Generation of squeezing for a charged oscillator and for a charged particle in a time-dependent electromagnetic field

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(Received 27 April 1992)

Recent papers [A. Bechler, Phys. Lett. A **130**, 481 (1988); M. S. Abdalla, Phys. Rev. A **44**, 2040 (1991)] investigate the generation of squeezing for a charged oscillator in a constant and uniform magnetic field. Here we pursue further this line by investigating the extended system of a charged particle oscillating in a uniform and arbitrary time-dependent magnetic field. Since we do not particularize the functional time dependence, our expressions for the variances of quadratures are quite general, so the possible occurrence and amount of squeezing will depend on that specification. The corresponding results mentioned above become a particularization of ours.

PACS number(s): 42.50.Dv, 84.30.Ng, 03.65.−w

I. INTRODUCTION

Squeezed states of the electromagnetic field and of harmonic oscillators (HO's) have received increasing theoretical [1] and experimental [2] attention in recent years. In this context, recent papers by Bechler [3] and Abdalla [4] investigate the time evolution and statistical properties of two-dimensional charged HO's in the presence of a constant magnetic field. It has been shown that [3,4] in such a situation the HO exhibits squeezing effects in a similar way as one finds for the radiation field interacting with some suitable nonlinear media [1].

In this paper we will pursue further this line by investigating the extended system of a charged HO, and of a charged particle, in the presence of a time-dependent (TD) electromagnetic field. As in the case of the charged HO in the presence of a constant magnetic field, the more general system of a charged HO in the presence of a TD electromagnetic field is a standard textbook problem. However, to our knowledge it has not been considered from the point of view of squeezing properties.

The present paper is arranged as follows. In Sec. II we define the system Hamiltonian. In Sec. III we quantize the system and obtain the Heisenberg equations of motion in terms of the ladder operators $\hat{a}$ and $\hat{a}^\dagger$ for the HO. In Sec. IV we solve the Heisenberg equations of motion for our system and calculate the fluctuations of the quadrature phase amplitudes. Section V contains comments and the conclusion.

II. CHARGED PARTICLE IN A TIME-DEPENDENT ELECTROMAGNETIC FIELD AND TIME-DEPENDENT HARMONIC OSCILLATOR

Let us consider [5] a particle, which may be an isotropic oscillator, of charge **e** and mass M moving in an axially symmetric magnetic field defined by the vector potential

$$\mathbf{A}=\tfrac{1}{2}B(t)\hat{\mathbf{k}}\times\mathbf{r} \tag{2.1}$$

and a scalar potential

$$\Phi=\frac{1}{2}\frac{e}{Mc^2}\eta(t)r^2 , \tag{2.2}$$

where (2.1) is valid if $B(t)$ is uniform. $\hat{\mathbf{k}}$ is a unit vector along the symmetry axis; $r^2=x^2+y^2$, x and y are the two Cartesian components perpendicular to the symmetry axis. $B(t)$ and $\eta(t)$ are arbitrary piecewise-continuous functions of time and c is the speed of light.

The Hamiltonian for the present system is given by [5]

$$\hat{H}=\frac{1}{2M}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2+e\Phi=\hat{H}_\perp+\hat{H}_\parallel , \tag{2.3}$$

where

$$\hat{H}_\perp=\left[\frac{1}{2M}(\hat{p}_x^2+\hat{p}_y^2)+\frac{M}{2}\tilde{\Omega}^2(t)(\hat{x}^2+\hat{y}^2)-\tfrac{1}{2}\omega_c(t)\hat{L}_z\right] , \tag{2.4}$$

$$\hat{H}_{\parallel}=\left[\frac{1}{2M}\hat{p}_z^2\right], \tag{2.5}$$

where $\hat{L}_z$ is the z component of the angular momentum: $L_z=(\mathbf{r}\times\mathbf{p})_z=xp_y-yp_x$, and

$$\tilde{\Omega}(t)=(e/Mc)\{[B(t)/2]^2+\eta(t)\}^{1/2}, \tag{2.6a}$$

$$\omega_c(t)=eB(t)/Mc . \tag{2.6b}$$

Note that $\hat{H}_{\parallel}$ in the Eq. (2.5) exhibits the axial motion of a particle in this field that is trivial, and we will ignore it by treating only the motion which is perpendicular to the symmetry axis. Note also that $[\hat{H}_{\perp},\hat{H}_{\parallel}]=0$, $[\hat{H}_{xy},\hat{L}_z]=0$, where

$$\hat{H}_{x,y}=\frac{1}{2M}(\hat{p}_x^2+\hat{p}_y^2)+\frac{M}{2}\tilde{\Omega}^2(t)(x^2+y^2) . \tag{2.7}$$

As is well known, $\hat{H}_{x,y}$ and $\hat{L}_z$ form a complete set of commuting observables [6].

