Two-peak passage-time distributions in transient CO_2 lasers near threshold

M. Ciofini, A. Lapucci, R. Meucci, Peng-ye Wang,* and F. T. Arecchi†
Istituto Nazionale di Ottica, Largo Enrico Fermi 6, 50125 Firenze, Italy
(Received 24 July 1991; revised manuscript received 18 February 1992)

We investigate two-peak passage-time statistical distributions in a Q-switched CO_2 laser using the excitation current as the control parameter. This two-peak statistics is due to the population-inversion noise when the initial laser state is set close to threshold. Such a phenomenon has been experimentally observed in a class-B laser when the passage time is shorter than the relaxation time of the population inversion. The time separation of the two peaks can be quantitatively related to the laser net gain.

PACS number(s): 42.55.Lt, 05.40.+j, 42.50.−p

I. INTRODUCTION

The transient evolution of a Q-switched laser depends on the initial state of the electric field. This initial condition is affected by different kinds of noise, such as spontaneous-emission fluctuations and population-inversion fluctuations. Noise effects are enhanced during the transient amplification process. For this reason transient lasers represent very useful systems for the noise analysis.

In the past few years several works have been provided about this subject both from theoretical [1–7] and experimental [1,8–11] points of view. In the particular case of delayed bifurcations [3] the different roles of additive and multiplicative noise were stressed.

Recently we investigated the effect of population-inversion noise on the transient statistics of a CO_2 laser [12]. The influence of this noise is important only when the laser is initially prepared very close to threshold (initial net gain close to zero) giving rise to two-peak passage-time distributions.

In this paper we report a detailed investigation of the time separation of the two peaks, showing that it scales linearly with the reciprocal of the net laser gain after the switch.

The outline of the paper is as follows. In Sec. II we report the experimental results. Section III contains a theoretical analysis and a comparison between experimental data, numerical simulations, and the theoretical expectations. Finally in Sec. IV we discuss the conditions leading to two-peak passage-time distributions.

II. EXPERIMENTAL RESULTS

The experimental setup consists of a single-mode CO_2 laser with an intracavity electro-optic modulator (EOM). The optical cavity is defined by a total reflecting spherical mirror and by a grating blazed for 10.6 μm (P20 line). The total reflecting mirror is mounted on a piezoelectric translator (PZT) in order to control the tuning between the cavity mode and the center of the CO_2 molecular line. Laser output is obtained via an intracavity ZnSe beam splitter with 5% reflectance. The discharge tube is terminated by Brewster windows. The EOM driver provides a voltage step signal with a rise time shorter than 40 ns. As the signal on the EOM is switched at $t=0$, a fast jump of the cavity-loss rate from a high value k_0 to a low value k_1 occurs. The corresponding laser output is detected with a Hg-Cd-Te photodiode having a response time shorter than 10 ns. For the laser field evolution we consider the transient time t_1 when the intracavity photon number n_1 is about one-fifth of the saturation value n_s so that the transient dynamics remains in the linear regime. The time t_1 is converted to a voltage signal by a time-to-amplitude converter. This signal is sent to a multichannel analyzer classifying the first passage-time distributions.

By adjusting the voltage applied to EOM we can set the initial state very close to the laser threshold. An example of an experimental two-peak first passage-time distribution is shown in Fig. 1.

Since the final laser net gain affects the passage-time distribution, we choose the discharge current as the control parameter. For each discharge current we set the voltage applied to EOM so that the initial state remains

