Resonant kink-impurity interactions in the ϕ^4 model

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We study kink-impurity interactions in the ϕ^4 model, extending our previous results [Zhang Fei, Y. Kivshar, and L. Vázquez, Phys. Rev. A **45**, 6019 (1992)] to the case where the kink has an internal degree of freedom which may be excited due to scattering. We demonstrate numerically that the kink may be reflected by an attractive impurity when its velocity lies in some "windows," and this effect is due to a resonant energy exchange between the kink translational mode, its internal mode, and the impurity mode. We observe additional features in the resonant interactions, e.g., quasiresonances at some intermediate velocities and "three-bounce" resonances. We also develop an analytical approach taking into account three dynamical variables and show that such a collective-coordinate model may explain qualitatively the resonance structures observed in numerical simulations.

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I. INTRODUCTION

Wave propagation in nonlinear disordered media has become an extensively studied subject in recent years [1–3]. In linear systems disorder generally creates Anderson localization, which means that the transmission coefficient of a plane wave decays exponentially with the system length (see, e.g., Refs. [4, 5] and references therein). However, nonlinearity may drastically modify transport properties of disordered systems when it contributes to create solitons [3]. As a step towards understanding the soliton transmission through disordered media, one has to study in detail the scattering of a soliton by a single impurity.

In the previous theoretical studies related to kink-impurity interactions the possible excitation of the impurity modes was not taken into account (see, e.g. Ref. [6] for a summary of the main results). However, in our recent studies [7, 8], we have observed that, due to the resonant energy exchange between the kink translational mode and the impurity mode, a sine-Gordon (SG) kink can be totally reflected by an attractive impurity if its initial velocity lies in some well-defined resonance "windows." As we have pointed out, this effect is quite similar to the resonance phenomena in kink-antikink collisions in some nonlinear Klein-Gordon equations [9–11].

In the present paper, we analyze the resonance structures in the kink-impurity interactions in the ϕ^4 model extending our previous study to the kinks with internal modes. By numerical simulations we also observe a set of resonance windows in which the kink can be reflected by the attractive impurity. We show clearly that both the kink internal mode and the impurity mode are excited during the scattering, and the resonance phenomena observed are due to an energy exchange between the kink translational mode, kink internal mode, and the impurity mode, so that it is impossible to explain the resonance structure by taking into account only one localized oscillating mode. Moreover, we observe two interesting features in the resonant interactions: the missing of some intermediate resonance windows and the possibility of "three-bounce" resonance windows in which the kink can escape after colliding with the impurity for three times. Finally, we show that a collective-coordinate approach with *three* dynamical variables (namely, the kink coordinate, the amplitude of the kink internal mode, and the amplitude of the impurity mode) can provide a good qualitative explanation to the resonance structures observed in numerical simulations.

II. MODEL

The model we deal with is the well-known ϕ^4 system, which includes a local inhomogeneity,

$$u_{tt} - u_{xx} + [1 - \epsilon\delta(x)](u^3 - u) = 0, \tag{1}$$

where $\delta(x)$ is the Dirac δ function. The "pure" ϕ^4 model [$\epsilon = 0$ in Eq.(1)] has numerous applications in solid-state physics; e.g., it can describe structural phase transitions in ferroelectric and ferromagnetic materials [12–15], proton conductivity in quasi-one-dimensional systems like biological macromolecules and hydrogen-bonded chains [16, 17], nonlinear excitations in polymer chains [18, 19], etc. The solitary wave solutions (kinks) of the model usually correspond to domain walls in the phase transitions or polymerization mismatches in the dynamics of macromolecules.

