Schrödinger-cat states at finite temperature: Influence of a finite-temperature heat bath on quantum interferences

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Recently several methods have been proposed for generation of superposition (Schrödinger-cat) states in microwave cavities. At microwave frequencies, thermal photons can significantly affect statistical properties of superposition states. In the present paper we study the influence of a thermal heat bath on nonclassical properties of quantum superposition states. We show that at nonzero temperature the loss of coherences is much faster than at zero temperature. Using the formalism of quasiprobability distributions and solving the corresponding Fokker-Planck equations, we describe the time evolution of the super-position states in phase space and derive the rate of the decay of quantum coherence. This decay rate depends on the separation between the component states and on the temperature of the heat bath. Moreover, we discuss in detail how the interaction with a nonzero-temperature heat bath leads to a transformation of a nonclassical state to a classical state. We show that the sensitivity of the quantum coherence to the presence of thermal photons can lead to some difficulties in the preparation of Schrödinger-cat states in microwave cavities unless the temperature of the microwave cavity is sufficiently low.

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I. INTRODUCTION

The quantum interference between states of light described by a pure superposition state gives rise to various nonclassical effects [1–3]. For instance, it has been shown that quadrature squeezing [1,3] (i.e., a reduction of quantum fluctuations below the level associated with a vacuum state [4]) and higher-order squeezing [1,5] emerge as a consequence of quantum interference between various components of superposition states. Quantum interference can also lead to sub-Poissonian photon statistics [1] (for a recent review of sub-Poissonian photon statistics see Ref. [6]) and oscillations in the photon-number distribution [1,2].

It has been shown that quantum superposition of coherent states can be produced in various nonlinear processes [7–12] and in quantum-nondemolition and back-evading measurements [13]. In particular, it has been shown by Yurke and Stoler [7] that in the presence of low dissipation, a nonlinear system (for instance, a Kerr-like medium [8]) may convert an initial coherent state $|\zeta\rangle$ into a quantum superposition of macroscopically distinguishable coherent states $|\zeta\rangle_{\mathrm{YS}}$ (for details see Ref. [7])

$$|\zeta\rangle_{\mathrm{YS}}=\frac{1}{\sqrt{2}}(|\zeta\rangle+e^{i\pi/2}|-\zeta\rangle)\;, \tag{1a}$$

where $|\zeta\rangle$ is a coherent state (CS) defined as

$$|\zeta\rangle=\hat{D}(\zeta)|0\rangle=\exp(-|\zeta|^2/2)\sum_{n=0}^{\infty}\frac{\zeta^n}{\sqrt{n!}}|n\rangle\;, \tag{1b}$$

and $\hat{D}(\zeta)=\exp(\zeta\hat{a}^\dagger-\zeta^*\hat{a})$ is a displacement operator with $\hat{a}^\dagger$ and $\hat{a}$ playing the role of the creation and annihilation operators of a photon in a single-mode field ($[\hat{a},\hat{a}^\dagger]=1$, we adopt units such that $\hbar=1$). The state (1a) is a superposition of two coherent states $|\zeta\rangle$ and $|-\zeta\rangle$, which are 180° out of phase with respect to each other. It is natural to refer to state (1a) as the Yurke-Stoler coherent state. It should be noted here that the Yurke-Stoler CS can be identified with a particular realization of a generalized coherent state introduced by Titulaer and Glauber [9] (see also Ref. [10]), who we believe were first to realize that state (1a) is of a thoroughly nonclassical nature because it cannot be represented by means of the P representation (Mandel [6] has explicitly shown how the P function acquires its singular character for the particular superposition states of interest here).

There have been described other methods to generate superposition states. For instance, dissipative optical bistability as a tool to obtain quantum superpositions has been proposed by Savage and Cheng [11]. Savage, Braunstein, and Walls [12] have suggested that quantum superposition states can be created by means of a single-atom dispersion. Another possibility to create superposition states is to use quantum-nondemolition techniques [13]. Phoenix and Knight [14] have shown that a single-mode electromagnetic field resonantly interacting with a single two-level atom described in the framework of the Jaynes-Cummings model [15] evolves into an almost pure state (see also recent papers by Gea-Banacloche [16]). Depending on the initial photon number of the coherent field and on the time of interaction between the atom and the field [17] this state can evolve into a state which is approximately described either as an even CS,

$$\begin{aligned}|\zeta\rangle_{\mathrm{even}}&=\mathcal{N}_{\mathrm{even}}^{1/2}(|\zeta\rangle+|-\zeta\rangle)\;,\\ \mathcal{N}_{\mathrm{even}}^{-1}&=2[1+\exp(-2|\zeta|^2)]\;,\end{aligned} \tag{2a}$$

or as an odd CS,

$$|\zeta\rangle_{\text{odd}} = \mathcal{N}_{\text{odd}}^{1/2}(|\zeta\rangle - |-\zeta\rangle) ,$$
$$\mathcal{N}_{\text{odd}}^{-1} = 2[1-\exp(-2|\zeta|^2)] . \tag{2b}$$

There is another possibility to go beyond the Jaynes-Cummings model to a more realistic model of a micromaser, describing a resonant interaction of a stream of polarized atoms with a single-mode cavity field. In particular, Meystre and co-workers [18] have made an extensive study of the production of macroscopic superpositions of micromaser states.

Recently Haroche and co-workers have proposed [19] a conceptually simple and elegant method to prepare superposition states confined in a micromaser cavity. The principle of this method (the so-called atomic phase detection quantum-nondemolition scheme) is based on a Ramsey-type experiment [20]: first, the two-level atom is prepared in a superposition of upper and lower states in the first Ramsey zone, then the atom is passed through the microwave cavity. The atomic transition frequency is *far* from resonance with the field frequency. Because of the detuning only one (upper) level of the atom interacts with the cavity field and it dephases this field by π. The lower level has no effect on the field. In the second Ramsey zone the atomic states are coherently mixed which results in the fact that the quantum-mechanical superposition is no longer carried by the atom but by the field. In other words, the atomic superposition has been transferred into a field superposition. The outcome of the experiment is that a field has either initial phase *or* a phase shifted by π, i.e., the field is prepared in either even or odd CS Eqs. (2).