Since $\hat{H}_{x,y}$ commutes with $\hat{L}_z$ we may set a transformation $|\psi(t)\rangle\rightarrow|\psi_L(t)\rangle=U(t)|\psi(t)\rangle=e^{i\xi(t)L_z}|\psi(t)\rangle$, with $\xi(t)=\frac{1}{2}\int_0^t\omega_c(t')dt'$, to obtain the Schrödinger equation

$$\hat{H}_{\perp}|\psi(t)\rangle=\left[\hat{H}_{x,y}+\frac{\omega_c(t)}{2}\hat{L}_z\right]|\psi(t)\rangle$$

$$=i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle , \tag{2.8}$$

which, in the Larmor frame, is

$$\hat{H}_{x,y}|\psi_L(t)\rangle=i\hbar\frac{\partial}{\partial t}|\psi_L(t)\rangle , \tag{2.9}$$

with $\hat{H}_{x,y}^{(L)}=\hat{H}_{x,y}$. Hence in the Larmor frame we will employ just the Hamiltonian (2.7).

If $\eta\neq 0(\eta=0)$ in Eq. (2.6), then the Hamiltonian (2.7) describes a charged oscillator (charged particle) in the presence of a TD electromagnetic field. Hence (2.7) is an extension of the system investigated by Bechler [3] and Abdalla [4].

III. QUANTIZATION AND HEISENBERG EQUATIONS FOR THE SYSTEM

Next, we introduce the ladder operators $\hat{a}_i,\hat{a}_i^{\dagger}$ (with $q_i=x,y$, $p_i=p_x,p_y$) through the usual canonical transformation

$$\hat{q}_i=\left[\frac{\hbar}{2M\omega_0}\right]^{1/2}\left[\hat{a}_i^{\dagger}+\hat{a}_i\right], \tag{3.1a}$$

$$\hat{p}_i=i\left[\frac{\hbar}{2}M\omega_0\right]^{1/2}\left[\hat{a}_i^{\dagger}-\hat{a}_i\right], \tag{3.1b}$$

with $[\hat{a}_i,\hat{a}_j^{\dagger}]=\delta_{i,j}$, $[\hat{a}_i,\hat{a}_j]=0=[\hat{a}_i^{\dagger},\hat{a}_j^{\dagger}]$.

The substitution of Eqs. (3.1) in Eq. (2.8) gives

$$\hat{H}_{x,y}(\hat{a},\hat{a}^{\dagger},t)=\hbar[f_1(t)(\hat{a}_x^{\dagger}\hat{a}_x+\tfrac{1}{2})+f_2(t)(\hat{a}_x^{\dagger 2}+\hat{a}_x^2)]$$
$$+\hbar[f_1(t)(\hat{a}_y^{\dagger}\hat{a}_y+\tfrac{1}{2})+f_2(t)(\hat{a}_y^{\dagger 2}+\hat{a}_y^2)] \tag{3.2}$$

whereas, for later application in this paper,

$$-\tfrac{1}{2}\omega_c(t)\hat{L}_z=\hbar[if_3(t)(\hat{a}_x^{\dagger}\hat{a}_y-\hat{a}_x\hat{a}_y^{\dagger})] , \tag{3.3}$$

where

$$f_1(t)=\tfrac{1}{2}\omega_0\left[\left[\frac{\tilde{\Omega}(t)}{\omega_0}\right]^2+1\right], \tag{3.4a}$$

$$f_2(t)=\tfrac{1}{4}\omega_0\left[\left[\frac{\tilde{\Omega}(t)}{\omega_0}\right]^2-1\right], \tag{3.4b}$$

