PHYSICAL REVIEW A VOLUME 46, NUMBER 7 1 OCTOBER 1992

Interaction of a two-level atom with a cavity mode in the bad-cavity limit

J. I. Cirac
Departamento de Fisica Aplicada, Facultad de Ciencias Quimicas, Paseo de la Universidad 4, 13071 Ciudad Real, Spain
(Received 20 February 1992)

The interaction of a two-level atom with a single mode of the radiation field is examined. The cavity mode is damped by coupling to a reservoir in a squeezed state, and the atom is also driven by an external laser field. A master equation for the atomic density operator in the bad-cavity limit is derived. The effect of the squeezing in the stationary population inversion and in the resonance spectrum of the light emitted by the atom into the background modes is studied. The results are compared with the numerical solution of the full master equation for the atom-plus-cavity mode-density operator.

PACS number(s): 42.50.Lc, 42.50.Dv, 42.50.Hz

I. INTRODUCTION

The behavior of atoms inside cavities has revealed interesting features in their interaction with the radiation field. It has been known for a long time that the dynamical behavior of an atom depends on the statistical properties of the electromagnetic field in which it is embeded. Inside a cavity these properties are different than in the free space, since the mode structure is different. This fact led Pourcel [1] to predict an enhancement in the spontaneous-emission rate when an atom is situated in a resonant cavity. Analogously, Kleppner [2] pointed out the possibility of inhibiting the spontaneous-emission process when atomic transitions are far from the cavity resonance. These modifications in the atomic decay rates have been demonstrated experimentally [3].

In the absence of dissipation the Jaynes-Cummings model [4] describes the dynamical behavior of a single two-level atom in a cavity. However, in most cases dissipation must be taken into account, since the atom might spontaneously emit in other modes (background modes) than the privileged cavity mode, and the cavity mode coupled to the atom also experiences losses via the cavity mirrors. Hence, the interaction of an atom with the electromagnetic field in a cavity is mainly governed by three parameters: the coupling constant between the atom and the cavity mode g, the cavity-mode loss rate κ, and the atomic spontaneous-decay rate into the background modes γ. The relative magnitude between these parameters determines the dynamical and stationary behavior of the atom in the cavity. For example, for $g \gg \kappa, \gamma$ (good-cavity limit [5]), the problem reduces to the damped Jaynes-Cummings model [6, 7], where damping is considered as a small perturbation, and therefore characteristic features of this model, such as collapses and revivals [8], vacuum Rabi splitting [9], and other spectral features [10] appear. In the opposite limit $\kappa \gg g, \gamma$ (bad-cavity limit), the problem essentially reduces to the spontaneous emission in the free space, since the field in the cavity may be regarded as a broadband electromagnetic reservoir. This situation can change dramatically if an external coherent field drives the atom simultaneously. In this case, the laser field induces a frequency shift between the levels of the combined atom-plus-laser field system (dressed levels). This can lead to various effects, such as a positive stationary population inversion [11, 12], changes in the atomic decay rates [13], photon antibunching, squeezing [5, 14], etc. On the other hand, the modifications in the atomic decay when an atom is damped by a broadband squeezed reservoir inside a cavity have been studied recently [15, 16]. In the bad-cavity limit and for very strong laser fields, the atom can be decoupled from the cavity, when some phase condition between the squeezed and the laser field is fulfilled. This is in contrast with what occurs when the atom is damped by a squeezed reservoir in the free space [17–20], where the decay of only one of the two components of the atomic polarization can be inhibited.

In this paper we derive a master equation for a two-level atom interacting with a cavity mode and driven by a laser field in the bad-cavity limit. The cavity mode is assumed to be coupled to an ideal broadband squeezed reservoir via the cavity mirrors and the spontaneous emission from the atom into the background modes in a broadband vacuum state is also considered. The condition $\kappa \gg g$ implies that the cavity-mode response to the squeezed reservoir is much faster than to that produced by its interaction with the atom. Then, the atom always "sees" the cavity mode in the state induced by the squeezed reservoir, which permits one to adiabatically eliminate the cavity-mode variables. This leads to a master equation for the atomic variables only, which can be exactly solved. In particular, we find that the stationary population inversion can be positive and it increases with the squeezing when some phase relation is fulfilled, reaching larger values than in the case in which the cavity mode is coupled to the normal vacuum [11]. We also give an analytical expression for the fluorescence spectrum emitted by the atom into the background modes, which mirrors the modifications in the atomic decay produced by the squeezed reservoir. All these results are illustrated with the numerical solution of the full master equation for the atom-plus-cavity mode-density operator.

The plan of this paper is as follows. In Sec. II the master equation for the atomic density operator is derived and discussed in terms of the dressed states of the atom-plus-laser field system. In Sec. III we analyze the stationary population inversion. The spectrum of the light emitted by the atom into the background modes is studied in Sec. IV. Section V contains a summary of our results. Finally, in the Appendix we present the method we used to perform the numerical solution of the full master equation for the atom-plus-cavity mode-density operator.

II. MASTER EQUATION IN THE BAD-CAVITY LIMIT

A. Model

We consider a single two-level atom coupled to a cavity mode and excited by an external laser field [21]. The master equation for the atom-plus-cavity mode-density operator ρ, in a frame rotating at the laser field frequency ω, is ($\hbar = 1$)

$$\frac{d\rho}{dt} = -i[H,\rho] + L_{sq}\rho + L_a\rho. \tag{2.1}$$

The Hamiltonian H contains the free atomic evolution and the interaction of the atom with the cavity mode and the laser field, and it is given by

$$H = H_{\Omega'} + V, \tag{2.2}$$

where

$$H_{\Omega'} = \frac{\Delta}{2}\sigma_z + \frac{\Omega}{2}(\sigma^+ + \sigma^-) \equiv \frac{\Omega'}{2}\sigma_w, \tag{2.3}$$

$$V = g(\sigma^+ a + a^\dagger \sigma^-). \tag{2.4}$$

Here, $\sigma^\pm$ and σ_z are atomic spin-$\frac{1}{2}$ operators, a and $a^\dagger$ the creation and annihilation operators for the cavity mode, g the atom-cavity mode coupling parameter, and $\Omega' = (\Omega^2 + \Delta^2)^{1/2}$ the generalized Rabi frequency of the atom-laser interaction.

Dissipation from the atom via spontaneous-emission into the background modes and from the cavity mode by coupling to a broadband reservoir in a squeezed vacuum state are included in (2.1) by the terms

$$L_a\rho = \frac{\gamma}{2}(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-), \tag{2.5}$$

and

$$\begin{aligned} L_{sq}\rho = {} & \kappa(N+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ & + \kappa N(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger) \\ & - \kappa M e^{i\phi}(2a^\dagger\rho a^\dagger - a^{\dagger 2}\rho - \rho a^{\dagger 2}) \\ & - \kappa M e^{-i\phi}(2a\rho a - a^2\rho - \rho a^2), \end{aligned} \tag{2.6}$$

respectively. γ is the atomic spontaneous-emission rate, 2κ the cavity damping rate, and N and M characterize the squeezing of the reservoir. The relative phase between the squeezed quadrature of the reservoir and the laser field is given by $\phi/2$, where $\phi = 0$ ($\phi = \pi$) corresponds to a driving field in phase with the maximally squeezed (unsqueezed) quadrature.

In the following we will concentrate in the case of an ideal squeezing, where the relationship $M^2 = N(N+1)$ is fulfilled. In this case, in the absence of coupling with the atom ($g = 0$) the cavity mode is only driven by the squeezed reservoir and therefore it ends up in a squeezed state with N being the mean photon number.

