BRIEF REPORTS

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than 4 printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.

Experimental observation of stochastic resonance in a magnetoelastic ribbon

M. L. Spano and M. Wun-Fogle
Naval Surface Warfare Center, Silver Spring, Maryland 20903

W. L. Ditto
Department of Physics, The College of Wooster, Wooster, Ohio 44691
(Received 23 March 1992)

We report the observation of stochastic resonance in a parametrically driven bistable magnetoelastic buckling-ribbon experiment. We have found that the parametric addition of white noise increases the signal-to-noise ratio of the response of the ribbon by 10 to 12 dB relative to that of the ribbon with no externally added noise. We have also observed a characteristic spectrum for the probability of residence times for this weakly bistable system.

PACS number(s): 05.20.−y, 75.80.+q, 75.50.Kj

INTRODUCTION

Recently attention has been focused on a statistical property of noise-driven multistable systems known as stochastic resonance. This phenomenon was first introduced by Benzi and co-workers [1] to account for the periodicity of the Earth's ice ages. Subsequent theoretical analyses have appeared which detail the features of this phenomenon [2–8]. One of the remarkable features of stochastic resonance is the increase of the signal-to-noise ratio of a periodically modulated system upon the addition of a random noise signal. Experimental results have demonstrated these increases of signal-to-noise ratios in analog circuits [2], a bi-directional ring laser [9], and in an electron-paramagnetic-resonance system [10]. Additionally, it has been claimed that systems undergoing stochastic resonance should exhibit a characteristic signature: a probability spectrum of residence times in the wells of a bistable system that is peaked at certain multiples of the drive period [11]. Remarkably this signature has been linked by Longtin, Bulsara, and Moss [8] to studies performed on single auditory nerve fibers in spider monkeys [12] and to experiments performed on single neutrons in the primary visual cortex of a cat [13].

The main contributions of this Brief Report are the following: (1) the observation of stochastic resonance in a mechanical system; (2) an experimental observation (apart from analog simulations in circuits) of the probability spectrum of residence times for a stochastic resonance system; and (3) the extension of the types of experimental systems exhibiting stochastic resonance to parametrically driven, highly asymmetrical experimental systems (although it should be noted that the laser system investigated by McNamara, Wiesenfeld, and Roy [9] is a parameterically driven system) which show novel behavior of the probability spectrum of residence times.

EXPERIMENT

The experiment consists of a gravitationally buckled, parameterically driven amorphous magnetoelastic ribbon. The ribbon material ($Fe_8B_{13.5}Si_{3.5}C_2$) belongs to a new class of amorphous magnetostrictive materials that have been found to exhibit very large reversible changes of Young's modulus $E(H)$ with the application of small magnetic fields [14]. The ribbon was clamped at the base to yield a free vertical length greater than the Euler buckling length, thus producing an initially bucked configuration. Young's modulus of the ribbon was varied by applying a vertical magnetic field having the form $H(t)=[H_{\rm dc}+H_{\rm ac}\sin(2\pi f t)+\xi(t)]$. The random component $\xi(t)$ was given by

$$\xi(t)=\sum_{+\infty}^{+\infty} a_k\{\theta(t-k\delta t)-\theta[t-(k+1)\delta t]\} ,$$

where θ is the unit step function, δt is the update interval, and the a_k are independent, zero-mean Gaussian, pseudorandom variables with variance σ^2. The update interval was set to be $\delta t=0.01$ s, a value much less than the ribbon's mechanical response time of 1.2 s ($1/e$ decay of the ribbon's response to a step change in the dc magnetic field). The a_k were produced by a 12-bit digital-to-analog