$$f_3(t)=eB(t)/2Mc=\tfrac{1}{2}\omega_c(t) . \tag{3.4c}$$

The Heisenberg equations of motion can be obtained from $i\hbar d\hat{a}_i/dt=[\hat{a}_i,\hat{H}_{x,y}]$ and $i\hbar d\hat{a}_i^{\dagger}/dt=[\hat{a}_i^{\dagger},\hat{H}_{x,y}]$. We then find from the Eq. (3.2)

$$\frac{d}{dt}\begin{bmatrix}\hat{a}_x\\ \hat{a}_x^{\dagger}\\ \hat{a}_y\\ \hat{a}_y^{\dagger}\end{bmatrix}=\begin{bmatrix}-if_1(t) & -2if_2(t) & 0 & 0\\ 2if_2(t) & if_1(t) & 0 & 0\\ 0 & 0 & -if_1(t) & -2if_2(t)\\ 0 & 0 & 2if_2(t) & if_1(t)\end{bmatrix}\begin{bmatrix}\hat{a}_x\\ \hat{a}_x^{\dagger}\\ \hat{a}_y\\ \hat{a}_y^{\dagger}\end{bmatrix} \tag{3.5}$$

which gives, in the subspace x [with $\underline{\tilde{M}}(t)=-if_1\hat{\underline{\sigma}}_z+2f_2\hat{\underline{\sigma}}_y$],

$$\frac{d}{dt}\begin{bmatrix}\hat{a}_x\\ \hat{a}_x^{\dagger}\end{bmatrix}=\underline{\tilde{M}}(t)\begin{bmatrix}\hat{a}_x\\ \hat{a}_x^{\dagger}\end{bmatrix}, \tag{3.6}$$

the same being valid for the subspace y, with $\hat{a}_x\rightarrow\hat{a}_y$, $\hat{a}_x^{\dagger}\rightarrow\hat{a}_y^{\dagger}$.

IV. SOLUTION OF HEISENBERG EQUATIONS: SQUEEZING

Now, considering the dependence on time of the parameters B and η [cf. Eq. (3.6)], the operators $\hat{a}$ and $\hat{a}^{\dagger}$, in the Heisenberg picture, follow from the calculation of the evolution operator

$$\hat{U}(t)=\hat{T}\left[\exp\int_0^t\tilde{M}(t')dt'\right], \tag{4.1}$$

namely, for the x subspace one has

$$\begin{bmatrix}\hat{b}_H(t)\\ \hat{b}_H^{\dagger}(t)\end{bmatrix}=\hat{U}^{\dagger}(t)\begin{bmatrix}\hat{a}_x\\ \hat{a}_x^{\dagger}\end{bmatrix}\hat{U}(t), \tag{4.2}$$

where $\hat{T}$ stands for the time-ordering operator [6]. However, the calculation of the evolution operator by Eq. (4.1) is a difficult task, and in a previous paper [7] we circumvented such difficulty by solving the Schrödinger equation for the Hamiltonian (3.2) by a method that uses, sequentially, a TD unitary transformation and the diagonalization of a TD invariant. From this solution we were able to obtain the evolution operator [7]

$$\hat{U}(t)=\hat{S}(\xi(t))\hat{R}(\Omega(t))\hat{S}^{\dagger}(\xi(0)) \tag{4.3}$$

where [with $\xi=r\exp(i\phi)$]

$$\hat{S}(\xi)=\exp\{\tfrac{1}{2}[\xi(\hat{a}^{\dagger})^2-\xi^*\hat{a}^2]\} \tag{4.4}$$

is the squeeze operator and

$$\hat{R}(\Omega(t))=\exp\left[-\frac{i}{2}\int_0^t[\Omega(t')-f_1(t')]dt' -i\left[\int_0^t\Omega(t')dt'\right]\hat{a}^{\dagger}\hat{a}\right] \tag{4.5}$$

is the rotation operator. The parameters r and ϕ are the solution of the coupled nonlinear differential equations [7],

$$\dot{r}=-2f_2(t)\sin[\phi(t)] , \tag{4.6a}$$

$$\dot{\phi}=-2f_1(t)-4f_2(t)[\coth 2r(t)]\cos\phi(t) . \tag{4.6b}$$

It is important to note that the initial conditions $r(0)$ and $\phi(0)$ are not necessarily null, which means that $\hat{S}(\xi(0))\neq\underline{1}$. In regard to the other operator $\hat{R}(\Omega)$, the parameter is

$$\Omega(t)=f_1(t)+2f_2(t)\tanh r(t)\cos\phi(t) , \tag{4.7}$$

and independently of any initial condition on r, ϕ, f_1, and f_2 we have $\hat{R}(\Omega(0))=\underline{1}$.