The Yurke-Stoler and even and odd coherent states belong to a wider class of quantum superposition states, which may be used for a study of the Schrödinger-cat paradox. Therefore these states have been called Schrödinger-cat states. They can be generally written in the following form:

$$|\psi\rangle = \mathcal{N}^{1/2}\left[\sum_{j=1}^{N} e^{i\varphi_j}|\zeta_j\rangle\right] ,$$
$$\mathcal{N}^{-1} = \sum_{j,k=1}^{N} e^{i(\varphi_j-\varphi_k)}\langle\zeta_k|\zeta_j\rangle . \tag{3}$$

The nonclassical properties of these states have been intensively studied recently and the influence of damping at *zero* temperature on quantum coherence has been analyzed by Walls and Milburn [21], Milburn and Holmes [22], Kennedy and Drummond [23], Agarwal and Adam [24], Vourdas and Wiener [25], Phoenix [26], and by others (for more references see Ref. [1]). In particular, it has been shown that the effect of dissipation is to wash out the oscillations of the photon-number distribution. The sensitiveness of the photon-number distribution (PND) to even a quite small dissipative coupling has the origin in the fact that the PND depends on all moments of the field observables. Generally it is true that higher moments decay more rapidly than lower moments and therefore the overall decay rate of the oscillations of the PND is high. On the other hand, as shown in Ref. [1], quantities such as quadrature squeezing are more robust against dissipation because they involve only lower moments of field operators.

The role of the influence of damping on quantum interferences was originally analyzed by Caldeira and Leggett [27] who have included the effect of dissipation by an influence-functional technique which is valid for the case of strong coupling as well as for the case of weak coupling. Unruh and Zhurek [28] have recently proposed an alternative way to describe the influence of the environment on a quantum system. They have studied a model of a harmonic oscillator interacting with a one-dimensional massless scalar field. From results of Caldeira and Leggett [27] and Unruh and Zurek [28], it follows that quantum interferences are in general destroyed much faster than is the relaxation time of the system (see also the paper by Joos and Zeh [29]).

The aim of this paper is to study in detail the influence of a thermal reservoir on the decay of superposition states. For a description of the time evolution of the quantum system initially prepared in the superposition state we will utilize the *exact* solution of the Fokker-Planck equation for the generalized quasidistribution of the system. The Fokker-Planck equation governing the time evolution of the quantum system follows directly from the master equation for the density operator in the Born-Markov approximation (for details see Ref. [30]) which is valid only for a weak damping, i.e., is not as general as the approach proposed by Caldeira and Leggett [27]. Nevertheless, for the quantum optical systems (for which the weak-damping approximation is justified) the *exact* solution of the Fokker-Planck equation for the initial superposition state can be found. This solution allows us to show clearly the effects of a nonzero-temperature heat bath on macroscopic superpositions. We will show that at nonzero temperature the loss of the quantum coherence can be very rapid and that at high temperatures Schrödinger-cat states are transformed into mixture states almost instantaneously. We will consider the experimental consequences of our results.

Recently Daniel and Milburn [31] have studied the dynamics of a nonlinear oscillator, modeling the interaction of a single-mode field with a Kerr-like medium, subject to damping at nonzero temperature. These authors have assumed the oscillator to be initially prepared in the coherent state, which means that they have analyzed the influence of a thermal heat bath on a production of the Yurke-Stoler states. In our paper we assume the field mode to be initially prepared in the superposition state (for instance, this superposition state can be generated in the experiment proposed by Haroche and co-workers [19]) and then we study the influence of damping on quantum coherences, i.e., our analysis is applicable to microwave experiments [15,18,19], in which the influence of damping is neglected during the time interval when an atom interacts with the cavity field and the superposition state of the field is produced.

II. THE MASTER EQUATION

The state of the quantum-mechanical system can be characterized by the density operator $\hat{\rho}$, which can be defined as

$$\hat{\rho}=\sum_i p_i|\Psi_i\rangle\langle\Psi_i| , \tag{4}$$

where p_i is the normalized probability ($\sum_i p_i=1; p_i>0$) that the system is described by the state vector $|\Psi_i\rangle$. The expectation value of an arbitrary operator $\hat{M}$ is given as $\langle\hat{M}\rangle=\mathrm{Tr}[\hat{\rho}\hat{M}]$. The density operator $\hat{\rho}$ is Hermitian and its trace is equal to unity. If the quantum-mechanical system is in a pure state, that is $\hat{\rho}=|\Psi\rangle\langle\Psi|$, then $\hat{\rho}^2=\hat{\rho}$ and $\mathrm{Tr}\hat{\rho}^2=1$. On the other hand, a statistical-mixture state is characterized by the $\hat{\rho}$ operator for which $\mathrm{Tr}\hat{\rho}^2<1$. The superposition state (3) is an example of the pure quantum-mechanical state characterized by the following density operator:

$$\hat{\rho}=\mathcal{N}\left[\sum_{i,j=1}^{N}\exp[i(\varphi_i-\varphi_j)]|\zeta_i\rangle\langle\zeta_j|\right]$$
$$=\mathcal{N}\left[\sum_{i=1}^{N}|\zeta_i\rangle\langle\zeta_i|+\sum_{\substack{i,j,=1\\ i\neq 1}}\exp[i(\varphi_i-\varphi_j)]|\zeta_i\rangle\langle\zeta_j|\right] , \tag{5}$$

while the density operator

$$\hat{\rho}=\sum_{i=1}^{N}p_i|\zeta_i\rangle\langle\zeta_i| , \quad \sum_{i=1}^{N}p_i=1 \tag{6}$$

describes a statistical mixture of coherent states $|\zeta_i\rangle$.