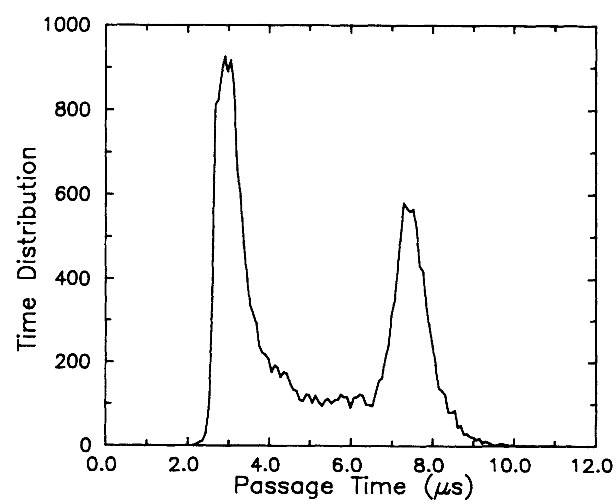

FIG. 1. First passage-time distribution with the initial laser state close to threshold. The discharge current is 3.70 mA, and the voltage step applied to the EOM is 581 V.

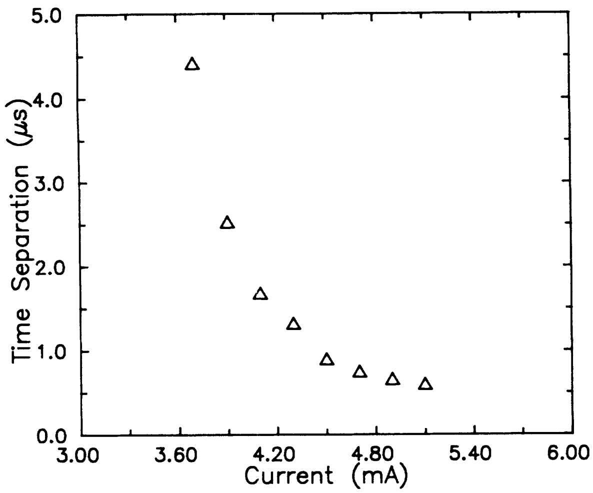

FIG. 2. Time separation between the two peaks vs the discharge current.

close to threshold. This also changes the initial cavity-loss rate to k_0. In Fig. 2 we show the time separation between the two peaks in the passage-time distributions as a function of the discharge current. It can be seen that the two peaks are closer for a larger current.

III. THEORETICAL ANALYSIS

Two-peak passage-time distributions can be theoretically treated dealing with two coupled differential equations for the complex field amplitude $E=|E|\exp(i\phi)$ and the population inversion Δ [13]:

$$\frac{dE}{dt}=-k(t)E+\frac{G\Delta E}{2}+(GN_2)^{1/2}\xi(t) , \quad (1)$$

$$\frac{d\Delta}{dt}=-\gamma(\Delta-\Delta_0)-2G\Delta|E|^2+(R)^{1/2}\zeta(t) . \quad (2)$$

k and γ are the two decay rates, G is the field-matter coupling constant, N_2 is the population of the upper laser level, and Δ_0 represents the population inversion provided by the pump mechanism. Both Eqs. (1) and (2) contain stochastic terms, a complex one $\xi(t)$ and a real one $\zeta(t)$, respectively, which are zero-average δ-correlated Gaussian processes accounting for noise. These processes are specified by

$$\langle \xi(t)\rangle=0, \quad \langle \xi^*(t)\xi(t')\rangle=\delta(t-t') ,$$

$$\langle \zeta(t)\rangle=0, \quad \langle \zeta(t)\zeta(t')\rangle=\delta(t-t') ,$$

where $\xi^*(t)$ represents the complex conjugate of $\xi(t)$.

The field noise is due to the spontaneous emission and its strength GN_2 is proportional to the upper laser level population N_2. The population noise (of strength R) accounts for several different processes such as fluctuations in plasma temperature or in gas flow and density.