Now, let us express master equation (2.1) in a slightly different form. To do this, we make the unitary transformation defined by

$$\hat{\rho} = S\rho S^\dagger, \tag{2.7}$$

where S is the usual squeeze operator [22] that transforms the annihilation and creation operators as

$$SaS^\dagger = \mu a + \nu a^\dagger,$$
$$Sa^\dagger S^\dagger = \nu^* a + \mu a^\dagger,$$

with $\mu = \sqrt{N+1}$ and $\nu = \sqrt{N}e^{i\phi}$. In the new picture, we find

$$\frac{d\hat{\rho}}{dt} = -i[H_{\Omega'} + \tilde{V}, \hat{\rho}] + L_{\text{vac}}\hat{\rho} + L_a\hat{\rho}, \tag{2.8}$$

where now

$$\tilde{V} = g(\tilde{\sigma}^+ a + a^\dagger\tilde{\sigma}^-) \tag{2.9}$$

gives the effective coupling atom-cavity mode, and

$$L_{\text{vac}}\hat{\rho} = \kappa(2a\hat{\rho}a^\dagger - a^\dagger a\hat{\rho} - \hat{\rho}a^\dagger a) \tag{2.10}$$

describes the cavity loss due to its coupling to an electromagnetic reservoir in a vacuum state. In (2.9), we have defined the atomic operators $\tilde{\sigma}^+$ and $\tilde{\sigma}^-$ as

$$\tilde{\sigma}^+ = \mu\sigma^+ + \nu^*\sigma^-, \tag{2.11a}$$

$$\tilde{\sigma}^- = \nu\sigma^+ + \mu\sigma^-. \tag{2.11b}$$

Note that in this picture the cavity mode is damped by the vacuum and the effect of the squeezed reservoir is transferred to the atom-cavity mode coupling through the replacement $\sigma^+ \to \tilde{\sigma}^+$. Hence, for $g = 0$ the final state of the cavity mode is now the normal vacuum.

Master equation (2.8) can be solved numerically by transforming it in an infinite hierarchy of ordinary differential equations by using characteristic technics [23] (see the Appendix) or, alternatively, by projecting it on a particular set of basis states [24]. However, here we are interested in an analytical expression in the bad-cavity limit.

B. Bad-cavity limit

In order to obtain the master equation in the bad-cavity limit we proceed now by transforming to a new "dissipation picture" defined by [7]

$$\tilde{\rho}(t) = e^{-L_{\text{vac}}t}\hat{\rho}(t). \tag{2.12}$$

In this picture, master equation (2.8) becomes

$$\begin{aligned} \frac{d\tilde{\rho}}{dt} = {} & -ige^{-\kappa t}(a[\tilde{\sigma}^+, \tilde{\rho}] + [\tilde{\sigma}^-, \tilde{\rho}]a^\dagger) \\ & -ige^{\kappa t}([a, \tilde{\rho}]\tilde{\sigma}^+ + \tilde{\sigma}^-[a^\dagger, \tilde{\rho}]) - i[H_{\Omega'}, \tilde{\rho}] + L_a\tilde{\rho} \\ \equiv {} & e^{-\kappa t}L_1\tilde{\rho} + e^{\kappa t}L_2\tilde{\rho} - i[H_{\Omega'}, \tilde{\rho}] + L_a\tilde{\rho}, \end{aligned} \tag{2.13}$$

where we have used the following relations:

$$L_{\rm vac}(a\rho) = a(L_{\rm vac}+\kappa)(\rho),$$
$$L_{\rm vac}(\rho a^\dagger) = [(L_{\rm vac}+\kappa)(\rho)]a^\dagger,$$
$$L_{\rm vac}([a,\rho]) = [a,(L_{\rm vac}-\kappa)\rho],$$
$$L_{\rm vac}([a^\dagger,\rho]) = [a^\dagger,(L_{\rm vac}-\kappa)\rho].$$

We are interested in the atomic observables, which only depend on the atomic density operator defined by tracing $\tilde\rho$ over field variables $\rho_a = {\rm Tr}_f\tilde\rho$. Tracing also Eq. (2.13) we have

$$\frac{d\rho_a}{dt} = {\rm Tr}_f\left(L_1 e^{-\kappa t}\tilde\rho\right) - i[H_{\Omega'},\rho_a] + L_a\rho_a, \qquad (2.14)$$

where we have used

$${\rm Tr}_f(L_2\tilde\rho) = -ig\{{\rm Tr}_f([a,\tilde\rho])\sigma^+ + \sigma^-{\rm Tr}_f([a^\dagger,\tilde\rho])\} = 0,$$

given that ${\rm Tr}_f(a\tilde\rho) = {\rm Tr}_f(\tilde\rho a)$. Note that ${\rm Tr}_f(L_1\tilde\rho)$ is not zero, since the trace is only taken over the field states.

Equation (2.14) is not a master equation for ρ_a, since in the first term it is the full density operator $\tilde\rho$ which appears. To eliminate this dependence, we first integrate formally Eq. (2.13) to obtain

$$e^{-\kappa t}\tilde\rho = e^{-\kappa t}e^{-iH_{\Omega'}t}\tilde\rho(0)e^{iH_{\Omega'}t} + e^{-\kappa t}L_a\tilde\rho$$
$$-ige^{-\kappa t}\int_0^t d\tau\, e^{-\kappa\tau}e^{-iH_{\Omega'}(t-\tau)}[L_1\tilde\rho(\tau)]e^{iH_{\Omega'}(t-\tau)}$$
$$-ig\int_0^t d\tau\, e^{-\kappa(t-\tau)}e^{-iH_{\Omega'}(t-\tau)}[L_2\tilde\rho(\tau)]e^{iH_{\Omega'}(t-\tau)}.$$

In the bad-cavity limit ($\kappa \gg g,\gamma$), after a short time $t \gtrsim \kappa^{-1}$, only the last term on the right-hand side of this expression becomes significant and therefore we can neglect the rest of them. The remaining term may also be written as

$$-ig\int_0^t d\tau\, e^{-\kappa\tau}e^{-iH_{\Omega'}\tau}[L_2\tilde\rho(t-\tau)]e^{iH_{\Omega'}\tau}.$$

The factors appearing in this integral give a nonvanishing contribution only for $\tau \lesssim \kappa^{-1}$; during this time the interaction atom-cavity mode and the spontaneous emission into the background modes are negligibles due to the smallness of the parameters g and γ when compared to κ. Hence, during this time, only the atom-laser interaction must be taken into account, i.e., we can make the substitution $\tilde\rho(t-\tau) \simeq e^{iH_{\Omega'}\tau}\tilde\rho(t)e^{-iH_{\Omega'}\tau}$. We can also extend the upper limit of the integral to infinity, given that we are interested in times $t \gtrsim \kappa^{-1}$. These approximations amount to making the usual Born-Markov approximations [25]. With all this, we have

$$e^{-\kappa t}\tilde\rho \simeq -ig\left([a,\tilde\rho]\sigma_{\Omega'} + \sigma_{\Omega'}^\dagger[a^\dagger,\tilde\rho]\right), \qquad (2.15)$$

where

$$\sigma_{\Omega'} = \int_0^\infty d\tau\, e^{-\kappa\tau}e^{-iH_{\Omega'}\tau}\tilde\sigma^+ e^{-iH_{\Omega'}\tau}. \qquad (2.16)$$