In the absence of perturbations, the ϕ^4 model has the kink solution,

$$u_k(x,t) = \sigma \tanh\left(\frac{x - X(t)}{\sqrt{2}\sqrt{1-V^2}}\right), \tag{2}$$

where $\sigma = \pm 1$ is the kink polarity, $X(t) = Vt + X_0$ is the kink coordinate, and $\dot{X} = V$ is its velocity. Linearizing Eq. (1) around the static kink ($V = 0$), one may obtain a linear equation that can be solved analytically. As is well known [9], the corresponding eigenvalue problem has two discrete modes with angular frequencies $\omega_0 = 0$ and $\omega_1 = \sqrt{3/2}$ (in this notation the continuous spectrum starts at $\omega = \sqrt{2}$). The zero-frequency mode corresponds to the translation of the kink. The mode with frequency $\omega_1 = \sqrt{3/2}$ describes localized deformations around the kink, and it can be considered as an internal oscillation of the kink shape. The normalized eigenfunction of this shape (internal) mode is

$$\eta(x) = (9/8)^{1/4} \tanh\left(\frac{x}{\sqrt{2}}\right) \cosh^{-1}\left(\frac{x}{\sqrt{2}}\right). \tag{3}$$

As is well known, the existence of the kink internal mode is very important to the resonance phenomena in kink-antikink collisions [9–11] because the internal mode may have energy exchange with the kink translational mode during the collisions. Further, it was shown analytically [20] that the ϕ^4 equation admits a long-lived wobbling kink solution, i.e., a kink with an excited internal mode.

In the presence of the impurity, the ϕ^4 equation (1) supports also a localized solution at the impurity site, the so-called *impurity mode*. To find the shape of the impurity mode we may linearize the ϕ^4 equation near the vacuum states $\phi = \pm 1$ obtaining the localized solution,

$$\Delta(x,t) = a_0 \cos(\Omega t + \theta) e^{-\epsilon|x|}, \tag{4}$$

where Ω and ϵ are connected by the dispersion relation

$$\Omega^2 = 2 - \epsilon^2. \tag{5}$$

Note that the impurity mode (4),(5) exists, provided $\epsilon > 0$ (an attractive impurity). As follows from Eq. (4), the impurity mode is periodic in time and localized in space (it falls off exponentially). The energy stored in the impurity mode is $\Omega^2 a_0^2 / 2\epsilon$.

III. NUMERICAL SIMULATION RESULTS

Let us consider the kink-impurity interactions in the ϕ^4 model (1). Suppose that there is a kink propagating toward the impurity (from far away) with a constant initial velocity V_i. When the kink is sufficiently close to the impurity, its steady-state motion will be changed because the impurity may either repulse ($\epsilon < 0$) or attract ($\epsilon > 0$) the kink. Moreover, the kink will emit radiation in the form of linear waves. Due to the kink scattering, the impurity mode as well as the kink internal mode may be excited. For such a nonlinear interaction process, it is impossible to give an exact analytical solution. So first, we will resort to numerical simulations.

We use a conservative numerical scheme to discretize

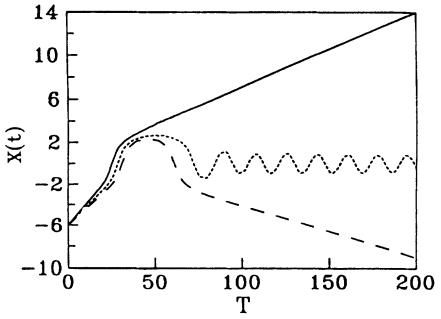

FIG. 1. The kink coordinate X vs time (at $\epsilon = 0.5$). The kink initial velocity V_i is taken from the three different regions: the region of pass (solid line), of capture (dotted line), and of reflection (dashed line).

Eq. (1), and carry out simulations for the case $\epsilon > 0$ (attractive impurity). The initial condition is taken as a kink starting at $X_0 = -6$ with the initial velocity $V_i > 0$ moving toward the impurity. The main ingredients of our simulation results are summarized as follows.

First, we take the impurity amplitude $\epsilon = 0.5$. In the numerical simulations, we find that there are three different regions of the kink initial velocity, namely, the regions of pass, of capture, and of reflection (see Fig. 1). The region of pass is a velocity interval $(V_c, 1)$, $V_c \approx 0.1850$ being the critical velocity, such that if the kink initial velocity is taken from it, the kink will pass the impurity and escape to the positive infinity. Such an interaction is inelastic because the kink loses a part of its kinetic energy to emit radiation and to excite its internal mode and the impurity mode (see Fig. 2). In this case there is a linear relationship between the squares of the kink initial velocity V_i and its final velocity V_f, i.e.,

$$V_f^2 = \alpha(V_i^2 - V_c^2), \tag{6}$$

where the constant parameter α (≈ 0.85 at $\epsilon = 0.5$) is determined empirically from numerical data.