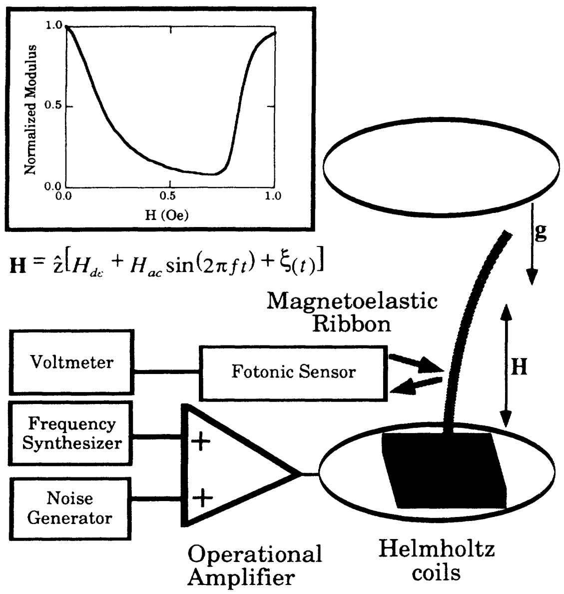

FIG. 1. Schematic of the magnetoelastic ribbon experiment. Inset: Normalized Young modulus for the ribbon vs magnetic field.

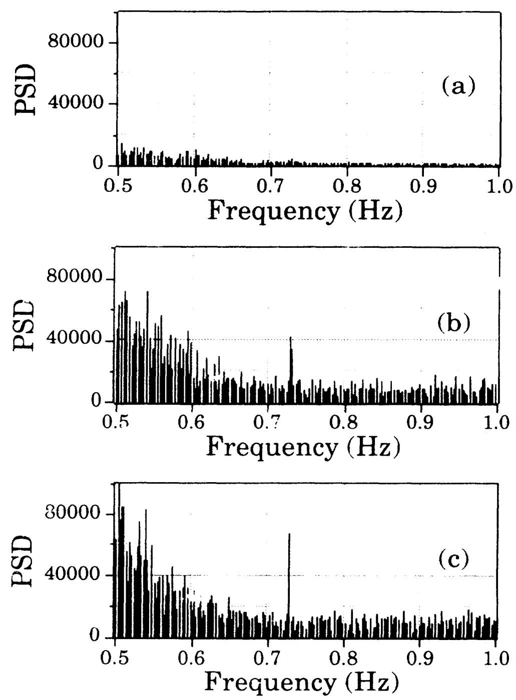

FIG. 2. Power spectra densities (PSD) obtained by a fast Fourier transform (FFT) on 20 filtered segments of 8192 points (with spectral range of 0 to 20 Hz of the FFT as dictated by the sampling rate of 23.4 ms), utilizing Hanning windowing with overlapping segments to minimize the spectral variance. The spectra at (a) input noise level of 2400 mV, (b) input noise level of 3800 mV, and (c) input noise level of 5000 mV. The maximum signal-to-noise ratio is demonstrated by the peak in (c) at the drive frequency $f=0.729$ Hz.

signal-processing board and passed standard statistical tests for independence and distribution. Consequently $\xi(t)$ closely approximated Gaussian white noise with a 3σ cutoff. An MTI Fotonic Sensor measured the curvature of the ribbon near its base. The data were time-series voltages $V(t)$ acquired from the output of the sensor. A schematic of the experiment is shown in Fig. 1. Other details of the experiment can be found in Refs. [15]–[17].

The sinusoidal modulation was set at $f=0.729$ Hz and $H_{ac}=0.23$ Oe and $H_{dc}=0.98$ Oe. These parameters provided a region where the ribbon dwelt exclusively in a buckled state on only one side of vertical (well B). The random noise signal $\xi(t)$ was then turned on and, above a threshold amplitude, switching occurred between buckled states in well B and well A. We have not compensated for the bias of the ribbon to favor one side (due to the manufacturing process of the ribbon) and, subsequently, when a transition from well B to well A is made, the ribbon almost immediately makes a transition back to well B. The ribbon spends less than 20% of the total time in well A because of the asymmetric potential.

The ribbon position was sampled every 23.4 ms for typical runs of 200 000 data points. The data were then filtered to assign a value of $+1$ if the ribbon was in the left buckled state (well B) and -1 if the ribbon was in the right buckled state (well A). A small amount of hysteresis was deliberately introduced during this filtering in order to prevent measurement noise from creating spurious transitions. The power spectrum was obtained by performing a fast Fourier transform on 20 filtered segments comprised of 8192 points each, utilizing a Hanning window and overlapping the segments one half of their length to obtain the smallest spectral variance [18]. Power spectra around the modulation frequency for different noise levels are shown in Fig. 2. The signal-to-noise ratio was seen to increase and then decrease with increasing noise, a characteristic of stochastic resonance, as shown in Fig. 3. An increase of 10 to 12 dB in the signal-to-noise ratio of our system was seen.