Substituting (2.15) in the term ${\rm Tr}_f(L_1 e^{-\kappa t}\tilde\rho)$ of Eq. (2.14) and after some lengthy algebra we find the following master equation for ρ_a:

$$\frac{d\rho_a}{dt} = -g^2\left([\tilde\sigma^+,\sigma_{\Omega'}^\dagger\rho_a] + [\rho_a\sigma_{\Omega'},\tilde\sigma^-]\right)$$
$$-i[H_{\Omega'},\rho_a] + L_a\rho_a. \qquad (2.17)$$

On the other hand, $\sigma_{\Omega'}$ can be expressed in a simpler form by taking into account that

$$e^{-iH_{\Omega'}\tau} = \cos\left(\tfrac{1}{2}\Omega'\tau\right) - i\sin\left(\tfrac{1}{2}\Omega'\tau\right)\sigma_w, \qquad (2.18)$$

resulting

$$\sigma_{\Omega'} = A_+\tilde\sigma^+ + A_-\sigma_w\tilde\sigma^+\sigma_w + iA_3[\tilde\sigma^+,\sigma_w], \qquad (2.19)$$

where

$$A_\pm = \frac{1}{2\kappa}\left(1 \pm \frac{\kappa^2}{\kappa^2+\Omega'^2}\right),$$

$$A_3 = \frac{1}{2}\frac{\Omega'}{\kappa^2+\Omega'^2}.$$

Finally, note that in the free-space limit ($\kappa\to\infty$, keeping g^2/κ constant) $A_- = A_3 = 0$, and it can be shown that the master equation (2.17) reduces to that found when there is no cavity, provided $\gamma_c = 2g^2/\kappa$ is identified with the damping rate in the squeezed reservoir.

C. Discussion

In view of Eq. (2.17) it is clear that the first term on its right-hand side describes the coupling between the atom and the squeezed reservoir via the cavity mode. To get a better insight into the physical meaning of this term, let us take matrix elements in the basis of the atom-plus-laser dressed states, defined by [26]

$$H_{\Omega'}|1\rangle = \tfrac{1}{2}\Omega'|1\rangle,$$
$$H_{\Omega'}|2\rangle = -\tfrac{1}{2}\Omega'|2\rangle,$$

where

$$|1\rangle = \sin\theta|g\rangle + \cos\theta|e\rangle, \qquad (2.20a)$$
$$|2\rangle = \cos\theta|g\rangle - \sin\theta|e\rangle, \qquad (2.20b)$$

with

$$\cot(2\theta) = \Delta/\Omega, \qquad (2.21)$$

and where $|g\rangle$ and $|e\rangle$ denote the ground and excited atomic levels, respectively. For the sake of simplicity we assume that the difference Ω' between the energies of these states is large compared to any other characteristic decay rate (i.e., the different emission lines are well separated). In this case, we can neglect the nonsecular couplings between populations and coherences [27] to find the evolution equation for the dressed levels populations $\pi_1 = \langle 1|\rho_a|1\rangle$ and $\pi_2 = \langle 2|\rho_a|2\rangle$ and the coherences $\sigma_{12} = \langle 1|\rho_a|2\rangle$, obtaining

$$\dot\pi_1 = -\Gamma_{1\to2}\pi_1 + \Gamma_{2\to1}\pi_2, \qquad (2.22a)$$
$$\dot\pi_2 = -\Gamma_{2\to1}\pi_2 + \Gamma_{1\to2}\pi_1, \qquad (2.22b)$$
$$\dot\sigma_{12} = -(i\Omega' + \Gamma_{\rm coh})\sigma_{12}, \qquad (2.22c)$$

where

$$\Gamma_{1\to 1} = |\langle 1|\tilde{\sigma}^+|1\rangle|^2 \gamma_c, \tag{2.23a}$$

$$\Gamma_{2\to 2} = |\langle 2|\tilde{\sigma}^+|2\rangle|^2 \gamma_c, \tag{2.23b}$$

$$\Gamma_{1\to 2} = |\langle 1|\tilde{\sigma}^+|2\rangle|^2 \gamma_c \kappa^2/(\kappa^2 + \Omega'^2), \tag{2.23c}$$

$$\Gamma_{2\to 1} = |\langle 2|\tilde{\sigma}^+|1\rangle|^2 \gamma_c \kappa^2/(\kappa^2 + \Omega'^2), \tag{2.23d}$$

and

$$\Gamma_{\rm coh} = \Gamma_{12} - K_{12}, \tag{2.24}$$

with

$$\Gamma_{12} = \tfrac{1}{2}(\Gamma_{1\to 1} + \Gamma_{2\to 2} + \Gamma_{1\to 2} + \Gamma_{2\to 1}),$$
$$K_{12} = \gamma_c \langle 1|\tilde{\sigma}^+|1\rangle^* \langle 2|\tilde{\sigma}^+|2\rangle.$$

Here, $\Gamma_{i\to j}$ is the transition rate from $|i\rangle$ to $|j\rangle$, while K_{12} is related to a radiative transfer of coherence. This result may be understood in the following way: We choose for the whole system atom-plus-cavity mode the basis $\{|i,n\rangle = |i\rangle \otimes |n\rangle; i = 1,2; n = 0,1,2,\ldots\}$, where $|i\rangle$ stands for the dressed levels defined in (2.20) and $|n\rangle$ denotes the Fock state of the cavity mode subspace with n photons. Note that this choice differs from that usually utilized in the Jaynes-Cummings model [10, 23], where the dressed states of the atom-plus-cavity mode are taken instead. The reason of our choice is based on the fact that the frequency shift due to the coupling of the atom to the cavity mode (which is of the order of $g\langle n\rangle^{1/2}$) is much smaller than that due to the coupling to the laser field (equal to Ω') in the limit considered here. For $g = 0$ the subspaces $|i,n\rangle$ with a given n remain decoupled in the course of time. However, for $g \neq 0$ transitions between different subspaces are likely to occur due to the coupling given by $\tilde{V}$ [see Eq. (2.9)]. Figure 1 shows the energy levels diagram and transitions between the states $|i,n\rangle$. From this figure it can be deduced that transitions $|i,n\rangle \to |i,n-1\rangle$ occur at the cavity-mode frequency ω, while transition $|1,n\rangle \to |2,n-1\rangle$ ($|2,n\rangle \to |1,n-1\rangle$) does at $\omega + \Omega'(\omega - \Omega')$. The transition rate $\Gamma_{i\to j}$ can be easily determined by using Fermi's golden rule, and therefore it is given by the product of the matrix element $|\langle i|g\tilde{\sigma}^+|j\rangle|^2$ by the density of modes function $\rho(\nu)$ of the cavity at the corresponding transition frequency. This function is given by

$$\rho(\nu) = \int_{-\infty}^{\infty} d\tau \langle [a(\tau), a^\dagger(0)]\rangle e^{i\nu\tau}. \tag{2.25}$$

For the case under study $\langle [a(\tau), a^\dagger(0)]\rangle = e^{-(i\omega+\kappa)\tau}$, and therefore the results (2.23) are found. Hence, we conclude that master equation (2.17) can be understood in terms of transitions between the states $|i,n\rangle$.

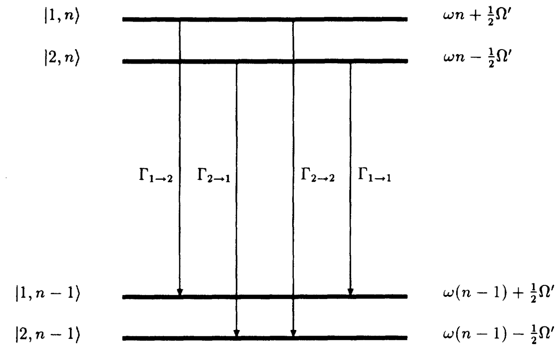

FIG. 1. Energy-level diagram of the states $|i,n\rangle$. Transitions between these states are represented by arrows.