Below the critical velocity V_c, the final result of the kink-impurity interaction is very sensitive to the initial kink velocity. More precisely, if the incoming velocity of the kink is smaller than V_c, the kink cannot escape from

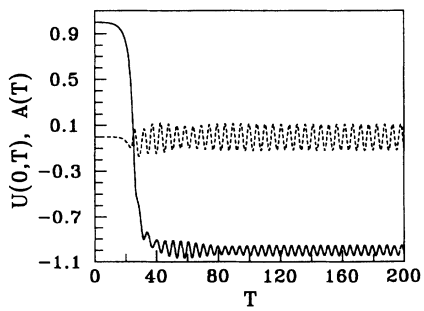

FIG. 2. Excitation of the impurity mode (solid line) and the kink internal mode (dashed) at $V_i = 0.2$. The amplitude of the internal mode is calculated by using a projection technique.

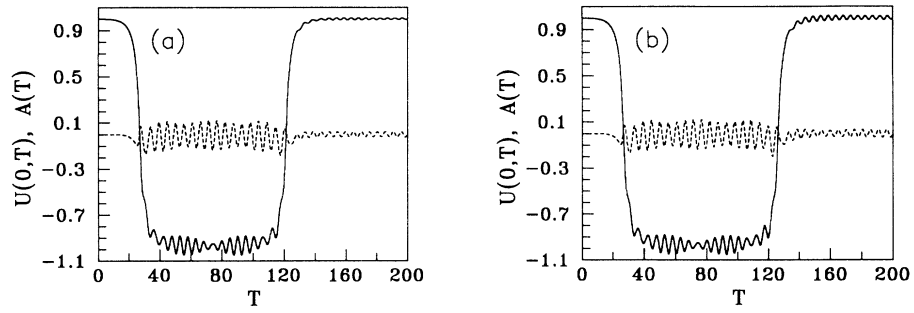

FIG. 3. The impurity displacement $u(0,t)$ (solid line), and the amplitude of the kink internal mode (dashed). The kink initial velocities are taken from the resonance windows: (a) $V_i = 0.182\,25$ and (b) $V_i = 0.182\,53$. Excitation and deexcitation of the two modes are clearly seen.

the impurity after the first interaction, but it will stop and then return to interact with the impurity again. For most of the initial velocities, the kink will lose its energy again in the second interaction and finally get trapped by the impurity. However, for some special initial velocities, the kink may escape to the negative infinity after the second interaction, i.e., the kink may be totally reflected by the attractive impurity (see, e.g., Fig. 3). Note that the kink reflection is possible only if its initial incoming velocity is situated in some well-defined narrow resonance "windows" (the regions of the kink reflection). By numerical simulation, we have found a series of such windows for different values of ϵ. The detailed results for $\epsilon = 0.5$ are presented in Table I and Fig. 4.

We notice that the reflection of the kink is realized by two steps: the first interaction removes kinetic energy from the kink and make it trapped by the impurity, while the second interaction retransfers enough energy to the kink translational motion and allows it to escape from the attractive impurity (Fig. 3). To explain this behavior one clearly needs a mechanism for transferring and restoring energy. As a matter of fact, the energy transfer process can be clearly seen in Figs. 3(a)–3(b), where the typical pictures of resonances are presented. From Fig. 3 we can observe the impurity mode and the kink internal mode ocillations between the two interactions, but after the kink reflection the oscillations almost vanish, which means that the energy in the local modes is retransferred back to the kink translational mode.