In this paper we consider the interaction of a single-mode bosonic state with a heat bath at finite temperature ($T\neq 0$). The master equation (in the Born-Markov approximation) describing the time evolution of the density operator $\hat{\rho}$ in the interaction picture can be written as [30–32]

$$\frac{\partial\hat{\rho}}{\partial t}=\frac{\gamma}{2}(\bar{n}+1)(2\hat{a}\hat{\rho}\hat{a}^\dagger-\hat{a}^\dagger\hat{a}\hat{\rho}-\hat{\rho}\hat{a}^\dagger\hat{a})$$
$$+\frac{\gamma}{2}\bar{n}(2\hat{a}^\dagger\hat{\rho}\hat{a}-\hat{a}\hat{a}^\dagger\hat{\rho}-\hat{\rho}\hat{a}\hat{a}^\dagger) , \tag{7}$$

where γ is the decay rate and $\bar{n}$ is the average number of thermal photons at the frequency ω of the cavity mode at the temperature T,

$$\bar{n}=\frac{1}{\exp(\hbar\omega/k_B T)-1} , \tag{8}$$

where k_B is the Boltzman constant. The first term of the right-hand side (rhs) of Eq. (7) describes the transfer through the decay of photons from the quantum system to the heat bath, while the second term corresponds to the transfer of excitations from the nonzero temperature heat bath to the quantum system (compare the ordering of the creation and annihilation operators). For $\bar{n}=0$ Eq. (7) reduces to the equation describing the decay of the quantum system to a zero-temperature reservoir [the second term in Eq. (7) is then obviously equal to zero]. As one may expect the stationary solution of Eq. (7) describes either a thermal ($\bar{n}\neq 0$) or vacuum ($\bar{n}=0$) state of the field mode under consideration.

It is not straightforward to solve the operator equation (7) for the density operator $\hat{\rho}$ directly. It is much more convenient to transfer this operator equation into a corresponding c-number Fokker-Planck equation for a quasiprobability distribution, which can be solved more easily.

III. TIME EVOLUTION OF QUASIPROBABILITY DISTRIBUTIONS

The state of the quantum-mechanical system is characterized by the density operator. Alternatively, the quantum system can be described by the complete set of expectation values of the system operators. In particular, the state of a field mode (harmonic oscillator) can be described by the mean values (moments) of the bosonic operators $\hat{a}$ and $\hat{a}^\dagger$. Generally, the moments of the bosonic operators are given in normally ordered $\langle(\hat{a}^\dagger)^m\hat{a}^n\rangle$, antinormally ordered $\langle\hat{a}^n(\hat{a}^\dagger)^m\rangle$, or symmetrically ordered $\langle\{(\hat{a}^\dagger)^m\hat{a}^n\}\rangle$ form, and can be evaluated with the help of the s-parametrized characteristic function (CF) $C(\xi,s)$ introduced by Cahill and Glauber [33]

$$C(\xi,s)=\mathrm{Tr}[\hat{\rho}\exp(\xi\hat{a}^\dagger-\xi^*\hat{a}+s|\xi|^2/2)] , \tag{9}$$

which for $s=1$ reduces to the normally ordered CF, for $s=-1$ to the antinormally ordered CF, and for $s=0$ to the CF of symmetrically ordered moments. Generally, the mean values can be evaluated as

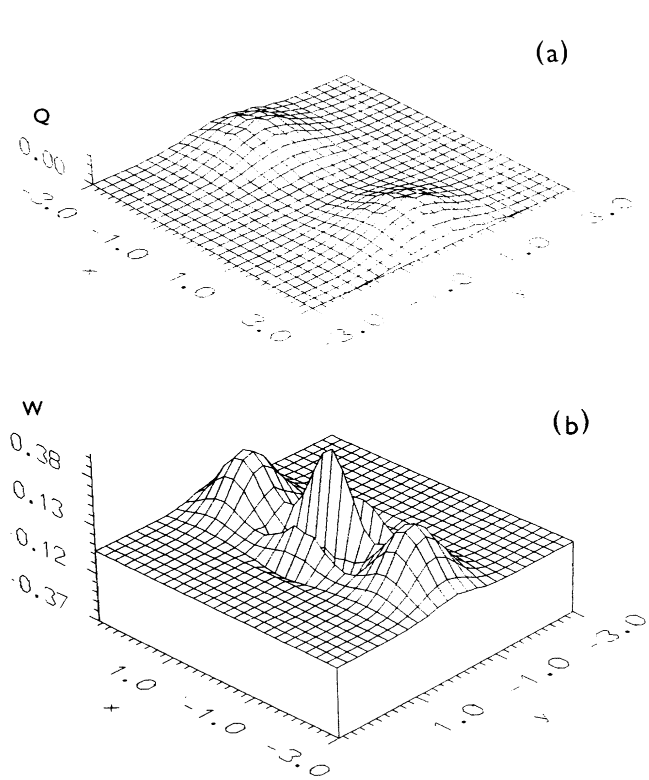

FIG. 1. The Q function (a) and the Wigner function (b) for the even CS with $\zeta=2$. We see that the quantum-interference term is more visible in the Wigner function but not in the Q function [$x=\mathrm{Re}(\alpha)$ and $y=\mathrm{Im}(\alpha)$].

$$\langle \{(\hat{a}^\dagger)^m \hat{a}^n\}_s \rangle = \left. \frac{\partial^{(m+n)}}{\partial \xi^m \partial(-\xi^*)^n} C(\xi,s) \right|_{\xi=0}, \quad (10)$$

where

$$\langle \{(\hat{a}^\dagger)^m \hat{a}^n\}_{s=-1} \rangle \equiv \langle \hat{a}^n (\hat{a}^\dagger)^m \rangle ,$$

$$\langle \{(\hat{a}^\dagger)^m \hat{a}^n\}_{s=1} \rangle \equiv \langle (\hat{a}^\dagger)^m \hat{a}^n \rangle ,$$

$$\langle \{(\hat{a}^\dagger)^m \hat{a}^n\}_{s=0} \rangle \equiv \langle \{(\hat{a}^\dagger)^m \hat{a}^n\} \rangle .$$

These mean values can be evaluated from the quasiprobability distributions $W(\alpha,s)$, which are defined as Fourier transforms of characteristic functions,

$$W(\alpha,s) = \frac{1}{\pi^2} \int d^2\xi \, C(\xi,s) \exp(\alpha \xi^* - \alpha^* \xi) \quad (11a)$$

and

$$\langle \{(\hat{a}^\dagger)^m \hat{a}^n\}_s \rangle = \int d^2\alpha \, \alpha^{*m} \alpha^n W(\alpha,s) . \quad (11b)$$

The quasiprobability distribution $W(\alpha,s=1)$ is the Glauber-Sudarshan P function [34], $W(\alpha,s=0)$ is the Wigner function [35], and $W(\alpha,s=-1) = \langle \alpha|\hat{\rho}|\alpha\rangle/\pi \equiv Q(\alpha)$ is the Q function [36].