III. STEADY-STATE POPULATION INVERSION

In a recent work, Savage [11] has predicted that the atomic population inversion can be positive, when a two-level atom interacts with a quantized cavity field damped by the vacuum. As discussed there, steady-state inversion is essentially a quantum-mechanical effect, and it is found for parameter ranges where the spontaneous-emission rate in the background modes, as well as the additional decay rate via the cavity mode, are small in comparison with the Rabi frequency of the external field. These conditions can be written as $\gamma, g \ll \kappa, \Omega'$, which agree with the regime of validity of the master equation deduced in the previous section. In the present section, starting from this master equation, we analyze how the population inversion is affected by the presence of an ideal squeezed reservoir.

An attempt to describe the inversion consists of using the dressed-state picture discussed above. The stationary population inversion $\langle \sigma_z\rangle_{\rm ss}$ can be written in terms of the dressed populations as

$$\langle \sigma_z\rangle_{\rm ss} = \cos(2\theta)(\pi_1^{\rm ss} - \pi_2^{\rm ss}), \tag{3.1}$$

since $\langle \sigma_{12}\rangle_{\rm ss}$ vanishes as a consequence of the secular approximation made in the derivation of Eqs. (2.22) (the subscript ss stands for the value taken in the steady state). On resonance $\cos(2\theta) = 0$ and this expression predicts saturation to $\langle \sigma_z\rangle_{\rm ss} = 0$. So, it does not account for the small deviations from this value due to the presence of the cavity, which are the final cause of the population inversion. Hence, in order to study the case $\langle \sigma_z\rangle_{\rm ss} > 0$ we need to include the nonsecular terms neglected in Eqs. (2.22), i.e., to take the whole master equation (2.17).

From Eq. (2.17), the equations of motion for the expectation values of the atomic operators can be found to be

$$\langle \dot{\sigma}^+\rangle = -\left\{\frac{\gamma}{2} + \frac{\gamma_c}{2}\frac{\kappa^2}{\kappa^2+\Omega'^2}\left(\frac{\Omega^2}{2\kappa^2}(2N+1+2Me^{-i\phi}) + (2N+1)\left[1 - i\frac{\Delta}{\kappa}\right]\right) - i\Delta\right\}\langle\sigma^+\rangle$$
$$+\gamma_c\frac{\kappa^2}{\kappa^2+\Omega'^2}\left(\frac{\Omega^2}{2\kappa^2}(2N+1+2Me^{-i\phi}) + 2Me^{-i\phi}\left[1 + i\frac{\Delta}{\kappa}\right]\right)\langle\sigma^-\rangle - i\frac{\Omega}{2}\langle\sigma_z\rangle + \frac{\gamma_c}{4}\frac{\Omega(\Delta+i\kappa)}{\kappa^2+\Omega'^2}, \tag{3.2a}$$

$$\langle \dot{\sigma}_z \rangle = -i\Omega \left(1 + \frac{\gamma_c}{2} \frac{\kappa}{\kappa^2 + \Omega'^2} \left[(2N + 1 - 2Me^{i\phi}) + i\frac{\Delta}{\kappa}(2N + 1 + 2Me^{i\phi}) \right] \right) \langle \sigma^+ \rangle$$
$$+ i\Omega \left(1 + \frac{\gamma_c}{2} \frac{\kappa}{\kappa^2 + \Omega'^2} \left[(2N + 1 - 2Me^{-i\phi}) - i\frac{\Delta}{\kappa}(2N + 1 + 2Me^{-i\phi}) \right] \right) \langle \sigma^- \rangle$$
$$- \left(\gamma + \gamma_c \frac{\kappa^2}{\kappa^2 + \Omega'^2} \left[\frac{\Omega^2}{2\kappa^2}[2N + 1 + 2M\cos(\phi)] + (2N + 1) \right] \right) \langle \sigma_z \rangle - \gamma - \gamma_c \frac{\kappa^2 + \Omega^2/2}{\kappa^2 + \Omega'^2}, \quad (3.2b)$$

where we have omitted that of $\langle \sigma^- \rangle = \langle \sigma^+ \rangle^*$. The stationary solution of these equations can be obtained analytically, though the final expression becomes very involved. Instead, we have numerically verified that optimum values of the inversion correspond to the choice $\Delta = 0$, which notably simplifies the algebra. Besides, two limits characterize the effect of the squeezing on the atomic dynamics, namely $\phi = 0, \pi$. Defining, as usual, $\sigma^\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ Eqs. (3.2) become

$$\langle \dot{\sigma}_x \rangle = -\gamma_x \langle \sigma_x \rangle, \quad (3.3a)$$
$$\langle \dot{\sigma}_y \rangle = -\Omega \langle \sigma_z \rangle - \gamma_y \langle \sigma_y \rangle + \Omega_0, \quad (3.3b)$$
$$\langle \dot{\sigma}_z \rangle = \Omega_1 \langle \sigma_y \rangle - \gamma_z \langle \sigma_z \rangle - \gamma_1, \quad (3.3c)$$

where we have defined

$$\gamma_x = \frac{\gamma}{2} + \frac{\gamma_c}{2} \frac{\kappa^2}{\kappa^2 + \Omega^2}[2N + 1 - 2\cos(\phi)M],$$
$$\gamma_y = \frac{\gamma}{2} + \frac{\gamma_c}{2}[2N + 1 + 2\cos(\phi)M],$$
$$\gamma_z = \gamma_x + \gamma_y,$$
$$\Omega_0 = \Omega \frac{\kappa\gamma_c/2}{\kappa^2 + \Omega^2},$$
$$\Omega_1 = \Omega \left[1 + \frac{\kappa\gamma_c/2}{\kappa^2 + \Omega^2}[2N + 1 - \cos(\phi)M] \right],$$
$$\gamma_1 = \gamma + \gamma_c \frac{k^2 + \Omega^2/2}{\kappa^2 + \Omega^2}.$$

and now $\cos(\phi)$ can take on the values 1 or -1.

These equations mainly depart from the usual Bloch equations in the squeezed vacuum [18] in two aspects. (i) The decay constant γ_x depends on the Rabi frequency Ω, which is a sign of the non-Markovian coupling between the atom and the cavity mode. As a consequence of this, the inhibition of the atomic decay has been predicted [15] when Ω is larger than κ and, at the same time, the squeezed field inhibits the decay of $\langle \sigma_y \rangle$. (ii) The equation of this last component of the Bloch vector is inhomogeneous, contrary to what happens in the free-space case. As we show below, this will lead to a positive population inversion.

The population inversion in the steady state is easily found to be

$$\langle \sigma_z \rangle_{ss} = \frac{\Omega_0\Omega_1 - \gamma_y\gamma_1}{\Omega\Omega_1 + \gamma_y\gamma_z}. \quad (3.4)$$

Thus, inversion occurs if

$$\Omega_0\Omega > \gamma_y\gamma_1. \quad (3.5)$$

From this result, we make the following observations.

(i) Inversion is only possible when $\Omega_0 \neq 0$. So, for example, in the free-space limit Ω_0 vanishes, and therefore, as expected, $\langle \sigma_z \rangle_{ss}$ is always negative. Moreover, in Eqs. (2.22) Ω_0 has been discarded as a consequence of the secular approximation. Hence, with these equations it is not possible to predict inversion, as mentioned above.