To analyze the problem quantitatively, we define the center of the kink $X(t)$ as the spatial point x at which the field function $u(x,t)$ is equal to zero. Let $T_{12}(V_i)$ be the time between the first and second interactions, V_i being the kink initial velocity. More precisely, $T_{12}(V_i)$ is the time difference between the first two instants at which the center of the kink is just at the impurity. Similarly, we define $T_{23}(V_i)$ as the time interval between the second and the third collisions.

It will be seen in the next section that the attractive potential created by the impurity falls off exponentially, so we can use a simple argument of classical mechanics as in Ref. [9] [see, e.g., Eq. (3.6) of Ref. [9]] to obtain an approximate formula to estimate $T_{12}(V_i)$,

$$T_{12}(V_i) = \frac{a}{\sqrt{V_c^2 - V_i^2}} + b, \tag{7}$$

where the parameters a and b are (empirically) determined by numerical data. At $\epsilon = 0.5$, we find $a \approx 3.0$, $b \approx -0.4$. Table I shows that this formula is rather good for the kink initial velocities lying in the interval $(0.136, 0.183)$.

In our previous work [7, 8] we showed that the resonance condition,

$$T_{12}(V) = nT + \tau, \tag{8}$$

is always satisfied in case of kink reflection, where $T = 2\pi/\omega_{\rm im}$ is the period of the impurity-mode oscillation, τ is an offset phase, and n is an integer. However, it is easy to verify that this resonance condition is not valid for the resonant kink-impurity interactions in the present model, and it is impossible to use a general formula, $V_n^2 = V_c^2 - \beta/(nT + \delta)^2$, to predict the resonance windows. Furthermore, from Table I it is seen that the average difference between $T_{12}(V_{n+1})$ and $T_{12}(V_n)$ is about 5.30, which is more or less close to the kink internal mode period $T_{\rm in} = 5.13$, but far away from the impurity mode period $T_{\rm im} = 4.75$. From Figs. 2 and 3(a)–3(b) we can

TABLE I. Two-bounce resonance windows in the kink-impurity interactions in the ϕ^4 model (1) at $\epsilon = 0.5$.

n	Resonance windows	Window centers V_n	$T_{12}(V_n)$ Numerical	$T_{12}(V_n)$ From Eq. (7)
5	(0.136, 0.1390)	0.1375	24.8	23.9
6	(0.1548, 0.1560)	0.1555	30.1	29.5
7	Quasiresonance	0.1648	35.6	35.3
8	Quasiresonance	0.1703	41.2	41.1
9	(0.1735, 0.1739)	0.1737	46.6	46.7
10	(0.1757, 0.1761)	0.1759	51.8	51.9
11	(0.1774, 0.1777)	0.1775	57.0	57.1
12	(0.178 65, 0.1788)	0.178 73	62.4	62.4
13	(0.179 63, 0.1797)	0.179 66	67.6	67.6
14	Quasiresonance	0.180 43	73.1	73.0
15	(0.181 02, 0.181 08)	0.181 05	78.6	78.5
16	(0.181 49, 0.181 58)	0.181 53	83.8	83.7
17	(0.181 89, 0.181 97)	0.181 92	89.0	88.9
18	(0.182 21, 0.182 28)	0.182 25	94.1	94.0
19	(0.182 51, 0.182 56)	0.182 53	99.1	99.2
20	(0.182 76, 0.182 79)	0.182 77	104.4	104.4

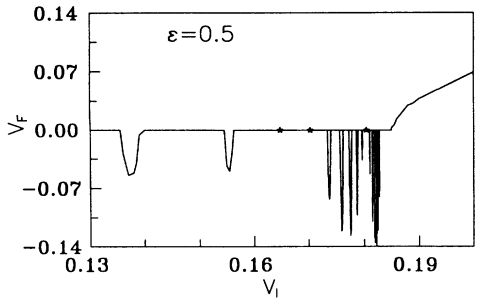

FIG. 4. Final kink velocity as a function of the initial kink velocity ($\epsilon = 0.5$). Zero final velocity means that the kink is captured by the impurity. The three stars indicate the quasiresonance velocities (missing windows).