First we consider the preparation phase for $t<0$ in order to define the field initial condition at $t=0$ when the cavity losses are changed. In this preparation stage $k(t)=k_0$, which is a constant value chosen so that the laser net gain is close to zero, i.e., near laser threshold. This implies that the mean intensity $\langle |E|^2\rangle$ is very small compared with the saturation value I_s estimated as $I_s=Z\gamma/2G$ ($Z\simeq 10$ is the number of rotational levels coupled to the two resonant levels [9]). Therefore the term proportional to $|E|^2$ in Eq. (2) is a small additive correction to the leading term $-\gamma(\Delta-\Delta_0)$ and thus it can be taken as a constant, since it contributes a very weak time dependence to Δ. With this approximation the Fokker-Planck equation associated with Eq. (2) can be easily solved, obtaining the stationary probability density

$$P(\Delta)=c_1\exp[-(\Delta-\bar{\Delta}_0)^2/2\sigma^2] , \quad (3)$$

where c_1 is a normalization constant, $\bar{\Delta}_0=\Delta_0/(1+|E|^2/I_s)\simeq\Delta_0(1-|E|^2/I_s)$, and $\sigma^2=(R/\gamma)(1-|E|^2/I_s)$. Equation (3) represents a Gaussian distribution around the deterministic solution $\bar{\Delta}_0$ of Eq. (2) so that the value of population inversion for $t=0$ can be written as

$$\Delta=\bar{\Delta}_0+\eta , \quad (4)$$

where η is a Gaussian noise distribution with zero mean. Combining Eq. (4) with Eq. (1) we obtain

$$\frac{dE}{dt}=(a_0+\mu)E-b|E|^2E+(GN_2)^{1/2}\xi(t) , \quad (5)$$

where $a_0=G\Delta_0/2-k_0$ is the laser net gain for $t<0$, $b=G^2\Delta_0/\gamma$, and $\mu=(G/2)\eta$ is the fluctuation due to population inversion.

Considering that the relaxation time of the population inversion $1/\gamma$ ($\simeq 10^{-4}$ s) is much larger than the duration of the transient field evolution (which is at most 10 μs as can be seen from Fig. 1), the stochastic term μ can be treated as a constant during the transient evolution of the field ruled by Eq. (5). Therefore we can define $\bar{a}_0=a_0+\mu$ as the effective initial net gain, which takes different values for each laser buildup process, so that μ acts as a multiplicative noise, besides the additive one $\xi(t)$. The steady-state solution of the Fokker-Planck equation associated with Eq. (5) is then

$$P(|E|,\phi)=c_2\exp\left[\left[\bar{a}_0\frac{|E|^2}{2}-b\frac{|E|^4}{4}\right]\Big/(GN_2)\right] , \quad (6)$$

where c_2 is a normalizing factor.

A plot of this distribution is shown in Fig. 3 for different μ, considering $a_0=0$. Each section with a constant μ represents the field probability density for a certain fixed value of population inversion. When $\mu<0$, the initial laser state is below threshold so that the most probable value $|E|_M$ for the field is $|E|_M=0$. If $\mu>0$ the initial laser state is above threshold and the probability density has a maximum for $|E|_M=(\mu/b)^{1/2}$. The $\mu=0$ section separates the regions of initial condition below and above threshold. As the starting point for the field can be everywhere on the whole surface of Fig. 3, the laser can start from below or above threshold during the repeated Q-switch operation which provides the passage-

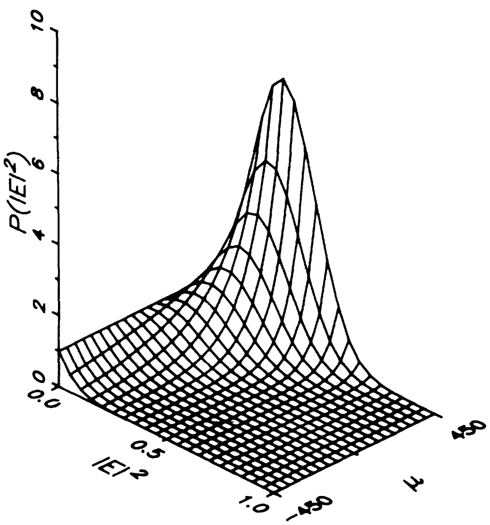

FIG. 3. Probability distribution $P(|E|^2)$ vs $|E|^2$ with $a_0=0$, $c_2=1$, $b=1.21\times10^{-4}$ s^{-1}, $GN_2=1.51\times10^9$ s^{-1} for different realizations of the stochastic process μ. Horizontal scales: μ is in s^{-1}, $|E|^2=1$ corresponds to 1.20×10^8 photons.

time distribution.