So now we are in a position to calculate algebraically the operators $\hat{a}$ and $\hat{a}^{\dagger}$ in the Heisenberg picture as

$$\hat{b}_H=\hat{U}^{\dagger}(t)\hat{a}\hat{U}(t)=u(t)\hat{a}+v(t)\hat{a}^{\dagger} , \tag{4.8a}$$

$$\hat{b}_H^{\dagger}=v^*(t)\hat{a}+u^*(t)\hat{a}^{\dagger} , \tag{4.8b}$$

where

$$u(t)=\cosh[r(t)]\cosh[r(0)]e^{-i\beta(t)} -\sinh[r(t)]\sinh[r(0)]e^{i[\beta(t)+\phi(t)-\phi(0)]} \tag{4.9a}$$

and

$$v(t)=\sinh[r(t)]\cosh[r(0)]e^{i[\beta(t)+\phi(t)]} -\cosh[r(t)\sinh[r(0)]e^{-i[\beta(t)-\phi(0)]} , \tag{4.9b}$$

with

$$\beta(t)=\int_0^t\Omega(t')dt' . \tag{4.10}$$

It can be verified that the unitary transformation $\hat{U}(t)$ is also a canonical transformation of the Bogoliubov-Valatin type, since $|u|^2-|v|^2=1$.

Now we go to the calculation of the fluctuations of the quadratures defined as

$$\hat{X}_{1,H}=\tfrac{1}{2}(\hat{b}_H^{\dagger}+\hat{b}_H) , \tag{4.11a}$$

$$\hat{X}_{2,H}=\frac{i}{2}(\hat{b}_H^{\dagger}-\hat{b}_H) . \tag{4.11b}$$

Calling the dynamic fluctuation for these quadrature operators

$$\langle\Delta\hat{X}_{i,H}^2\rangle=\langle\hat{X}_{i,H}^2\rangle-\langle\hat{X}_{i,H}\rangle^2 , \tag{4.12}$$

$i=1,2$, we consider the mean in an arbitrary TD state. So, Eq. (4.12) can be written in terms of the static fluctuations,

$$\langle\Delta\hat{a}^2\rangle=\langle\hat{a}^2\rangle-\langle\hat{a}\rangle^2 , \tag{4.13a}$$

$$\langle\Delta(\hat{a}^{\dagger})^2\rangle=\langle(\hat{a}^{\dagger})^2\rangle-\langle\hat{a}^{\dagger}\rangle^2 , \tag{4.13b}$$

$$\langle\Delta\hat{a}^{\dagger}\hat{a}\rangle=\langle\hat{a}^{\dagger}\hat{a}\rangle-\langle\hat{a}^{\dagger}\rangle\langle\hat{a}\rangle , \tag{4.13c}$$

as

$$\begin{aligned}\langle\Delta\hat{X}_{1,H}^2\rangle&=\tfrac{1}{4}[\langle\Delta\hat{b}_H^2\rangle+\langle\Delta(\hat{b}_H^{\dagger})^2\rangle+2\langle\Delta(\hat{b}_H^{\dagger}\hat{b}_H)\rangle+1]\\&=\tfrac{1}{4}[2|u+v^*|^2\langle\Delta(\hat{a}^{\dagger}\hat{a})\rangle+(u+v^*)^2\langle\Delta\hat{a}^2\rangle\\&\quad+(u^*+v)^2\langle\Delta(\hat{a}^{\dagger})^2\rangle+|u+v^*|^2]\end{aligned} \tag{4.14a}$$

and

$$\begin{aligned}\langle\Delta\hat{X}_{2,H}^2\rangle&=\tfrac{1}{4}[2|u-v^*|^2\langle\Delta(\hat{a}^{\dagger}\hat{a})\rangle-(u-v^*)^2\langle\Delta\hat{a}^2\rangle\\&\quad-(u^*-v)^2\langle\Delta(\hat{a}^{\dagger})^2\rangle+|u-v^*|^2] .\end{aligned} \tag{4.14b}$$