One can convert the master equation (7) into a linear differential equation for the Q function (see, for instance, Refs. [30,31,37] and references therein):

$$\frac{\partial Q(\alpha,t)}{\partial t} = \left\{ \gamma + \frac{\gamma}{2}\left[\alpha \frac{\partial}{\partial \alpha} + \alpha^* \frac{\partial}{\partial \alpha^*} \right] + \gamma(\bar{n}+1) \frac{\partial^2}{\partial \alpha \partial \alpha^*} \right\} Q(\alpha,t) , \quad (12)$$

which can be identified as a generalized Fokker-Planck equation [31,32]. This equation for the Q function with an initial condition

$$Q(\alpha,0) = \exp(-|\alpha|^2) \sum_{m,n=0}^{\infty} \frac{\alpha^m (\alpha^*)^n}{(m!n!)^{1/2}} h_{m,n}(0) ,$$
$$h_{m,n}(0) = \frac{1}{\pi} \langle n|\hat{\rho}|m\rangle , \quad (13)$$

can be solved exactly (see Appendix) to give the solution

$$Q(\alpha,t) = \frac{e^{-|\alpha|^2}}{\bar{n}(1-e^{-\gamma t})+1} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \alpha^m \alpha^{*n} \left[\frac{e^{-\gamma t/2}}{\bar{n}(1-e^{-\gamma t})+1} \right]^{m+n}$$
$$\times \sum_{l=0}^{\infty} \left[1 - \frac{e^{-\gamma t}}{\bar{n}(1-e^{-\gamma t})+1} \right]^l \frac{(m+l)!(n+l)!}{m!n!l!} h_{m+l,n+l}(0)$$
$$\times F(-m,-n,l+1;4\bar{n}(\bar{n}+1)\sinh^2(\gamma/2)t) , \quad (14)$$

where $F(x,y,z;h)$ is the hypergeometric function [38].

If we assume the field mode to be initially prepared in the superposition state (ζ is real)

$$|\psi\rangle = \mathcal{N}^{1/2}(|\zeta\rangle + e^{i\varphi}|-\zeta\rangle) , \quad \mathcal{N}^{-1} = 2[1+\cos\varphi \exp(-2\zeta^2)] , \quad (15a)$$

then the Q function of the initial state is given by the relation

$$Q(\alpha,0) = \frac{\mathcal{N}}{\pi} e^{-\alpha_i^2} [e^{-(\alpha_r-\zeta)^2} + e^{-(\alpha_r+\zeta)^2} + 2e^{-\zeta^2} e^{-\alpha_r^2} \cos(\varphi + 2\zeta\alpha_i)] , \quad (15b)$$

and the matrix element $h_{m,n}(0)$ by

$$h_{m,n}(0) = \frac{\mathcal{N} e^{-\zeta^2}}{\pi(m!n!)^{1/2}} \zeta^{n+m} [1 + (-1)^n e^{i\varphi} + (-1)^m e^{-i\phi} + (-1)^{n+m}] . \quad (16)$$

In this case from Eq. (14) we find the explicit expression for the Q function at $t \geq 0$,

$$Q(\alpha,t) = \frac{\mathcal{N}}{\pi(\bar{n}_t+1)} \exp\left[-\frac{\alpha_i^2}{\bar{n}_t+1} \right] \left\{ \exp\left[-\frac{(\alpha_r - e^{-\gamma t/2}\zeta)^2}{\bar{n}_t+1} \right] + \exp\left[-\frac{(\alpha_r + e^{-\gamma t/2}\zeta)^2}{\bar{n}_t+1} \right] \right.$$
$$\left. + 2\exp\left[-\left[2 - \frac{e^{-\gamma t}}{\bar{n}_t+1} \right] \zeta^2 \right] \exp\left[-\frac{\alpha_r^2}{\bar{n}_t+1} \right] \cos\left[\frac{2e^{-\gamma t/2}}{\bar{n}_t+1} \zeta \alpha_i + \varphi \right] \right\} , \quad (17)$$

where α_r and α_i denote the real and imaginary parts of α respectively, i.e., $\alpha = (\alpha_r, \alpha_i)$. The parameter $\bar{n}_t$ is defined as

$$\bar{n}_t = \bar{n}[1 - \exp(-\gamma t)] , \quad (18)$$

which is, in fact, the number of photons transferred from the nonzero temperature heat bath to the quantum system. Equation (17) at $t=0$ gives a picture of the Q function exhibiting the "central" interference term [the term in the rhs of Eq. (17) containing the cos function], which has its maximum at the origin of the phase space. This term arises as a direct consequence of the quantum interference between coherent states $|\zeta\rangle$ and $|-\zeta\rangle$. The coherent components of the superposition state under consideration are described as two Gaussian peaks around $\alpha_r = \pm\zeta$. We should emphasize here that these two peaks can be observed also when the field is initially prepared in the mixed state described by the density operator

$$\hat{\rho} = \tfrac{1}{2}[|\zeta\rangle\langle\zeta| + |-\zeta\rangle\langle-\zeta|] . \tag{19}$$

In this case the Q function at $t \geq 0$ reads

$$Q(\alpha,t) = \frac{1}{2\pi(\bar{n}_t+1)}\exp\left[-\frac{\alpha_i^2}{\bar{n}_t+1}\right]\left\{\exp\left[-\frac{(\alpha_r - e^{-\gamma t/2}\zeta)^2}{\bar{n}_t+1}\right] + \exp\left[-\frac{(\alpha_r + e^{-\gamma t/2}\zeta)^2}{\bar{n}_t+1}\right]\right\} , \tag{20}$$

and we see that it does not contain an interference term. From Eq. (17) it follows that the interference term in the Q function contains a factor $\exp(-\zeta^2)$ which means that even for relatively small values of ζ this interference term is significantly suppressed. The amplitude of the Q function at the origin $\alpha=(0,0)$ (i.e., the amplitude of the interference term) is much smaller than the amplitude at $\alpha=(\pm\zeta,0)$ corresponding to the component states $|\zeta\rangle$ and $|-\zeta\rangle$ [see Fig. 1(a) describing the Q function of the even CS with $\zeta=2$]. Therefore the Q function is not very convenient for a pictorial description of quantum-interference effects. As seen below the interference term in the corresponding Wigner function is much more pronounced and therefore we will use this function to describe the influence of the thermal heat bath on the quantum-interference effects.