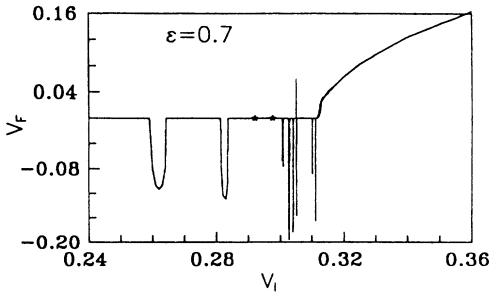

FIG. 6. The same as in Fig. 4 but at $\epsilon = 0.7$. The two stars indicate the quasiresonance velocities (missing windows). Additionally, there is a very narrow "three-bounce" resonance window at $V_i = 0.305$, for which the kink is transmitted (with final velocity being positive) after the third collision.

observe clearly the excitation and deexcitation of the kink internal mode whose amplitude is considerably large, so it seems that the kink *internal mode* is more *important* than the impurity mode in the resonant interactions.

Another surprising feature in the resonance structure is that there are some intermediate windows that do not show up, but are replaced by quasiresonances. Analyzing the numerical data of Table I, and applying the resonance condition (8) (with T=5.13), we expected to observe the next two windows near velocities V_7 and V_8, such that $T_{12}(V_7) \approx 35.4$, $T_{12}(V_8) \approx 40.7$. Although a much smaller velocity step ($\Delta V = 10^{-5}$) has been used to scan these windows, we have not found real kink reflection. Instead, we have only observed quasiresonances, i.e., if the kink initial velocity is close to some velocities (see Table I), the second interaction will cause the kink to be reflected farther away from the impurity, or equivalently speaking, the time between the second and third interactions, $T_{23}(V_i)$, will be larger (see Fig. 5), yet the kink cannot escape to infinity. Therefore, the resonance structures observed here are quite different from those for the SG model in which the resonance windows come out one by one and they can be easily predicted by a simple formula.

We have also investigated the kink-impurity interactions in the case of $\epsilon = 0.7$, and observed the similar phenomena as described above. In this case, the critical velocity V_c is found to be about 0.3124, above which the kink will pass the impurity inelastically and escape to infinity. Below the critical velocity, eight two-bounce resonance windows are observed (see Fig. 6). In particular, we also observed high-order resonance windows which are very close to the critical velocity. In Fig. 7 we can see that between the first and the second interaction there are a lot of small bumps that show the impurity oscillations modulated by the kink internal-mode oscillations. We also observe that there are two intermediate-resonance windows replaced by quasiresonance effects (as in the case of $\epsilon = 0.5$). Most strikingly, here we observe a "three-bounce" resonance window (see Figs. 6 and 8), in which the kink is transmitted after colliding with the impurity for three times. Note that such a higher bounce resonance phenomenon has not been observed in the SG model [7, 8].

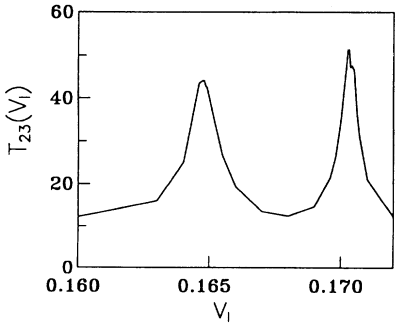

FIG. 5. The time between the second and the third interaction of the kink with the impurity, $T_{23}(V_i)$, vs the kink initial velocity. The quasiresonance phenomena are shown by the two peaks in the curve.

IV. COLLECTIVE-COORDINATE APPROACH

We would like to mention that the inelastic interaction of a kink with an attractive impurity in the ϕ^4 model was discussed by Belova and Kudryavtsev [21]. They observed only one resonance window and they tried to explain the phenomenon with the help of a collective-coordinate approach, taking into account the kink translational mode and its internal mode. However, as we have shown above, the resonances are found to be a *joint* effect of the kink internal mode and the impurity mode. There-

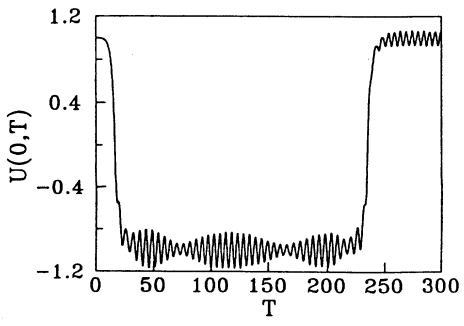

FIG. 7. The impurity displacement $u(0,t)$ vs time, showing a higher-order resonance at $V_i = 0.3111$ and $\epsilon = 0.7$.