(ii) Inversion can occur for $N = 0$, since Ω_0 does not depend on the mean number of squeezed photons N. In fact, for $N = 0$ Eq. (3.4) reduces to Eq. (4.9) of Ref. [12], where a detailed study of the case of a vacuum reservoir is given. However, for $\cos(\phi) = -1$, $\langle \sigma_z \rangle_{ss}$ is an increasing function of N. This can be seen by differentiating (3.4) with respect to N, and checking that the result is positive. Then, we conclude that by increasing the squeezing one could obtain a larger inversion.

(iii) The population inversion cannot be raised arbitrarily by varying the parameter N. It is bounded from above

$$\langle \sigma_z \rangle_{ss} < \frac{\Omega_0}{\Omega} = \frac{g^2}{\kappa^2 + \Omega^2}, \quad (3.6)$$

which is very small in the limit of validity of Eq. (2.17), that is, for $g \ll \kappa$. In view of (3.6) maximum values of $\langle \sigma_z \rangle_{ss}$ are expected for strong atom-cavity mode couplings, where a solution of the exact master equation (2.8) is requiered. We have solved this equation numerically (see the Appendix) obtaining, for example, that for $N = 1$ the maximum value is $\langle \sigma_z \rangle_{ss} = 0.104$ when $g = 1.15\kappa$, $\Omega = 2.7\kappa$, and $\gamma = 0$. This value is larger than the maximum value of 0.07 given in Ref. [12] for the case $N = 0$. We have also verified that for large values of N the population inversion is saturated to zero.

To illustrate the validity of master equation (2.17), derived under the Born-Markov approximation, we have plotted in Fig. 2 the population inversion in the stationary state given by (3.4) (dashed lines) and the numerical solution of the exact master equation (2.8) (solid lines). The values of the paramaters are $g = 1$, $\gamma = 0.1$, $N = 1$, $\Omega = 5$ [Fig. 2(a)] and $\Omega = 2$ [Fig. 2(b)]. As expected, for low κ, the analytic approximation (3.4) and the exact solution give different results. However, as κ increases ($\gtrsim 10$) the population inversion tends to the values predicted by the master equation derived in the bad-cavity limit. In this figure, it is also shown that for $\cos(\phi) = -1$ [curve (1)] the inversion is always larger than for $\cos(\phi) = 1$ [curve (2)], in agreement with the above discussion. On the other hand, it is for small values of κ ($\simeq g$) when $\langle \sigma_z \rangle_{ss}$ reaches its maximum value, being also larger when $\Omega = 2$ than for $\Omega = 5$, since as Ω

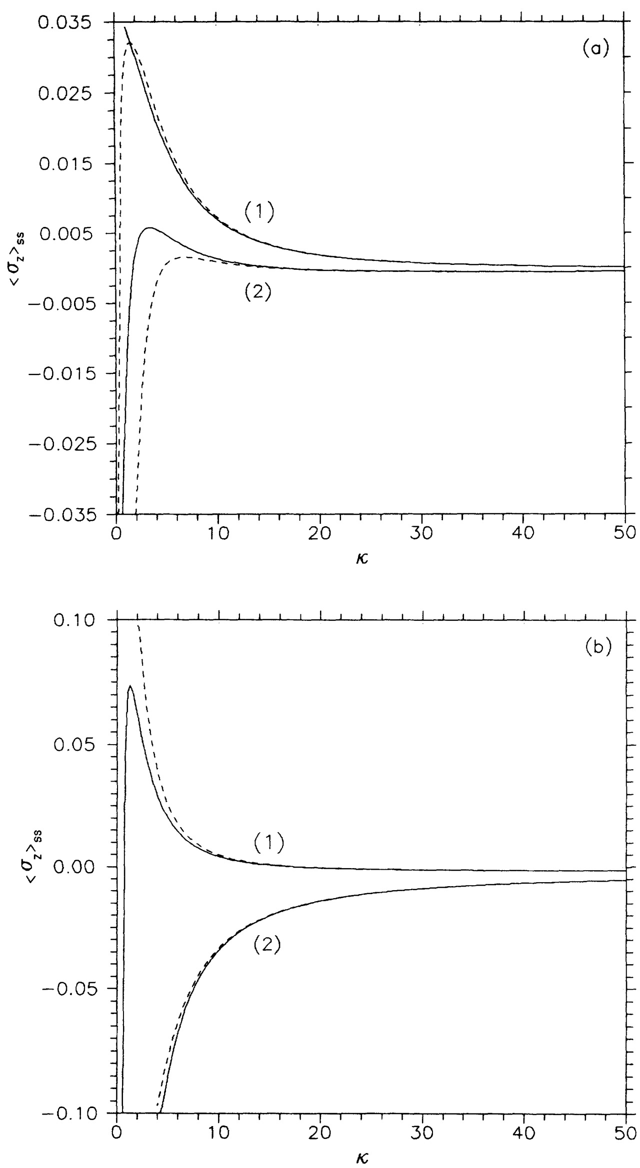

FIG. 2. (a) Steady-state population inversion as a function of the cavity loss rate κ, for $\Omega = 5$, $g = 1$, $\gamma = 0.1$, $N = 1$, and $\cos(\phi)$: -1 (1) and 1 (2). The solid line gives the numerical solution of the exact mater equation and the dashed line, the result given by the analytic approximation. (b) Same as (a) for $\Omega = 2$.

increases the laser photons tend to saturate the atomic transition.

Figure 3 displays the dependence on the mean number of squeezed photons N, obtained by solving numerically Eq. (2.8) for $g = 1$, $\gamma = 0.1$, $\kappa = 10$, $\cos(\phi) = -1$, and $\Omega = 2$ (solid line), $\Omega = 3$ (dashed line), and $\Omega = 4$ (dotted line). The figure shows that $\langle\sigma_z\rangle_{ss}$ increases with N, reaching a maximum value, in agreement with Eq. (3.4). Note that the maximum slope of the curves occurs for $N = 0$, which implies that a little amount of squeezing would increase the inversion notably. Note also that it

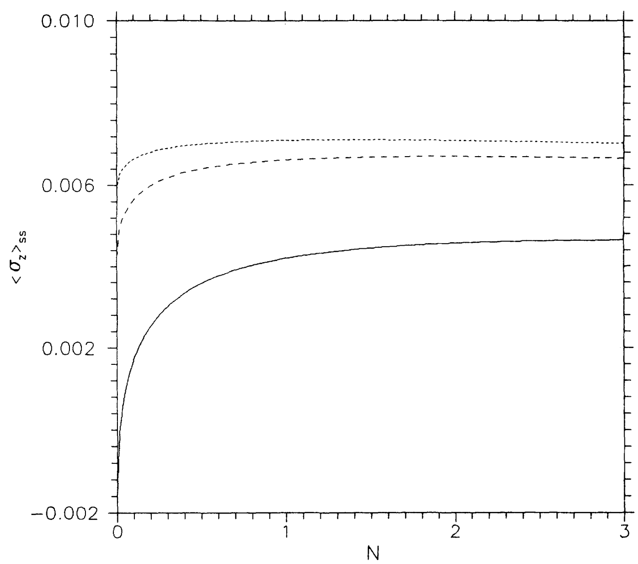

FIG. 3. Steady-state population inversion as a function of N for $g = 1$, $\kappa = 10$, $\gamma = 0.1$, $\cos(\phi) = -1$, and Ω: 2 (solid line), 3 (dashed line), and 4 (dotted line).

is precisely for small values of Ω where the effect of the squeezing of the field becomes more important. So, for $\Omega = 2$, by squeezing the electromagnetic reservoir one can change the population inversion in the stationary state from negative to positive.