An additional signature for the existence of stochastic

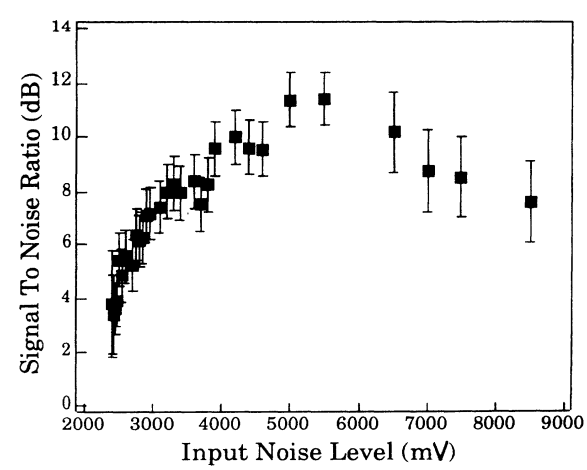

FIG. 3. Signal-to-noise ratio vs input noise. The signal-to-noise ratio increases and then decreases with increasing noise, a signature of stochastic resonance.

resonance is the sequence of maxima that appear in the probability density of residence times. The residence time is defined as the amount of time spent in one well. Consider the following system, which qualitatively reproduces the ribbon dynamics:

$$r\ddot{x}+\dot{x}-[b+pS_p(t)]x+x^3+c=aS_a(t) , \tag{1}$$

where

$$S_p(t)=q_p \sin(\omega_p t)+\sigma_p F_p(t) ,$$

$$S_a(t)=q_a \sin(\omega_a t)+\sigma_a F_a(t) ,$$

where x represents the position of the ribbon at the base, and $F_p(t)$ and $F_a(t)$ are Gaussian white noise with variance equal to 1.

The canonical system displaying stochastic resonance is a symmetric overdamped system with *additive* driving and *additive* noise [i.e., $r=0$, $p=0$, and $c=0$ in Eq. (1)]. In such a system undergoing stochastic resonance, there will be well-defined peaks in the residence-time histogram located at odd multiples of half the period of the modulating signal [2]. One can understand this structure by looking at a transition from well A to well B in the ribbon and counting the time spent in each well. If a transition from well B to well A takes place and if the reverse transition $A \rightarrow B$ does not take place at the next most favorable time (one half period later), then the system must wait one complete period of the modulation for the subsequent most favorable time for the $A \rightarrow B$ transition to take place. A total time equal to $3T_0/2$ would elapse since the original $B \rightarrow A$ transition. The magnetoelastic ribbon considered in this experiment is more accurately modeled as an asymmetric *parametrically* driven system with momentum (i.e., $r>0$, $p\neq 0$, $c\neq 0$, and $a\approx 0$) and *parametric* noise [19]. If it is assumed that there is no additive noise, then a nonzero momentum is required for interwell motion to occur. The asymmetry in the system is such that when the ribbon does make a transition $B \rightarrow A$ (where B is the large well and A is the small well), it almost always makes an immediate transition back to well B. In this case the locations of the peaks in the histogram for the large well will approach multiples of T_0 when measuring either the time in the well or the time between exits from the well. This is demonstrated experimentally in Fig. 4, where the residence time histogram is plotted for well B. It is seen that the peaks are located at integer multiples of the modulation period T_0.

Histograms qualitatively similar to Fig. 4 have been obtained by means of digital simulation of Eq. (1) with momentum, parametric modulation, noise, asymmetric wells, and no additive modulation or noise. The power spectrum of $x(t)$ (before applying a two state filter) yielded a peak at the driving frequency. The peak occurs because of the modulation *and* because of the asymmetry in the potential wells. In the symmetric case, the direction of switching between wells is not coherent and the motion due to the modulation within one well is exactly out of phase with the motion within the other well, the result being that there is no net contribution to the power spectrum at the driving frequency. The power spectrum taken after applying a two-state filter to $x(t)$ shows no peak at the driving frequency. This indicates that the peak in the power spectrum of $x(t)$ before the two-state filter was due solely to the motion within the large well and not to the switching between wells. Therefore the digital simulations reproduce the qualitative results shown in the experimental residence-time histogram of Fig. 4 but do not yield peaks in the power spectrum which would support the experimental data of Fig. 2.

