magnitude of the degeneracy lift induced by symmetry breaking and close to the relaxation frequency of the laser.

Let us now focus our attention on the region of strong-mode coupling. When the cavity length is fixed near the critical length to reduce the mode spacing ($\Delta\nu_T < 1$ MHz) at large N, a wide spot with an irregular shape is obtained. When the average symmetry is lost, the homodyne spectrum fluctuates and its components no longer associate in clusters. In these conditions, one observes a random succession of temporary lockings to form large irregular-shaped clusters distant either by 500 kHz or harmonics thereof. This time-dependent spectrum is a signature of a very slow dynamics compared to the characteristic time of the laser. Such a slowing down is typical of interactions of a very large amount of modes [11]. Reducing the cavity aperture restores order and simpler well-organized patterns, such as a ring with a central dot or a double ring, are observed. The temporal

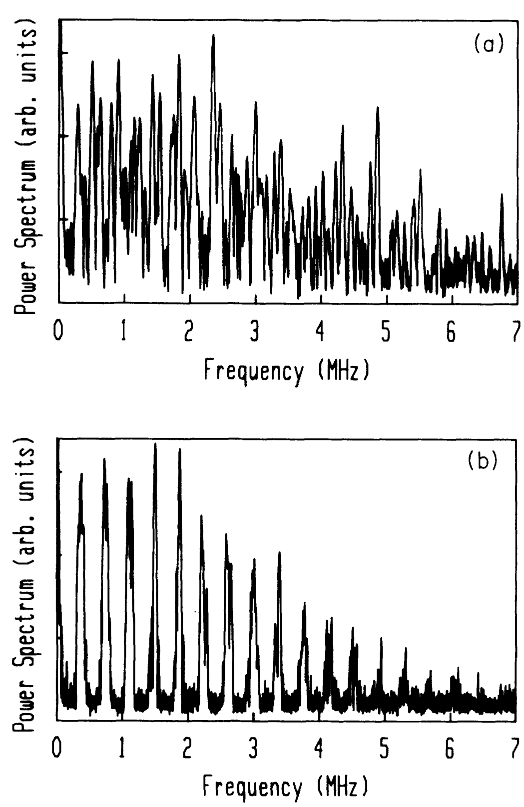

FIG. 4. Local temporal homodyne radio-frequency spectrum of the output beam of the CO_2 laser ($\Delta\nu_T = 19$ MHz): (a) spectrum of a blurred six-ring pattern ($N=20$) (b) spectrum of a sharp four-ring pattern ($N=13$).

spectrum of these structures was always found to be periodic after fine adjustment of the cavity mirrors.

Since this region is near instability, the cavity becomes very sensitive to mechanical vibrations and to air turbulence. In order to check the influence of mode degeneracy on pattern formation, we have investigated situations where the quasidegeneracy of modes is not related to marginal stability of the cavity. These situations are obviously met when $\Delta\nu_T$ is a submultiple (with $n>2$) of the free spectral range. We have studied successively the $n=3$ ($\Delta\nu_T=36$ MHz) and $n=4$ ($\Delta\nu_T=22.5$ MHz) cases. In each case the lens was translated on a wide range and we have found that the pattern loses its regularity as soon as the degeneracy condition is fulfilled, but recovers order on each side of this critical position. The reduction of the transverse size of the system appears to be an order factor as a regular structure is restored if the aperture is progressively reduced. The homodyne spectrum follows the same kind of evolution. We have also observed that the set of spectral components which appears at $\Delta\nu_T$ is subjected to a strong frequency pulling connected to mode interaction. For example, in the $n=3$ case, the center frequency of the components is shifted by more than 7 MHz as the aperture is increased from 6 to 24 mm.

In conclusion, owing to an experimental setup designed to reach large effective Fresnel numbers, we have observed previously unnoticed, nonstationary large-scale well-ordered patterns. They may be considered as a generalization of the phase-singularity crystals discovered by Brambilla *et al.* in the Na_2 laser [8], which contained only a few phase singularities. Their corresponding cluster-shaped spectra is a generalization to a largely multimode system, of the cooperative frequency locking reported by Lugiato, Oldano, and Narducci. Hexagonal or pentagonal structural defects have been observed by changing the limit conditions on the beam edge through a slight detuning of the cavity. One property of these patterns that may be emphasized is their robustness, since apart from N and $\Delta\nu_T$, the other control parameters have a negligible influence on them.

The Laboratoire de Spectroscopie Hertzienne is "Unité de Recherche Associée au Centre National de la Recherche Scientifique (URA249)."

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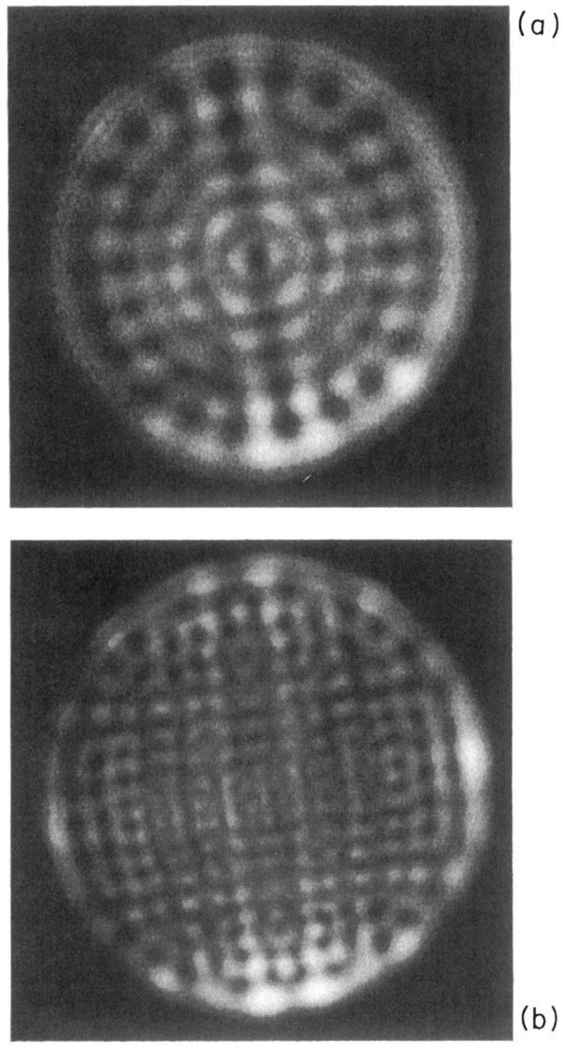

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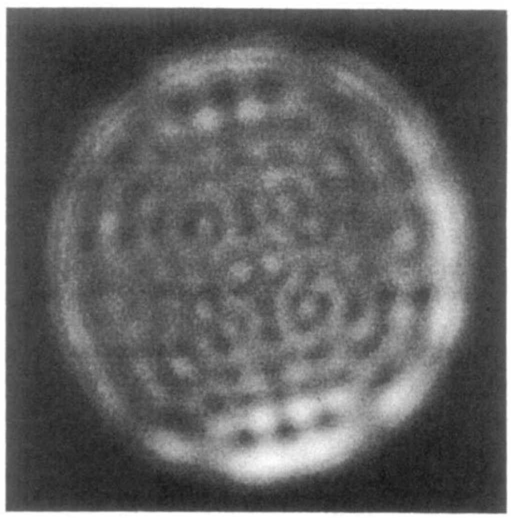

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