IV. THE TIME EVOLUTION OF THE WIGNER FUNCTION

From the explicit expression for the Q function we can derive other quasiprobability distributions. Using the Fourier transform Eq. (11) we can obtain from the Q function the characteristic function $C(\xi,s=-1,t) \equiv C^{(a)}(\xi,t)$ for the antinormally ordered moments of the bosonic operators,

$$C^{(a)}(\xi,t) = 2\mathcal{N}\exp[-(\bar{n}_r+1)|\xi|^2]\{\cos(2e^{-\gamma t/2}\zeta\xi_i) + \exp(-2\zeta^2)\cosh(2\zeta e^{-\gamma t/2}\xi_r + i\varphi)\} . \tag{21}$$

The characteristic functions $C(\xi,s)$ for various values of the parameter s are related as follows:

$$C(\xi,s=-1) = \exp(-|\xi|^2/2)C(\xi,s=0) = \exp(-|\xi|^2)C(\xi,s=1) , \tag{22}$$

where $C(\xi,s=0)$ and $C(\xi,s=1)$ are the characteristic functions for the Wigner and the P functions, respectively. Once the explicit expression for the symmetrically ordered characteristic function is known from Eqs. (21) and (22) we can find the Wigner function which we write as a sum of two terms,

$$W(\alpha,t) = \sum_{i=1}^{2} W_M^{(i)}(\alpha,t) + W_I(\alpha,t) , \tag{23}$$

where $W_I(\alpha,t)$ corresponds to the interference term

$$W_I(\alpha,t) = \frac{4\mathcal{N}}{\pi(2\bar{n}_t+1)}\exp\left[-\frac{2|\alpha|^2}{2\bar{n}_t+1}\right]\exp\left[-2\left[1-\frac{e^{-\gamma t}}{2\bar{n}_t+1}\right]\zeta^2\right]\cos\left[\frac{4e^{-\gamma t/2}}{2\bar{n}_t+1}\zeta\alpha_i\right] , \tag{24}$$

and $W_M^{(i)}(\alpha,t)$ separately describes the Wigner function (up to the normalization constant) of the composition states $|\zeta\rangle$ $(i=1)$ and $|-\zeta\rangle$ $(i=2)$, whose sum is the Wigner function of the mixture state (19)

$$W_M^{(i)}(\alpha,t) = \frac{2\mathcal{N}}{\pi(2\bar{n}_t+1)}\exp\left[-\frac{2\alpha_i^2}{2\bar{n}_t+1}\right] \times\exp\left[-\frac{2(\alpha_r \mp e^{-\gamma t/2}\zeta)^2}{2\bar{n}_t+1}\right] . \tag{25}$$

The Wigner function (23) of the even CS at $t=0$ is plotted in Fig. 1(b). The interference term which is responsible for nonclassical effects is much more visible compared with the Q function [see Fig. 1(a)].

We notice here that the values of the Wigner function (23) for the initial moment $t=0$ at $\alpha=(0,0)$ and $\alpha=(\pm\zeta,0)$ are

$$W(\alpha,t)|_{\alpha=(0,0),t=0} = \frac{4\mathcal{N}}{\pi}[1+e^{-2\zeta^2}] = \frac{2}{\pi} , \tag{26a}$$

$$W(\alpha,t)|_{\alpha=(\pm\zeta,0),t=0} = \frac{2\mathcal{N}}{\pi}[1+2e^{-2\zeta^2}+e^{-8\zeta^2}] , \tag{26b}$$

which means that for large enough ζ we can approximate the Wigner function at $\alpha=(0,0)$ and $\alpha=(\pm\zeta,0)$ as follows:

$$W(\alpha,t)|_{\alpha=(0,0),t=0} \simeq W_I(\alpha,t)|_{\alpha=(0,0),t=0} = \frac{4\mathcal{N}}{\pi} \tag{27a}$$

and

$$W(\alpha,t)|_{\alpha=(\pm\zeta,0),t=0} \simeq W_M^{(i)}(\alpha,t)|_{\alpha=(\pm\zeta,0),t=0} = \frac{2\mathcal{N}}{\pi} . \tag{27b}$$

From above it follows that the total Wigner function locally around $\alpha=(0,0)$ and $\alpha=(\pm\zeta,0)$ at initial moments of the evolution can be approximated by the functions $W_I(\alpha,t)$ and $W_M^{(i)}(\alpha,t)$, respectively.

To understand properly the decay of the quantum coherences we evaluate the first derivative of the Wigner function at $t=0$

$$\frac{\partial}{\partial t}W(\alpha,t)|_{t=0}=\sum_{i=1}^{2}\frac{\partial}{\partial t}W_M^{(i)}(\alpha,t)|_{t=0}+\frac{\partial}{\partial t}W_I(\alpha,t)|_{t=0}\ , \tag{28}$$

where

$$\frac{\partial}{\partial t}W_M^{(i)}(\alpha,t)|_{t=0} =2\gamma W_M^{(i)}(\alpha,t)|_{t=0}\{-\bar{n}+2\alpha_i^2\bar{n}+2\bar{n}(\alpha_r\mp\zeta)^2 \mp\zeta(\alpha_r\mp\zeta)\} \tag{29}$$

and

$$\frac{\partial}{\partial t}W_I(\alpha,t)|_{t=0} =2\gamma W_I(\alpha,t)|_{t=0}\{-\bar{n}+2|\alpha|^2\bar{n}-(2\bar{n}+1)\zeta^2 +[\zeta\alpha_i+4\bar{n}\zeta\alpha_i]\tan(4\zeta\alpha_i)\}\ . \tag{30}$$

At the origin of the phase space (i.e., for $\alpha_r=\alpha_i=0$), we find

$$\frac{\partial}{\partial t}W(\alpha,t)|_{t=0,\alpha=(0,0)} =4\gamma W_M^{(i)}(\alpha,t)|_{t=0,\alpha=(0,0)}(-\bar{n}+2\bar{n}\zeta^2+\zeta^2) -2\gamma W_I(\alpha,t)|_{t=0,\alpha=(0,0)}(\bar{n}+\zeta^2+2\bar{n}\zeta^2) \tag{31}$$

from which it follows that we can, for ζ large enough, approximate the Wigner function in the following way:

$$\frac{\partial}{\partial t}W(\alpha,t)|_{t=0,\alpha=(0,0)}\simeq\frac{\partial}{\partial t}W_I(\alpha,t)|_{t=0,\alpha=(0,0)} =-\frac{8\gamma\mathcal{N}}{\pi}(\bar{n}+\zeta^2+2\bar{n}\zeta^2)\ . \tag{32}$$