IV. RESONANCE FLUORESCENCE SPECTRUM

In this section we study the spectrum of the light radiated by the atom into the background modes, which are in the normal vacuum state. Starting from master equation (2.17) we derive an analytical expression, which permits us to study the effect produced by the cavity in the spectrum. In particular, we find that all three peaks in the Mollow triplet can be narrowed at the same time by increasing the laser intensity and for certain phase relation between the laser field and the ideal squeezed reservoir. This effect has been already predicted by Parkins [15] and Cirac and Sanchez-Soto [16] in the interaction of a two-level atom with a continuum-of-cavity modes in a squeezed state, and occurs as a consequence of the dynamical inhibition of the atomic decay.

The spectrum can be calculated as

$$S(\nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tau\, e^{-i(\nu-\omega)\tau}\,\langle\sigma^+(\tau)\sigma^-(0)\rangle_{ss}, \quad (4.1)$$

where σ^+ and σ^- are defined in a rotating frame at frequency ω. With this definition, the integral of $S(\nu)$ over all frequencies equals the population of the excited state $\mathrm{Tr}(|e\rangle\langle e|\rho_a)$.

Here, we are concerned with the case in which the spectrum consists of three well-separated lines ("Mollow's triplet"), that is, when Ω' is large compared to all the atomic decay rates. In this situation, the dressed-state formalism provides a good approach and a simple explanation [28]. Hence, let us express the two-time correlation function appearing in (4.1) in terms of the dressed operators, defined as

$$\pi_w = |2\rangle\langle 2| - |1\rangle\langle 1|, \tag{4.2a}$$

$$\sigma_{12} = |2\rangle\langle 1|, \tag{4.2b}$$

$$\sigma_{21} = |1\rangle\langle 2|, \tag{4.2c}$$

where $|1\rangle$ and $|2\rangle$ are the dressed states introduced in (2.20). The result is

$$\begin{aligned}\langle\sigma^+(\tau)\sigma^-(0)\rangle_{ss} = {} & \sin(\theta)\cos(\theta)\langle\pi_w(\tau)\sigma^-(0)\rangle_{ss} \\ & + \cos^2(\theta)\langle\sigma_{21}(\tau)\sigma^-(0)\rangle_{ss} \\ & - \sin^2(\theta)\langle\sigma_{12}(\tau)\sigma^-(0)\rangle_{ss}. \end{aligned} \tag{4.3}$$

The first term gives rise to the central band of the spectrum, while the other two yield the sidebands. The correlation functions appearing in (4.3) can be evaluated by using the quantum-regression theorem [29], valid when the response time of the cavity mode (of the order of κ^{-1}) is much smaller than that of the atom (of the order of $\Gamma_{i\to j}^{-1}$) [30]. This condition coincides with that assumed in the derivation of master equation (2.17), and therefore, in the limit under study is always fulfilled. Then, from Eqs. (2.22) we have

$$\begin{aligned}\frac{d}{d\tau}\langle\pi_w(\tau)\sigma^-(0)\rangle_{ss} = {} & -\Gamma_w\langle\pi_w(\tau)\sigma^-(0)\rangle_{ss} \\ & -(\Gamma_{1\to 2} - \Gamma_{2\to 1})\langle\sigma^-\rangle_{ss}, \end{aligned} \tag{4.4a}$$

$$\frac{d}{d\tau}\langle\sigma_{12}(\tau)\sigma^-(0)\rangle_{ss} = -(i\Omega' + \Gamma_{coh})\langle\sigma_{12}(\tau)\sigma^-(0)\rangle_{ss}, \tag{4.4b}$$

$$\frac{d}{d\tau}\langle\sigma_{21}(\tau)\sigma^-(0)\rangle_{ss} = (i\Omega' - \Gamma_{coh})\langle\sigma_{21}(\tau)\sigma^-(0)\rangle_{ss}, \tag{4.4c}$$

where $\Gamma_w = \Gamma_{1\to 2} + \Gamma_{2\to 1}$. By solving these equations and Fourier transforming, one obtains the well-known result [26]

$$\begin{aligned}S(\nu) = {} & I_{el}\delta(\nu-\omega) + I_0 \mathrm{L}(\nu-\omega,\Gamma_w) \\ & + I_+ \mathrm{L}(\nu-\omega-\Omega',\Gamma_{coh}) \\ & + I_- \mathrm{L}(\nu-\omega+\Omega',\Gamma_{coh}), \end{aligned} \tag{4.5}$$

where

$$\mathrm{L}(\nu-\nu',\Gamma) = \frac{1}{\pi}\frac{\Gamma}{\Gamma^2 + (\nu-\nu')^2} \tag{4.6}$$

is a Lorentzian function of width Γ centered at ν', and

$$I_{el} = \tfrac{1}{4}\sin^2(2\theta)(\pi_1^{ss} - \pi_2^{ss})^2, \tag{4.7}$$

$$I_0 = \sin^2(2\theta)\pi_1^{ss}\pi_2^{ss}, \tag{4.8}$$

$$I_\pm = \tfrac{1}{4}[1 \pm \cos(2\theta)]^2\pi_{1,2}^{ss}, \tag{4.9}$$

are the intensities emitted in the elastic component of the spectrum, in the central band and in the sidebands, respectively. π_1^{ss} and π_2^{ss} stand for the population in the steady state, which can be derived from the detailed balance condition ($\pi_1^{ss}\Gamma_{1\to 2} = \pi_2^{ss}\Gamma_{2\to 1}$), resulting in

$$\pi_1^{ss} = \frac{\Gamma_{2\to 1}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}}, \tag{4.10}$$

$$\pi_2^{ss} = \frac{\Gamma_{1\to 2}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}}. \tag{4.11}$$

The widths appearing in (4.5) can be calculated from (2.23) giving

$$\begin{aligned}\Gamma_w = {} & \gamma_c\frac{\kappa^2}{\kappa^2 + \Omega'^2} \\ & \times\{(2N+1) - \sin^2(2\theta)[N + \tfrac{1}{2} + M\cos(\phi)]\}, \end{aligned} \tag{4.12}$$

$$\Gamma_{coh} = \tfrac{1}{2}\Gamma_w + \gamma_c\sin^2(2\theta)[N + \tfrac{1}{2} + M\cos(\phi)]. \tag{4.13}$$

The incoherent part of the spectrum is then composed of a central band of width Γ_w and height I_0/Γ_w and two sidebands, symmetrically situated around the central one, of width Γ_{coh} and heights I_+/Γ_{coh} and I_-/Γ_{coh}, respectively.

As what occurs in the free-space limit, the sidebands are different when $\Delta \neq 0$, since $I_+ \neq I_-$. One of them can even disappear when either $\Gamma_{1\to 2}$ or $\Gamma_{2\to 1}$ are zero, that is, when dressed-population trapping occurs [31]. In this case, also the central band disappears, since one of the dressed populations in the stationary state vanishes. The condition for this to occur is either $\langle 1|\tilde{\sigma}^+|2\rangle = 0$ or $\langle 2|\tilde{\sigma}^+|1\rangle = 0$, that can also be expressed in a different form as

$$\tan^2\theta = \sqrt{(N+1)/N} \tag{4.14a}$$

or

$$\tan^2\theta = \sqrt{N/(N+1)}. \tag{4.14b}$$

Note that these conditions only depend on the squeezing parameter N and the ratio Δ/Ω.