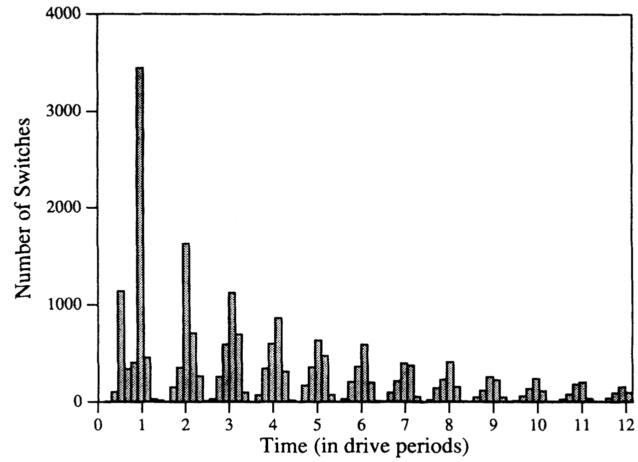

FIG. 4. Residence-time histogram for the large well (well B). Note the peaks at multiples of the driving period T_0.

CONCLUSION

We have observed stochastic resonance in a parametrically driven asymmetric magnetomechanical ribbon. This system showed a rise in the signal-to-noise ratio of 10 to 12 dB as a function of injected noise, which rise is a characteristic of stochastic resonance. However, observations of the residence-time histogram showed peaks at multiples of the modulation period versus the predicted (*for additive noise*) odd half-integer multiples of the drive period. Preliminary simulations of dynamical equations featuring purely parametric driving and noise (which equations qualitatively model this system) reproduced the qualitative aspects of the residence time histogram data but failed to yield a peak in the power spectrum at the driving frequency. We speculate that this disagreement may be due to the fact that, while magnetic forces on the ribbon are negligible to first order and while the driving force is predominately due to gravity and is controlled by the changes in Young's modulus, small magnetic forces (which would introduce additive driving and additive noise) may appear when the ribbon is considered in higher-order approximation.

ACKNOWLEDGMENTS

We gratefully acknowledge A. R. Bulsara, E. W. Jacobs, and R. D. Boss for all of their help. Additionally we would like to thank F. Moss and K. B. Hathaway for their inspiration, stimulating discussions, and technical expertise. This work was supported by the Office of Naval Research, the Naval Research Laboratory, and the Naval Surface Warfare Center's Independent Research Program.