The above expression describes the rapidity of the loss of the quantum coherence. We can conclude from it that for $\bar{n}=0$ (zero-temperature heat bath) the rapidity of loss of the coherence depends only on the value of $\gamma\zeta^2$, that is on the separation between the components of the superposition state and the spontaneous-damping rate γ. In this case during the initial moments of the evolution the peak of the interference term of the Wigner function [i.e., $W_I(\alpha,t)$ at $\alpha=(0,0)$] is decaying according to the relation

$$W_I(\alpha,t)|_{\alpha=(0,0)}\simeq\frac{4\mathcal{N}}{\pi}\exp(-2\gamma\zeta^2t)\equiv w(t)\ , \tag{33}$$

which is in agreement with the results by Milburn and Walls [21], Phoenix [26], and Bužek and co-workers [1]. On the other hand, if the quantum superposition state is decaying into a nonzero-temperature heat bath ($\bar{n}>0$), then the evolution of the Wigner function at $\alpha=(0,0)$ for small t is governed by the relation:

$$W_I(\alpha,t)|_{\alpha=(0,0)}\simeq\frac{4\mathcal{N}}{\pi}\exp[-2(\gamma\zeta^2+\gamma\bar{n}+2\gamma\bar{n}\zeta^2)t] \equiv w(t)\exp[-2\gamma\bar{n}(1+2\zeta^2)]\ , \tag{34}$$

which means that at nonzero temperature the quantum coherence is lost *much faster* than at zero temperature. How much faster depends on the actual values of $\bar{n}$ and ζ. For large values of $\bar{n}$ the coherence is lost almost instantaneously. Qualitatively one can understand this result as follows: The decay rate of the quantum coherence at zero temperature during the initial moments of the time evolution is equal to $\gamma\zeta^2$. On the other hand, the system relaxation rate at nonzero temperature, due to the stimulated emission, equals to $\gamma'\equiv\gamma(\bar{n}+1)$. Combining these two facts, one can expect that the decay rate of the quantum coherence at nonzero temperature should be $\gamma'\zeta^2=(\bar{n}+1)\gamma\zeta^2$ which is in qualitative agreement with Eq. (34). The rapid decay of the interference term of the Wigner function can be seen in Fig. 2. From this picture we see that the even CS is transformed under the influence of the thermal heat bath into the mixed state. We will turn our attention to this point later when we analyze the oscillations of the photon-number distribution of the even CS decaying into the thermal reservoir.

It is interesting to note that the time derivative of the Wigner function at $\alpha=(\pm\zeta,0)$ for small t (during the first moments of the evolution) can be approximated as

$$\frac{\partial}{\partial t}W(\alpha,t)|_{t=0,\alpha=(\pm\zeta,0)} \simeq\frac{\partial}{\partial t}W_M^{(i)}(\alpha,t)\bigg|_{t=0;\alpha=(\pm\zeta,0)}=-\frac{4\mathcal{N}}{\pi}\gamma\bar{n}\ . \tag{35}$$

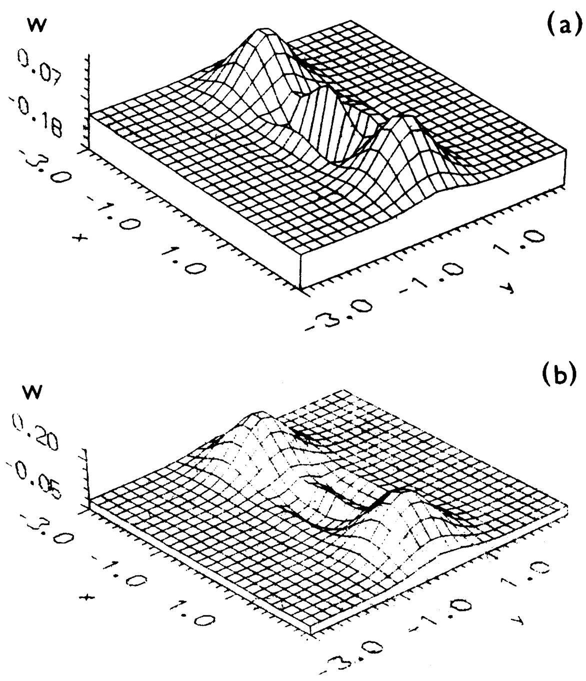

FIG. 2. The Wigner function at $\gamma t=0.1$ of the field mode initially prepared in the even CS with $\zeta=2$. The field mode interacts (a) with a zero-temperature heat bath ($\bar{n}=0$) and (b) with a nonzero-temperature heat bath ($\bar{n}=1$). We see that the interference term in the latter case ($\bar{n}\neq0$) is much more suppressed than the former case of zero temperature ($\bar{n}=0$).

For $\bar{n}>0$ the maximum amplitude of the part of the Wigner function corresponding to the component state $|\zeta\rangle$ ($|-\zeta\rangle$) is slowly decreasing and as seen from the expression, Eq. (25), simultaneously the width of this part of the Wigner function becomes broader. This is due to the interaction with thermal photons. If $\bar{n}=0$ then the width of the peaks of the Wigner function corresponding to $|\pm\zeta\rangle$ is constant. The maximum amplitude of $W_M^{(i)}(\alpha,t)$ is constant as well. The coherent state remains such under dissipation, with its mean value decaying but its variance being unchanged. In a heat bath the initial coherent-state variances will *increase* to reflect the greater fluctuations of the thermal state to which it must tend in the equilibrium, i.e., in the stationary limit, $t\to\infty$, the field mode interacting with the thermal heat bath will reach the thermal state described by the Wigner function

$$W(\alpha,t)|_{t\to\infty}=\frac{2}{\pi(1+2\bar{n})}\exp\left[-\frac{2|\alpha|^2}{1+2\bar{n}}\right], \tag{36}$$

which for $\bar{n}=0$ corresponds to the Wigner function of the vacuum state.