In the following, we concentrate on the modifications on the spectrum due to the presence of the cavity. For the sake of simplicity we will consider the resonant case ($\Delta = 0$), where $\pi_1^{ss} = \pi_2^{ss} = \frac{1}{2}$. In this case, the height of the central peak becomes $1/(4\Gamma_w)$, and that of the sidebands $1/(8\Gamma_{coh})$, which are now symmetric. For $\Omega \ll \kappa$ the same results than in the absence of cavity [18] are found. First, for $\cos(\phi) = 1$, $\Gamma_{coh} \gg \Gamma_w$, i.e., the central peak is very narrow and high, a consequence of the inhibition of the dressed-population decay. Contrarily, for $\cos(\phi) = -1$, $\Gamma_{coh} \simeq \Gamma_w/2$, and the sidebands have double the width and the same height as the central one. It is for $\Omega \gtrsim \kappa$ where additional results appear. In this case, for $\cos(\phi) = 1$, Γ_w becomes smaller than in the free-space case, i.e., the central peak is narrower than before, while the sidebands remain practically unchanged. When $\cos(\phi) = -1$, increasing the Rabi frequency Ω, $\Gamma_w/2$ becomes as small as we please. Γ_{coh} also decreases, but the inequality $\Gamma_{coh} > \Gamma_w$ is fulfilled. Then, in this case all the peaks of the spectrum are narrower than in the free-space case, being the central one higher than the sidebands.

In Fig. 4 we have plotted the right part of the spectrum obtained by numerically solving master equation (2.8) for $\cos(\phi) = -1$ [Fig. 4(a)] and $\cos(\phi) = 1$ [Fig. 4(b)], and $\kappa = 5, 10, 100$ (solid, dashed, and dotted lines, respectively), with $\gamma_c = 2g^2/\kappa = 0.2$, $\gamma = 0.1$, $N = 2$, and $\Omega = 10$. When the driving field is in quadrature with the squeezed reservoir [Fig. 4(a)], the sideband and the central band become narrower and higher with decreasing κ. The height of the central band increases more

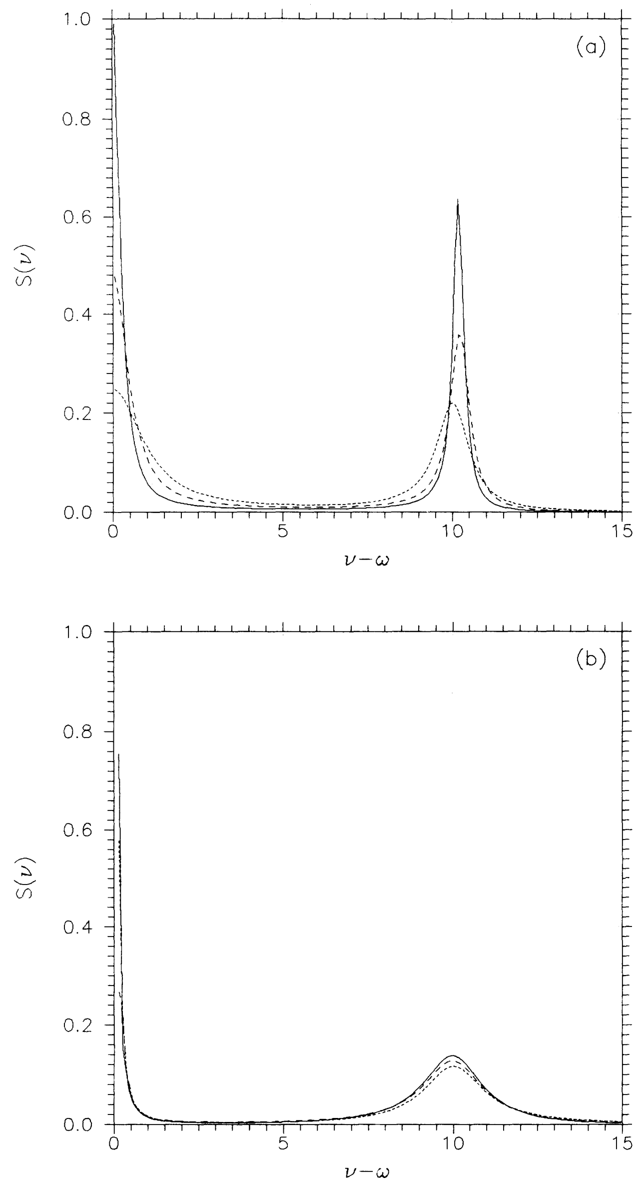

FIG. 4. (a) Right part of the spectrum emitted by the atom into the background modes for $\cos(\phi) = -1$, $\gamma = 0.1$, $\Omega = 10$, $N = 2$, and κ: 5 (solid line), 10 (dashed line), 100 (dotted line). (b) Same as in (a) but with $\cos(\phi) = 1$. In all three curves $\gamma_c = 2g^2/\kappa = 0.1$.

than that of the sidebands. In the opposite case [Fig. 4(b)] the sideband remains practically unaltered when κ is decreased.

Finally, it is worth mentioning that the resonance spectrum in the good-cavity limit has been studied recently [32]. It has been found that the sideband width strongly depends on the phase difference between the laser field and the squeezed reservoir, and it is determined by the width of the population distribution of the atom-plus-cavity mode-dressed states.

V. CONCLUSIONS

In this paper we have studied the problem of a two-level atom interacting with a cavity mode and driven by an external laser field in the bad-cavity limit. The cavity mode is damped by coupling to a reservoir in an ideal squeezed state, and the spontaneous emission from the atom to other modes (background modes) than the priviliged cavity mode is also considered. We have derived a master equation for the atomic density operator, valid in the bad-cavity limit, which can be easily interpreted in terms of transitions between coupled states formed by the dressed states of the atom-plus-laser system and the Fock states of the cavity mode. In steady state, positive population inversion is found in the absence of squeezing, in agreement with previous results. When the driving field is in quadrature with the maximally squeezed quadrature of the damping reservoir, the population inversion increases with the mean number of squeezed photons. However, inversion cannot be increased arbitrarily by raising the squeezing of the reservoir. We have found, for instance, that the maximum value of the population inversion for $N = 1$ is $\langle\sigma_z\rangle_{ss} = 0.1$. Finally, the spectrum of resonance fluorescence has been calculated. As κ decreases and for intense laser field the central band and the sidebands of the Mollow's triplet get narrower, when again, the driving field and the squeezed field are in quadrature. This is a direct effect of the atom-cavity decoupling predicted earlier.

ACKNOWLEDGMENTS

I would like to thank Dr. Sanchez-Soto, Dr. H. Ritsch, and, especially, Professor P. Zoller for helpful discussions.