When the initial state is set far above ($a_0 \gg 0$) or below ($a_0 \ll 0$) threshold the fluctuation μ is much smaller than the absolute value of a_0 and the initial state surface cannot cross the laser threshold. In this case, the population noise is less important and the two-peak characteristic in the passage-time distribution disappears.

After the preparation stage, the cavity decay rate is switched to the lower value k_1 starting at $t=0$. The switching takes place in a time much shorter than the transient time so that it may be considered as a step process. Anyway a finite rise time of the modulator voltage does not affect the time separation between the two peaks, as we have directly verified by adding a capacitor on the modulator input to increase the switching time from 40 to 400 ns and checking the invariance of the peak separation.

We consider the time t_1 necessary to reach a certain photon number n_1 below the saturation value n_s. Since the mean transient time $\langle t_1 \rangle$ ($\simeq 10^{-6}$ s) is much shorter than the relaxation time $1/\gamma$, the population inversion can be considered as maintaining its initial value $\Delta(t=0)$ during the transient time. Moreover, as $n_1 \ll n_s$ the laser dynamics can be taken as linear up to t_1 [7,9] although the cubic nonlinearity is included in Eq. (5) to avoid the diverging of $P(|E|,\phi)$ whenever $\bar{a}_0>0$.

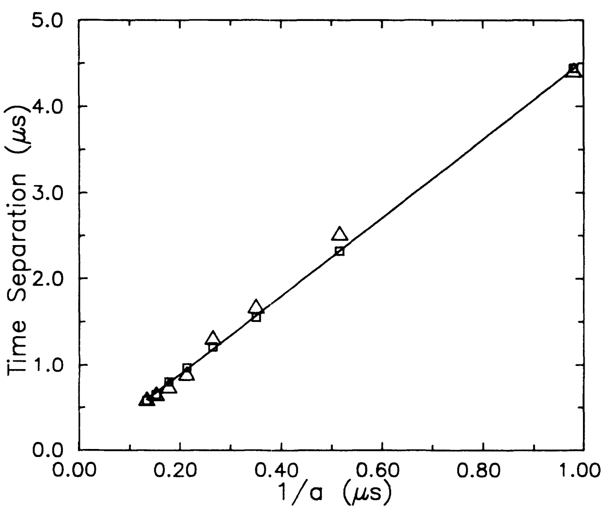

FIG. 4. Linear dependence of the time separation Δt on the inverse final net gain $1/a$. Triangles denote experimental points; squares denote numerical results. The solid line is the best fit of the simulation. The following parameter values are used in the simulation: $G=4.6\times10^{-8}$ s^{-1}, $k_0=1.7\times10^7$ s^{-1}, $k_1=1.6\times10^7$ s^{-1}, $R^{1/2}=1.57\times10^{14}$ s^{-1}, $\gamma=2.0\times10^4$ s^{-1}, and $n_1=3.84\times10^{11}$.