V. OSCILLATIONS OF THE PHOTON-NUMBER DISTRIBUTION

One of the most transparent nonclassical effects emerging from the quantum-interference between two coherent states is the presence of oscillations in the photon-number distribution [1,2]. The photon-number distribution $\wp(l)$ is defined as

$$\wp(l)\equiv\langle l|\hat{\rho}|l\rangle , \tag{37}$$

and can be evaluated from the Wigner function $W(\alpha)$ using the relation

$$\wp(l)=\pi\int d^2\alpha\, W(\alpha)W_l(\alpha) , \tag{38}$$

where $W_l(\alpha)$ is the Wigner function of the number state $|l\rangle$ [33],

$$W_l(\alpha)=\frac{2(-1)^l}{\pi}\exp(-2|\alpha|^2)\mathcal{L}_l(4|\alpha|^2) \tag{39}$$

and $\mathcal{L}_l(x)$ is the Laguerre polynomial of order l [38].

Using the explicit expression for the Wigner function (23) of the superposition state (15a), we find the following expression for the photon-number distribution:

$$\wp(l,t)=\frac{2\mathcal{N}l!e^{-\zeta^2}}{\bar{n}_t+1}\sum_{j=0}^{l}\frac{1}{j!(l-j)!^2}\left[\frac{\bar{n}_t}{1+\bar{n}_t}\right]^j\left[\frac{\zeta^2e^{-\gamma t}}{(\bar{n}_t+1)^2}\right]^{(l-j)}$$
$$\times\left\{\exp\left[\left[1-\frac{e^{-\gamma t}}{\bar{n}_t+1}\right]\zeta^2\right]+(-1)^{(l-j)}\cos\varphi\exp\left[-\left[1-\frac{e^{-\gamma t}}{\bar{n}_t+1}\right]\zeta^2\right]\right\}. \tag{40}$$

From this expression we can easily derive the photon-number distribution of the initial superposition state at $t=0$. If we consider the field initially prepared in the even CS (i.e., $\varphi=0$), then

$$\wp(l,t=0)=\frac{2\exp(-|\zeta|^2)}{1+\exp(-2|\zeta|^2)}\frac{|\zeta|^{2l}}{l!}, \quad \text{if } l=2m ,$$
$$\wp(l,t=0)=0 \quad \text{if } l=2m+1 , \tag{41}$$

which means that $\wp(l,t=0)$ exhibits significant oscillations [see Fig. 3(a)]. These oscillations are very similar to those which are typical for the squeezed vacuum [2] and they can serve as a good indication that the state under consideration is nonclassical. The photon-number distribution of the statistical mixture (19) is Poissonian, i.e., $\wp(l,t)=\exp(-|\zeta|^2)|\zeta|^{2l}/l!$, without oscillations.

As seen from Eq. (40) the photon-number distribution $\wp(l,t)$ can be expressed in the form

$$\wp(l,t)=\sum_{i=1}^{2}\wp_M^{(i)}(l,t)+\wp_I(l,t) , \tag{42}$$

where

$$\wp_M^{(i)}(l)=\pi\int d^2\alpha\, W_M^{(i)}(\alpha)W_l(\alpha) , \tag{43a}$$

and

$$\wp_I(l)=\pi\int d^2\alpha\, W_I(\alpha)W_l(\alpha) . \tag{43b}$$

The term $\wp_I(l)$, which is related to the quantum-interference part of the Wigner function and can be negative, is responsible for the oscillatory behavior of the photon-number distribution. As we have shown earlier, the quantum-interference part of the Wigner function, i.e., $W_I(\alpha,t)$, is decaying much faster than the functions $W_M^{(i)}(\alpha,t)$. This results in the fact that the oscillations of the photon-number distribution are very sensitive to the influence of the heat bath.

In Fig. 3(b) we plot the photon-number distribution $\wp(l,t)$ of the initial even CS ($\zeta=2$) at $t=0.01/\gamma$ for two values of $\bar{n}$. From this picture we see that even with one thermal photon (dotted line) the oscillations are smeared out in a short time and the photon-number distribution rapidly converges to that for the statistical mixture. When an ideal heat bath at zero temperature is considered (solid line) the oscillations can be observed for a longer time. We will turn to this point later in Sec. VII.

VI. QUADRATURE SQUEEZING

Another nonclassical effect which has its origin in the quantum interference between coherent states [1–3] is quadrature squeezing [4]. In order to study quadrature squeezing of a single-mode field we introduce two quadrature operators $\hat{a}_1$ and $\hat{a}_2$ corresponding to the creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ of the field mode under consideration

$$\hat{a}_1=\frac{\hat{a}+\hat{a}^\dagger}{2}\ ,\quad \hat{a}_2=\frac{\hat{a}-\hat{a}^\dagger}{2i} \tag{44}$$

and

$$[\hat{a}_1,\hat{a}_2]=2iC\ ,\quad C=\tfrac{1}{4}\ . \tag{45}$$

One of the consequences of the commutation relation (45) is the uncertainty relation for the variances of the quadrature operators,

$$\langle(\Delta\hat{a}_1)^2\rangle\langle(\Delta\hat{a}_2)^2\rangle\geq C^2=\tfrac{1}{16}\ , \tag{46}$$

where the variance of the operator $\hat{a}_i$ is defined as $\langle(\Delta\hat{a}_i)^2\rangle=\langle\hat{a}_i^2\rangle-\langle\hat{a}_i\rangle^2$ and is related to the normally ordered variance $\langle:(\Delta\hat{a}_i)^2:\rangle$ as follows:

$$\langle(\Delta\hat{a}_i)^2\rangle=C+\langle:(\Delta\hat{a}_i)^2:\rangle\ . \tag{47}$$