APPENDIX

In this appendix we transform the exact master equation (2.8) into a infinite hierarchy of ordinary differential equations, which we have solved numerically to produce the figures. Following Refs. [20, 23], we define the characteristic functions by

$$u(\sigma^j, \lambda_1, \lambda_2, t) = \mathrm{Tr}\left[e^{i\lambda_1 a^\dagger} e^{i\lambda_2 a} \sigma^j \hat{\rho}(t)\right], \tag{A1}$$

where $\{\sigma^j, j = 0,1,2,3\} = \{1, \sigma_z, \sigma^+, \sigma^-\}$. The evolution equation for these characteristic functions can be found with the help of Eq. (2.8). The result is

$$\begin{aligned}\frac{d}{dt}u(\sigma^j) = &-g\left[\frac{\partial}{\partial\lambda_2}u([\sigma^j,\tilde{\sigma}^+]) + \frac{\partial}{\partial\lambda_1}u([\sigma^j,\tilde{\sigma}^-]) + \lambda_1 u(\tilde{\sigma}^+\sigma^j) - \lambda_2 u(\sigma^j\tilde{\sigma}^-)\right]\\ &-i\frac{\Omega'}{2}u([\sigma^j,\sigma_w]) - \kappa\left[\lambda_1\frac{\partial}{\partial\lambda_1} + \lambda_2\frac{\partial}{\partial\lambda_2}\right]u(\sigma^j) + \frac{\gamma}{2}u([\sigma^+,\sigma^j]\sigma^- + \sigma^+[\sigma^j,\sigma^-]).\end{aligned} \tag{A2}$$

Expanding in Taylor's series

$$u(\sigma^j,\lambda_1,\lambda_2)=\sum_{n,m=0}^{\infty}\lambda_1^n\lambda_2^m u_{n,m}^j, \tag{A3}$$

we have

$$\dot{u}_{n,m}^0=-g(\mu u_{n-1,m}^2+\nu^* u_{n-1,m}^3-\nu u_{n,m-1}^2-\mu u_{n,m-1}^3)-\kappa(n+m)u_{n,m}^0, \tag{A4a}$$

$$\begin{aligned}\dot{u}_{n,m}^1=&\,g(\mu u_{n-1,m}^2+\nu^* u_{n-1,m}^3-\nu u_{n,m-1}^2-\mu u_{n,m-1}^3)\\&-2g[(m+1)(\mu u_{n,m+1}^2-\nu^* u_{n,m+1}^3)+(n+1)(\nu u_{n+1,m}^2-\mu u_{n+1,m}^3)]\\&-i\Omega(u_{n,m}^2-u_{n,m}^3)-[\gamma+\kappa(n+m)]u_{n,m}^1-\gamma u_{n,m}^0,\end{aligned} \tag{A4b}$$

$$\begin{aligned}\dot{u}_{n,m}^2=&-\frac{g}{2}[\nu^*(u_{n-1,m}^0-u_{n-1,m}^1)+\mu(u_{n,m-1}^0+u_{n,m-1}^1)]\\&-g[(m+1)\nu^* u_{n,m+1}^1+(n+1)\mu u_{n+1,m}^1]-i\frac{\Omega}{2}u_{n,m}^1-\left[\frac{\gamma}{2}+\kappa(n+m)+i\Delta\right]u_{n,m}^2,\end{aligned} \tag{A4c}$$

$$\begin{aligned}\dot{u}_{n,m}^3=&\,\frac{g}{2}[\nu(u_{n,m-1}^0-u_{n,m-1}^1)-\mu(u_{n-1,m}^0+u_{n-1,m}^1)]\\&+g[(m+1)\mu^* u_{n,m+1}^1+(n+1)\nu u_{n+1,m}^1]+i\frac{\Omega}{2}u_{n,m}^1-\left[\frac{\gamma}{2}+\kappa(n+m)-i\Delta\right]u_{n,m}^3.\end{aligned} \tag{A4d}$$

To obtain the population inversion in the steady state we have solved these equations with $\dot{u}_{n,m}=0$ by assuming $u_{n,n}=0$ for $n>n_0$ and checking that the result did not change when n_0 was increased. We have also used this method, together with the quantum regression theorem, to find the stationary spectrum numerically.

---

[1] E. M. Pourcel, Phys. Rev. **69**, 681 (1946).
[2] D. Kleppner, Phys. Rev. Lett. **47**, 233 (1981).
[3] P. Goy, J. M. Raimond, M. Gross, and S. Haroche, Phys. Rev. Lett. **50**, 1903 (1983); R. G. Hulet, E. S. Hilfer, and D. Kleppner, *ibid.* **55**, 2137 (1985); W. Jhem, A. Anderson, E. A. Hind, D. Mschede, L. Moi, and S. Haroche, *ibid.* **58**, 666 (1987); D. Heinzen, J. J. Childs, J. E. Thomas, and M. S. Feld, *ibid.* **58**, 1320 (1987).
[4] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).
[5] P. R. Rice and H. J. Carmichael, IEEE J. Quantum Electron. **24**, 1351 (1988).
[6] S. Schadev, Phys. Rev. A **29**, 2627 (1984); R. R. Puri and G. S. Agarwal, *ibid.* **35**, 3433 (1987); J. Eiselt and H. Risken, *ibid.* **43**, 346 (1991); Opt. Commun. **72**, 351 (1989).
[7] S. M. Barnett and P. L. Knight, Phys. Rev. A **33**, 2444 (1986).
[8] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980).
[9] J. J. Sanchez-Mondragon, N. B. Narozhny, and J. H. Eberly, Phys. Rev. Lett. **51**, 550 (1983); G. S. Agarwal, *ibid.* **53**, 1732 (1984).
[10] C. M. Savage, Phys. Rev. Lett. **63**, 1376 (1989).
[11] C. M. Savage, Phys. Rev. Lett. **60**, 1828 (1988).
[12] M. Lindberg and C. M. Savage, Phys. Rev. A **38**, 5182 (1988).
[13] M. Lewenstein, T. W. Mossberg, and R. J. Glauber, Phys. Rev. Lett. **59**, 775 (1987); M. Lewenstein and T. W. Mossberg, Phys. Rev. A **37**, 2048 (1988).
[14] H. J. Carmichael, Phys. Rev. Lett. **55**, 2790 (1985).
[15] A. S. Parkins, Phys. Rev. A **42**, 4352 (1990).
[16] J. I. Cirac and L. L. Sanchez-Soto, Phys. Rev. A **44**, 1948 (1991).
[17] C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).
[18] H. J. Carmichael, A. S. Lane, and D. F. Walls, Phys. Rev. Lett. **58**, 2539 (1987); J. Mod. Opt. **34**, 821 (1987).
[19] A. S. Parkins and C. W. Gardiner, Phys. Rev. A **37**, 3867 (1988).
[20] H. Ritsch and P. Zoller, Phys. Rev. Lett. **61**, 1097 (1988); Phys. Rev. A **38**, 4657 (1988).
[21] For a discussion on how to accomplish experimentally the situation described by master equation (2.1) see, for example, S. Savage, Quantum Opt. **2**, 89 (1991), and references therein.
[22] See the special issues on squeezed states in J. Opt. Soc. Am. B **4**, 1450 (1987) and J. Mod. Opt. **34**, 709 (1987).
[23] J. I. Cirac, H. Ritsch, and P. Zoller, Phys. Rev. A **44**, 4541 (1991).
[24] C. M. Savage and H. J. Carmichael, IEEE J. Quantum Electron. **24**, 1495 (1988).
[25] See, for example, G. S. Agarwal, *Quantum Optics* (Springer-Verlag, Berlin, 1974).
[26] C. Cohen-Tannoudji and S. Reynaud, in *Multiphoton Processes*, edited by J. Eberly and P. Lambropoulos (Wiley, New York, 1978).
[27] For a detailed analysis on the dressed states see C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Processus d'Interaction Entre Photons et Atomes* (InterEditions, Paris, 1988).
[28] C. Cohen-Tannoudji and S. Reynaud, J. Phys. B **10**, 345 (1977).
[29] C. W. Gardiner, *Quantum Noise* (Springer-Verlag, Berlin, 1991).
[30] J. M. Courty and S. Reynaud, Opt. Commun. **72**, 93 (1989).
[31] J. M. Courty and S. Reynaud, Europhys. Lett. **10**, 237 (1989).
[32] C. M. Savage, Quantum Opt. **2**, 89 (1991).