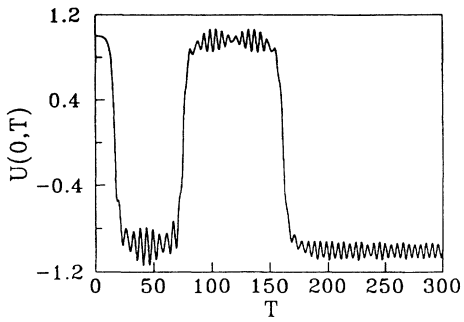

FIG. 8. The impurity displacement $u(0,t)$ vs time, showing a three-bounce resonance at $V_i = 0.305$ ($\epsilon = 0.7$). The kink is transmitted after the third collision.

fore the resonance structures must be explained with a generalized collective-coordinate approach.

The fundamental approximation involved in this approach is the replacement of the (infinitely) many degrees of freedom of a kink in a continuum field equation by a (finite) number of collective coordinates. Such a reduction is usually helpful for one to extract the main physical mechanism underlying the observed nonlinear phenomena.

There are different ways to obtain the equations of motion for the collective coordinates (see, e.g., [6]). However, it has been recognized that the Lagrangian approach is very useful for studying the kink dynamics [7, 8, 21]. The first step in this approach is to derive an effective Lagrangian by inserting a certain *ansatz* into the system Lagrangian. Then the equation of motion for the collective coordinates may be derived in a standard variational way. In our case, Eq. (1) is defined by the Lagrangian

$$L = \int_{-\infty}^{\infty} dx \left[\frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 - \frac{1}{4}[1-\epsilon\delta(x)](u^2-1)^2 \right]. \tag{9}$$

Since the impurity mode and the internal mode are excited in the scattering and they can be considered as small-amplitude oscillating states, we propose the following more general ansatz:

$$u(x,t) = \tanh\left(\frac{x-X(t)}{\sqrt{2}}\right) + A(t)\eta(x) + a(t)e^{-\epsilon|x|}, \tag{10}$$

where $\eta(x)$ is the kink internal (shape) mode given by Eq. (3), $A(t)$ being its amplitude, and $a(t)e^{-\epsilon|x|}$ represents the impurity mode (4). Substituting Eq. (10) into the system Lagrangian (9), in the lowest-order approximation we obtain the following (reduced) effective Lagrangian:

$$L_{\text{eff}} = \frac{M_k}{2}\dot{X}^2 - U(X) + \frac{1}{2}\dot{A}^2 - \frac{\omega_1^2}{2}A^2 - AF(X) + (1/\epsilon)\left(\frac{1}{2}\dot{a}^2 - \frac{\Omega^2}{2}a^2\right) - aG(X) - aAD(X), \tag{11}$$

where $\omega_1 = \sqrt{3/2}$ and $\Omega = \sqrt{2-\epsilon^2}$ are the frequencies of the kink internal and the impurity modes, respectively, and $M_k = 2\sqrt{2}/3$ is the kink mass. The functions are

$$U(X) = -\frac{\epsilon}{4\cosh^4(X/\sqrt{2})}, \tag{12}$$

$$F(X) = (9/8)^{1/4}\frac{\epsilon\tanh^2(X/\sqrt{2})}{\cosh^3(X/\sqrt{2})}, \tag{13}$$

$$G(X) = -\frac{\epsilon\tanh(X/\sqrt{2})}{\cosh^2(X/\sqrt{2})}, \tag{14}$$

$$D(X) = (9/8)^{1/4}\frac{2\epsilon\tanh(X/\sqrt{2})}{\cosh(X/\sqrt{2})}. \tag{15}$$