In particular if the mean is considered in a (initial) coherent state, then

$$\langle\Delta\hat{X}_{1,H}^2\rangle=\tfrac{1}{4}|u+v^*|^2 , \tag{4.15a}$$

$$\langle\Delta\hat{X}_{2,H}^2\rangle=\tfrac{1}{4}|u-v^*|^2 , \tag{4.15b}$$

and a direct calculation shows that

$$\begin{aligned}|u+v^*|^2=&\cosh[2(r-r_0)]+2\sinh(2r)\sinh(2r_0)\sin^2\left[\beta+\frac{\phi-\phi_0}{2}\right]+\sinh(2r)\cosh(2r_0)\cos\phi\\&-\sinh(2r_0)[\cosh^2(r)\cos(2\beta-\phi_0)+\sinh^2(r)\cos(2\beta+2\phi-\phi_0)]\end{aligned} \tag{4.16a}$$

and

$$\begin{aligned}|u-v^*|^2=&\cosh[2(r-r_0)]+2\sinh(2r)\sinh(2r_0)\sin^2\left[\beta+\frac{\phi-\phi_0}{2}\right]-\sinh(2r)\cosh(2r_0)\cos\phi\\&+\sinh(2r_0)[\cosh^2(r)\cos(2\beta-\phi_0)+\sinh^2(r)\cos(2\beta+2\phi-\phi_0)] .\end{aligned} \tag{4.16b}$$

Now, considering the static situation, when $B=B_0$ and $\eta=\eta_0$ in Eq. (4.6), $\dot{r}=0$ and $\dot{\phi}=0$, this implies $\phi=\phi_0=\pi$, $\tanh(2r_0)=2f_2/f_1$, and we obtain from Eqs. (4.15) and (4.16)

$$\langle\Delta\hat{X}^2_{1,H}\rangle=\frac{1}{4}\left[\cos^2(\Omega t)+\left[\frac{2f_2-f_1}{\Omega}\right]^2\sin^2(\Omega t)\right] \tag{4.17a}$$

and

$$\langle\Delta\hat{Y}^2_{2,H}\rangle=\frac{1}{4}\left[\cos^2(\Omega t)+\left[\frac{2f_2+f_1}{\Omega}\right]^2\sin^2(\Omega t)\right], \tag{4.17b}$$

with $\beta=\Omega t$ and $\Omega=(f_1^2-4f_2^2)^{1/2}=\tilde{\Omega}$ [cf. Eq. (4.17)]. The results in Eqs. (4.17) may be compared with those corresponding in Refs. [3] and [4].

To conclude this analysis, we emphasize the fact that the fluctuations are determined once the system of Eq. (4.6) is solved and $r(t)$ and $\phi(t)$ are available either by analytical methods or numerically. Now, if we pass from the Larmor frame to the laboratory frame, the x and y components of the operators in the Heisenberg picture are given by

$$\hat{a}_{x,H}=\hat{b}_x(t)\cos F_3(t)+\hat{b}_y(t)\sin F_3(t), \tag{4.18a}$$

$$\hat{a}^\dagger_{x,H}=\hat{b}^\dagger_x(t)\cos F_3(t)+\hat{b}^\dagger_y(t)\sin F_3(t), \tag{4.18b}$$

where $\hat{b}_x$, $\hat{b}^\dagger_x$, $\hat{b}_y$, and $\hat{b}^\dagger_y$ are as defined in Eqs. (4.8) but for operators $(\hat{a}_x,\hat{a}^\dagger_x)$ and $(\hat{a}_y,\hat{a}^\dagger_y)$, respectively. The function $F_3(t)$ is

$$F_3(t)=\int_0^t f_3(t')dt'. \tag{4.19}$$

Following the same calculation for the fluctuations of the x subspace, we now obtain [compare with Eqs. (4.14)]

$$\begin{aligned}\langle\Delta\hat{X}^2_{1,H}\rangle=\tfrac{1}{4}\{&[2|u+v^*|^2\langle\Delta(\hat{a}^\dagger_x\hat{a}_x)\rangle+(u+v^*)^2\langle\Delta\hat{a}^2_x\rangle+(u^*+v)^2\langle\Delta(\hat{a}^\dagger_x)^2\rangle+|u+v^*|^2]\cos^2F_3\\&+[2|u+v^*|^2\langle\Delta(\hat{a}^\dagger_y\hat{a}_y)\rangle+(u+v^*)^2\langle\Delta\hat{a}^2_y\rangle+(u^*+v)^2\langle\Delta\hat{a}^2_y\rangle+|u+v^*|^2]\sin^2F_3\}\end{aligned} \tag{4.20a}$$