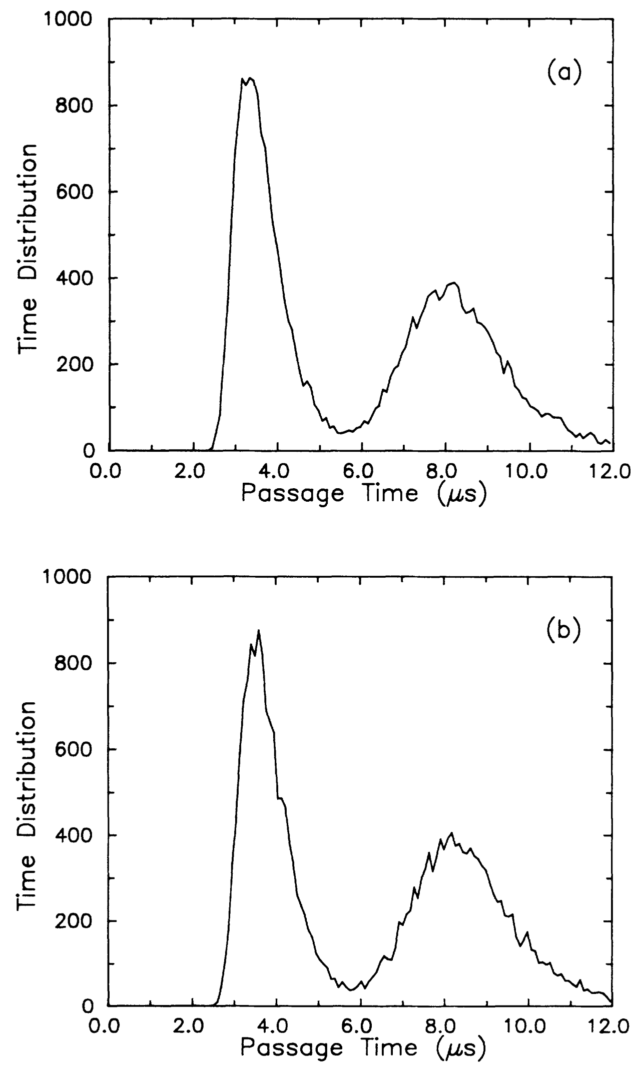

FIG. 5. Numerical simulation of the first passage-time distribution corresponding to the experimental conditions of Fig. 1; (a) simulation of Eq. (5) with $\langle\mu^2\rangle^{1/2}=6.0\times10^4$ s^{-1}, and (b) simulation of Eqs. (1) and (2).

Based on the above considerations, for a laser starting from below threshold, we obtain [7,9]

$$\langle t_1 \rangle = \frac{1}{2a}[\ln(n_1) - \ln(N_0) - \psi(1)] , \tag{7}$$

where $a = G\Delta_0/2 - k_1$ is the final net gain, and $\psi(1) \simeq -0.577$ is the digamma function. N_0 is a total mean photon number accounting for the mean initial noise and the noise along the amplification process. In our case the noise along the amplification process can be neglected due to the fact that the initial state is very close to threshold [7,9]. We should notice that Eq. (7) describes the mean transient time at a certain value of $\bar{a}_0$ (which determines a corresponding value of N_0), while $\bar{a}_0$ is changing during the whole statistical procedure. However, due to the fact that $n_1 \gg N_0$, we can assume the variation in $\ln(N_0)$ to be negligible compared with $\ln(n_1)$. Therefore the term $\ln(n_1/N_0)$ can be treated as a constant.

When the laser starts from above threshold, the initial distribution can be considered as a δ function and the transition occurs between two coherent states. From a direct integration of the linear equation for the photon number we easily obtain

$$t'_1 = \frac{1}{2a}[\ln(n_1) - \ln(N_1)] , \tag{8}$$

where N_1 is the mean photon number of the initial coherent state. The fluctuation in $\bar{a}_0$ provides a corresponding spread in N_1 which in turn induces a spread in t'_1. To consider this fluctuation we take the average over t'_1:

$$\langle t'_1 \rangle = \frac{1}{2a}[\ln(n_1) - \langle \ln(N_1) \rangle] . \tag{9}$$

The separation of the two peaks of the first passage-time distribution can be calculated from Eqs. (7) and (9):

$$\Delta t = \langle t_1 \rangle - \langle t'_1 \rangle = [\langle \ln(N_1) \rangle - \ln(N_0) - \psi(1)]\frac{1}{2a} . \tag{10}$$

From Eq. (10) we see that the time separation between the two peaks is inversely proportional to the final net gain a, which is proportional to the excitation current i via the relation $\Delta_0 = Mi$, where $M = 2\times 10^{14}\,\mathrm{mA}^{-1}$ [10,12]. The results of numerical simulations of Eqs. (1) and (2) (see Fig. 4) confirm the validity of Eq. (10), also showing that the dependence of the prefactor on discharge current is very weak. To confirm further our approximations, we report in Fig. 5 a typical passage-time distribution obtained by numerical simulation of Eq. (5) [Fig. 5(a)] and of the full stochastic model [Fig. 5(b)]. The agreement between the two distributions, both in peak positions and widths, shows that the reduction of the complete description of Eqs. (1) and (2) to our simplified model of Eq. (5) is fully justified, as far as transient phenomena are concerned.