The equations of motion derived from the Lagrangian (11) are

$$\begin{aligned} &M_k\ddot{X} + U'(X) + AF'(X) + aG'(X) + aAD'(X) = 0, \\ &\ddot{A} + \omega_1^2 A + F(X) + aD(X) = 0, \\ &(1/\epsilon)[\ddot{a} + \Omega^2 a] + G(X) + AD(X) = 0. \end{aligned} \tag{16}$$

The system (16) describes a particle coupled with *two* oscillators, which correspond to the kink internal mode and the impurity mode, respectively. The two oscillators are also coupled through the term $aAD(X)$. The system (16) may be useful, e.g., to estimate analytically the amplitudes of the impurity mode and the kink internal mode excited due to the kink scattering in the way earlier used for the SG model [22] (see also the recent paper [23] where the similar calculations for the ϕ^4 model have been done with the help of the perturbation theory).

We have found that the system (16) can describe most of the features of the kink-impurity interactions observed in direct numerical simulations. To see this, we have simulated the system (16) under the suitable initial conditions corresponding to the problem of kink-impurity interactions. At $\epsilon = 0.5$, we find that there is a critical velocity $V_c \approx 0.26586$, above which the particle will pass the potential well inelastically, because it loses energy to excite the two oscillators. The final velocity of the particle satisfies the relation $V_f^2 = \alpha(V_i^2 - V_c^2)$, with $\alpha \approx 0.726$.

If the initial velocity of the particle is smaller than the critical value V_c, the particle cannot escape to infinity after the first interaction, instead it will stop and then return to interact with the oscillators again. Usually the particle will lose its energy again, and it remains trapped in the potential well during the simulation time $0 \le T \le 300$. This corresponds to the kink capture by the impurity. However, if the initial velocity of the particle is taken from some well-defined "windows," the particle will escape to $-\infty$ after the second interaction (see Fig. 9), this effect corresponds to the reflection of the kink by the impurity.

Just as in the previous section, here we notice that the time between the first and second interactions can be estimated by $T_{12}(V_i) \approx 2.801/\sqrt{V_c^2 - V_i^2} - 1.0$. However, in this case the resonance windows *cannot* be predicted by a standard formula because the resonance condition (8) no longer holds. Nevertheless, by careful numerical simulations, we have detected 13 two-bounce resonance

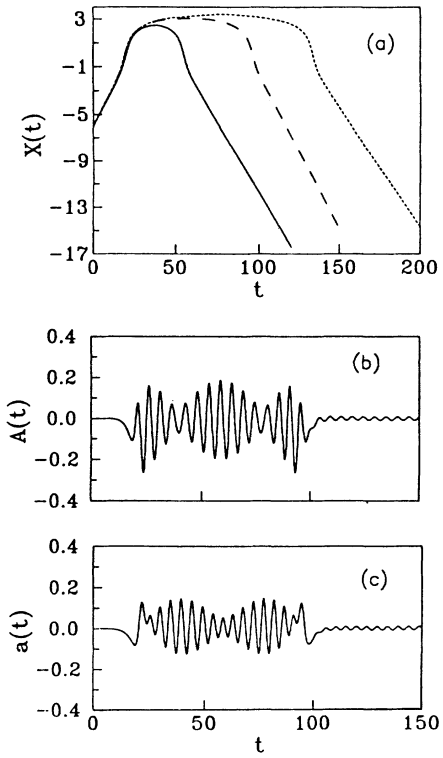

FIG. 9. Resonant reflection of the particle described by Eqs. (16). (a) The particle coordinate $X(t)$ vs time at initial velocities $V_i = 0.252$ (solid line), 0.2634 (dashed), 0.264 75 (dotted). (b),(c) Excitation and deexcitation of the two oscillators in the case of resonance, $V_i = 0.2634$, showing a typical resonant energy exchange process.

windows (see Fig. 10). Most surprisingly, we observe the qualitatively similar structure of the resonance windows; e.g., some (intermediate) two-bounce resonance windows are missing. More precisely, at the initial velocities near $V_i = 0.2609$, 0.2620, 0.264 33, and 0.264 54, we expected to observe reflection, but actually only quasiresonance effects are observed. Furthermore, from Fig. 10 we can see that the missing windows have a certain influence on their neighboring windows, i.e., in the neighboring windows the corresponding final velocities of the particle are smaller. In this respect, it is clear that Fig. 10 is qualitatively like Figs. 4 and 6.