and

$$\begin{aligned}\langle\Delta\hat{X}_{2,H}\rangle=\tfrac{1}{4}\{&[2|u-v^*|^2\langle\Delta(\hat{a}^\dagger_x\hat{a}_x)\rangle-(u-v^*)^2\langle\Delta\hat{a}^2_x\rangle-(u^*-v)^2\langle\Delta(\hat{a}^\dagger_x)^2\rangle+|u-v^*|^2]\cos^2F_3\\&+[2|u-v^*|^2\langle\Delta(\hat{a}^\dagger_y\hat{a}_y)\rangle-(u-v^*)^2\langle\Delta\hat{a}^2_y\rangle-(u^*-v)^2\langle\Delta(\hat{a}^\dagger_y)^2\rangle+|u-v^*|^2]\sin^2F_3\}.\end{aligned} \tag{4.20b}$$

If we consider the mean in an (initial) coherent state, $\langle\Delta(\hat{a}^\dagger\hat{a})\rangle=\langle\Delta\hat{a}^2\rangle=\langle\Delta(\hat{a}^\dagger)^2\rangle=0$ for x and y components, we recover the results in Eq. (4.14). From this we conclude that if the initial state is a coherent state, then the fluctuations of the quadratures are frame independent.

V. COMMENTS AND CONCLUSION

In this paper we have calculated the fluctuations in quadrature-phase amplitudes for a charged HO, and also for a charged particle, in the presence of a TD electromagnetic field. The present model constitutes an extension of a previous one investigated by Bechler [3] and Abdalla [4], for a charged HO in the presence of a constant magnetic field.

For our extended system, the results for quadrature fluctuations are obtained in Eq. (4.14) or, equivalently, in Eqs. (4.20). To obtain these results we have employed a method of a previous work [7] that uses, sequentially, a TD unitary transformation and the diagonalization of a TD invariant. Since we have assumed throughout this paper that the TD electromagnetic field is arbitrary, then our results in Eqs. (4.14) and (4.16), are just formal.

Here particular models will emerge from the specification of $B(t)$ and $\eta(t)$ in Eq. (2.6) that yield a particular TD Hamiltonian [cf. Eq. (2.4)] with $\tilde{\Omega}(t)$ and $\omega_c(t)$ already determined from $B(t)$ and $\eta(t)$ [cf. Eq. (2.6)]. Once this is done, our procedure leads one to the solution of the system (4.6), which is obtained either analytically or numerically. For the case of a static field [put $B=B_0$, $\eta=\eta_0$ in Eq. (2.6)] one recovers the results in the literature, as we have mentioned below Eq. (4.16).

The application of the present method in the investigation of squeezing and other statistical properties for some particular examples of TD electromagnetic fields is in progress and will be the subject of a future publication. An interesting example is provided by an external field where $B(t)=B_0\cos\omega t$, $\eta=\eta_0$. In this case we note that Eq. (2.3) includes both the rotating and counter-rotating components of the electromagnetic field, and this may constitute an appropriate format for the investigation of the rotating-wave approximation, an up-to-date problem [8]. Another interesting example is the trapping problem [9], where the TD Hamiltonian is given by [10] $\hat{H}(\hat{q},\hat{p},t)=\hat{p}^2/2M+(M/2)W^2(t)\hat{q}^2$, with the frequency $W(t)$ expressing the time dependence of an externally applied field. In general, for any example of a TD Hamiltonian, the quantum states are not stationary states or energy eigenstates of any kind. In the words of Glauber [10], "finding them thus represents somewhat novel problems in the context of quantum mechanics, and developing simple means for doing that possesses a certain methodological interest in its own right."

As a final remark, we mention that an alternative approach for the same system, using the ladder operators introduced by Rajagopal and Marshall [11], has also been developed [12]. In this case, as in others [10], one is led to a frame (which could be called the principal frame) where, to a certain extent [13], the problem can be solved without specifying the time dependence of the field at all.

ACKNOWLEDGMENTS

The authors wish to thank Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil for partial financial support.

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[13] Squeezing only refers to the variance of two quadrature components of the oscillator; thus, it does not fully characterize these states.