IV. CONCLUSIONS

We have studied the influence of the population-inversion noise in a CO_2 laser. The effects of such fluctuations are enhanced when the initial state of the transient process is set close to threshold, giving rise to two-peak first passage-time distributions. This particular phenomenon can be observed in class-*B* lasers characterized by a slow relaxation rate of the population inversion with respect to the buildup time of the field. In fact, in this case the transient process keeps memory of the starting point, which can be below or above threshold due to the fluctuation of population inversion. Two-peak passage-time distributions can also be expected in a class-*A* laser near threshold with slowly fluctuating pump noise, provided the noise correlation time is longer than the transient time.

The separation of the two peaks provides a quantitative description of this statistical feature. Our results confirm a linear dependence of the time separation on the inverse value of the final laser gain.

This work was done with partial support of the EC, Contract No. SC1-CT91-0697 (TSTS).

---

*Permanent address: Institute of Physics, Academia Sinica, Beijing, China.

†Also at Department of Physics, University of Firenze, Italy.

[1] F. T. Arecchi and A. Politi, Phys. Rev. Lett. **45**, 1219 (1980); F. T. Arecchi, A. Politi, and L. Ulivi, Nuovo Cimento B **71**, 119 (1982).

[2] G. Broggi, A. Colombo, L. A. Lugiato, and P. Mandel, Phys. Rev. A **33**, 3635 (1986).

[3] H. Zeghlache, P. Mandel, and C. Van den Broeck, Phys. Rev. A **40**, 286 (1989).

[4] M. James, F. Moss, P. Hänggi, and C. Van den Broeck, Phys. Rev. A **38**, 4690 (1988).

[5] N. G. Stocks, R. Mannella, and P. V. E. McClintock, Phys. Rev. A **42**, 3356 (1990).

[6] F. de Pasquale, J. M. Sancho, M. San Miguel, and P. Tartaglia, Phys. Rev. Lett. **56**, 2473 (1986); M. C. Torrent, F. Sagués, and M. San Miguel, Phys. Rev. A **40**, 6662 (1989).

[7] M. C. Torrent and M. San Miguel, Phys. Rev. A **38**, 245 (1988).

[8] R. Roy, A. W. Yu, and S. Zhu, Phys. Rev. Lett. **55**, 2794 (1985); S. Zhu, A. W. Yu, and R. Roy, Phys. Rev. A **34**, 4333 (1986).

[9] F. T. Arecchi, R. Meucci, and J. A. Roversi, Europhys. Lett. **8**, 225 (1989); F. T. Arecchi, W. Gadomski, R. Meucci, and J. A. Roversi, Phys. Rev. A **39**, 4004 (1989).

[10] M. Ciofini, R. Meucci, and F. T. Arecchi, Phys. Rev. A **42**, 482 (1990).

[11] A. Mecozzi, S. Piazzolla, A. D'Ottavi, and P. Spano, Phys. Rev. A **38**, 3136 (1988); P. Spano, A. Mecozzi, and A. Sapia, Phys. Rev. Lett. **64**, 3003 (1990); S. Balle, P. Colet, and M. San Miguel, Phys. Rev. A **43**, 498 (1991).
[12] M. Ciofini, R. Meucci, Peng-ye Wang, and F. T. Arecchi (unpublished).
[13] F. T. Arecchi, in *Instabilities and Chaos in Quantum Optics,* edited by F. T. Arecchi and R. G. Harrison (Springer-Verlag, Berlin, 1987), p. 9.