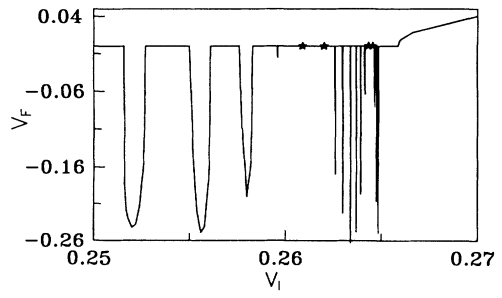

FIG. 10. Structure of two-bounce resonances in the collective-coordinate system (16) [cf. Figs. 4 and 6]. The four stars indicate the missing two-bounce resonance windows.

It is not difficult to understand why some two-bounce windows may disappear in the system (16). Since the mechanism underlying the resonance is an energy exchange between the particle and the two oscillators, the resonance process can be divided into two steps: First, an incident particle loses its kinetic energy to excite the oscillators and get trapped by the potential well. Second, the particle comes back to the interaction region and has energy exchange again with the excited oscillators. Whether the particle can escape or not after the second interaction will depend on how much net energy the particle may obtain from the two oscillators. Because the frequencies of the oscillators are generally not commensurable, it may happen that even if the particle can restore energy from one, it may transfer energy to another. As a result, the particle cannot obtain enough kinetic energy to escape from the potential well. Therefore, instead of true resonances, only quasiresonance effects are observed at certain initial velocities.

Since the system (16) is Hamiltonian, it easily allows higher bounce (e.g., three-bounce) resonances, which means that a kink can be transmitted or reflected after colliding with the impurity for a number of times. However in real interactions these resonances are strongly influenced by radiation, so that they were not found in the sine-Gordon model [7, 8]. Nevertheless, due to the interplay between several oscillating modes such resonances are possible in the nonlinear systems supporting kinks with internal degrees of freedom, and in the present paper, we indeed observe a three-bounce resonance for the ϕ^4 model (see Fig. 8).

V. CONCLUSIONS

In conclusion, we have studied the kink scattering by an attractive impurity in the ϕ^4 model and demonstrated that below a certain threshold velocity the kink is not necessarily to be captured by the attractive impurity due to radiative losses; instead, the kink may still escape to infinity if its initial velocity is located in some resonance windows. We have shown that this effect is a result of the resonant energy exchange between the kink translational mode, its internal mode, and the impurity mode, and we have determined numerically the structure of two-bounce windows. In comparison with the SG model [7, 8], we have found different features in the kink-impurity interactions for the present model; e.g., the missing of some intermediate windows that are replaced by quasi-resonance effects, "three-bounce" resonances in which the kink can be transmitted after colliding with the impurity for three times. We have demonstrated that the collective-coordinate approach, taking into account three dynamical variables (namely, the kink coordinate, the amplitude of the kink internal-mode oscillations, and the amplitude of the impurity-mode oscillations), is suitable to describe properly the resonance structures in the kink-impurity interactions observed in the ϕ^4 model.

Finally, we would like to point out again that the resonant effects observed in the kink-impurity interactions in

the ϕ^4 model, as well as in the SG model [7, 8], are analogous to the resonance phenomena in the kink-antikink collisions in some nonlinear Klein-Gordon equations. The main physical mechanism of the resonances is an energy exchange between the kink translational motion and one, or several localized oscillating modes. Such a mechanism has been clearly shown in the direct numerical simulations and the collective-coordinate approach developed in the present paper.

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