[1] R. Benzi, S. Sutera, and A. Vulpiani, J. Phys. A **14**, L453 (1981); R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, Tellus **34**, 10 (1982); R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, SIAM J. Appl. Math. **43**, 565 (1983).
[2] T. Zhou and F. Moss, Phys. Rev. A **41**, 4255 (1990).
[3] B. McNamara and K. Wiesenfeld, Phys. Rev. A **39**, 4854 (1989).
[4] P. Jung and P. Hanggi, Europhys. Lett. **8**, 505 (1989); Phys. Rev. A **44**, 8032 (1991).
[5] R. Fox, Phys. Rev. A **39**, 4148 (1989).
[6] P. Jung, Z. Phys. B **76**, 521 (1989).
[7] P. Presilla, F. Marchesoni, and L. Gammaitoni, Phys. Rev. A **40**, 2105 (1989).
[8] A. Longtin, A. Bulsara, and F. Moss, Phys. Rev. Lett. **67**, 656 (1991).
[9] B. McNamara, K. Wiesenfeld, and Rajarshi Roy, Phys. Rev. Lett. **60**, 2626 (1988).
[10] L. Gammaitoni, M. Martinelli, and L. Pardi, Phys. Rev. Lett. **67**, 1799 (1991).
[11] T. Zhou, F. Moss, and P. Jung, Phys. Rev. A **42**, 3161 (1990).
[12] J. E. Rose, J. F. Brugge, D. J. Anderson, and J. E. Hind, J. Neurophysiol. **30**, 769 (1967).
[13] R. M. Siegel, Physica (Amsterdam) D **42**, 385 (1990).
[14] H. T. Savage and C. Adler, J. Magn. Magn. Mater. **58**, 320 (1986); H. T. Savage and M. L. Spano, J. Appl. Phys. **53**, 8092 (1982).
[15] H. T. Savage, W. L. Ditto, P. A. Braza, M. L. Spano, S. N. Rauseo, and W. C. Spring III, J. Appl. Phys. **67**, 5619 (1990).
[16] W. L. Ditto, S. Rauseo, R. Cawley, C. Grebogi, G.-H. Hsu, E. Kotelich, E. Ott, H. T. Savage, R. Segnan, M. L. Spano, and J. A. Yorke, Phys. Rev. Lett. **63**, 923 (1989); W. L. Ditto, M. L. Spano, H. T. Savage, S. N. Rauseo, J. Heagy, and E. Ott, *ibid.* **65**, 533 (1990); W. L. Ditto, S. N. Rauseo, and M. L. Spano, *ibid.* **65**, 3211 (1990).
[17] J. C. Sommerer, W. L. Ditto, C. Grebogi, E. Ott, and M. L. Spano, Phys. Rev. Lett. **66**, 1947 (1991).
[18] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes* (Cambridge University Press, Cambridge, England, 1986), pp. 420–429.
[19] J. Heagy and W. L. Ditto, J. Nonlinear Sci. **1**, 423 (1991).

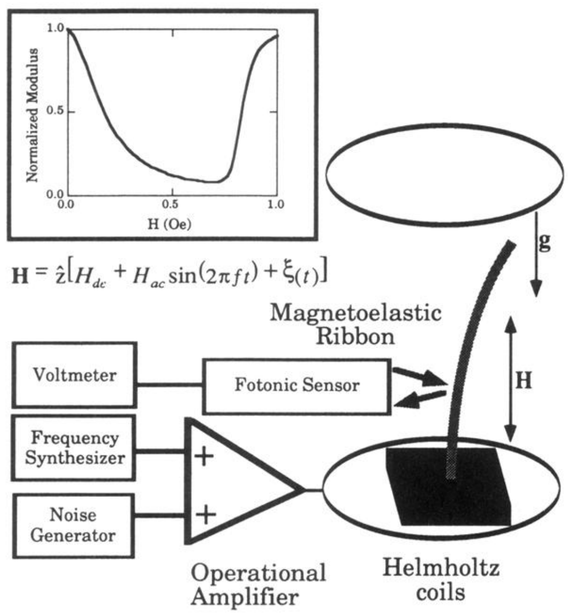

FIG. 1. Schematic of the magnetoelastic ribbon experiment. Inset: Normalized Young modulus for the ribbon vs magnetic field.

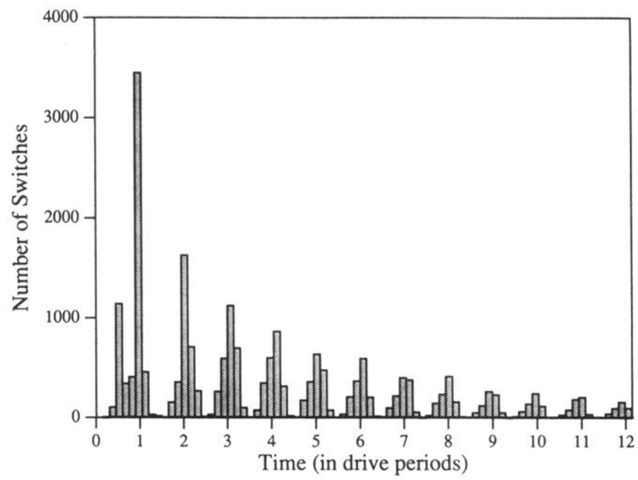

FIG. 4. Residence-time histogram for the large well (well *B*). Note the peaks at multiples of the driving period T_0.