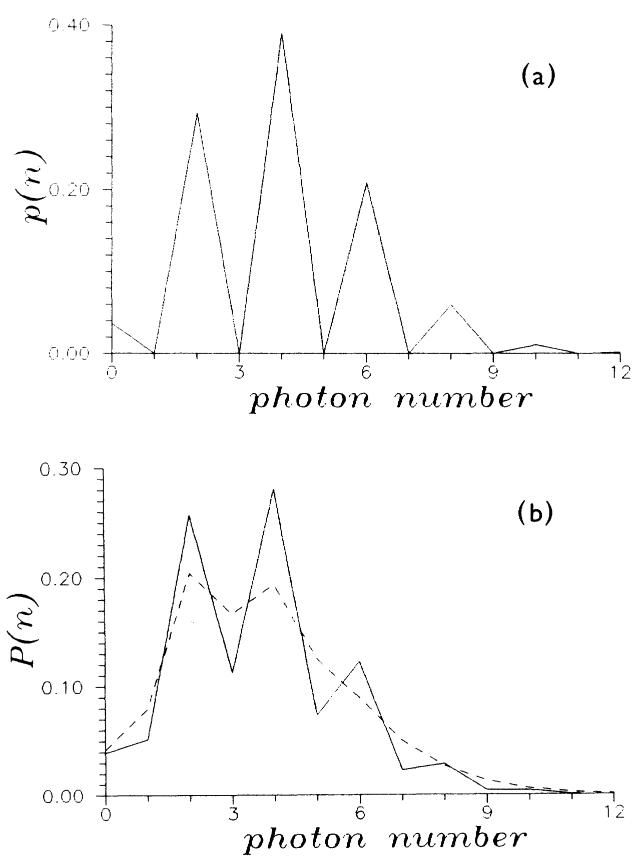

FIG. 3. (a) The photon-number distribution $\wp(l,0)$ for the initial even CS with $\zeta=2$ (a). We see typical oscillations emerging from the quantum-interference between component states of the even CS. (b) The photon-number distribution $\wp(l,t)$ at $\gamma t=0.01$ of the field mode initially prepared in the even CS. The field mode interacts with a zero-temperature heat bath (solid line) and with a nonzero-temperature heat bath of $\bar{n}=1$ (dotted line). We see that the oscillations of the PND are much more pronounced in the case of the zero-temperature heat bath.

The state for which the equality in Eq. (46) holds is called the minimum uncertainty state (MUS). For instance, the vacuum and the coherent states of light are examples of the MUS. For these states the variances in both quadratures are equal to $\frac{1}{4}$. The state is called squeezed if the variance of the quadrature operator is less than the vacuum fluctuations (i.e., less than $\frac{1}{4}$). Alternatively, one can say that the state is squeezed if the normally ordered variance is less than zero. It is not necessary for the squeezed state to be a MUS.

The variances of the quadrature operators can be written as

$$\langle(\Delta\hat{a}_1)^2\rangle=\tfrac{1}{4}+\tfrac{1}{2}[\langle\hat{a}^\dagger\hat{a}\rangle+\mathrm{Re}\langle\hat{a}^2\rangle-2(\mathrm{Re}\langle\hat{a}\rangle)^2]\ , \tag{48a}$$

$$\langle(\Delta\hat{a}_2)^2\rangle=\tfrac{1}{4}+\tfrac{1}{2}[\langle\hat{a}^\dagger\hat{a}\rangle-\mathrm{Re}\langle\hat{a}^2\rangle-2(\mathrm{Im}\langle\hat{a}\rangle)^2]\ , \tag{48b}$$

from which it follows that squeezing can appear only if the expectation values $\langle\hat{a}\rangle$ and/or $\langle\hat{a}^2\rangle$ are nonzero (of course this is not a sufficient condition for observation of squeezing). The nonzero values of $\langle\hat{a}\rangle$ and/or $\langle\hat{a}^2\rangle$ are associated with the off-diagonal terms of the density matrix in the number-state basis.

To measure the degree of quadrature squeezing we can introduce two squeezing parameters $S_i^{(2)}$,

$$\begin{aligned}S_1^{(2)}&=\frac{\langle(\Delta\hat{a}_1)^2\rangle-C}{C}\\&=\frac{1}{C}\langle:(\Delta\hat{a}_1)^2:\rangle\\&=2[\langle\hat{a}^\dagger\hat{a}\rangle+\mathrm{Re}\langle\hat{a}^2\rangle-2(\mathrm{Re}\langle\hat{a}\rangle)^2]\end{aligned} \tag{49a}$$

and

$$\begin{aligned}S_2^{(2)}&=\frac{\langle(\Delta\hat{a}_2)^2\rangle-C}{C}\\&=\frac{1}{C}\langle:(\Delta\hat{a}_2)^2:\rangle\\&=2[\langle\hat{a}^\dagger\hat{a}\rangle-\mathrm{Re}\langle\hat{a}^2\rangle-2(\mathrm{Im}\langle\hat{a}\rangle)^2]\ .\end{aligned} \tag{49b}$$

The squeezing condition now reads $S_i^{(2)}<0$, and the maximum squeezing corresponds to $S_i^{(2)}=-1$ or, equivalently $\langle:(\Delta\hat{a}_i)^2:\rangle=-\frac{1}{4}$.

Using the relation (11b) we can evaluate the mean values of the field operators from quasiprobability distributions. In particular, if the field is initially prepared in the superposition state (15a) then the mean value of the number operator at time t has the form

$$\langle\hat{n}\rangle=2\mathcal{N}\{[1+\bar{n}_t+\zeta^2\exp(-\gamma t)]+e^{-2\zeta^2}[1+\bar{n}_t-\zeta^2\exp(-\gamma t)]\cos\varphi\}-1 \tag{50}$$

from which we can find the mean photon number of the superposition state (15a) at time $t=0$

$$\langle\hat{n}\rangle=\zeta^2\frac{1-\cos\varphi e^{-2\zeta^2}}{1+\cos\varphi e^{-2\zeta^2}}\ . \tag{51}$$

On the other hand in the stationary limit $t \to \infty$, when the field is in the equilibrium with the thermal heat bath, we find that

$$\langle \hat{n} \rangle|_{t\to\infty} = \bar{n} \ . \tag{52}$$

Analogously, with the use of Eq. (11b) we calculate the time development of the squeeze parameters $S_i^{(2)}(t)$ when the field is initially in the superposition of coherent states (15a)

$$S_1^{(2)}(t) = 4\mathcal{N}[(1+\bar{n}_t)(1+e^{-2\zeta^2}\cos\varphi)+2\zeta 2e^{-\gamma t}]-2 \ , \tag{53a}$$

$$S_2^{(2)}(t) = 4\mathcal{N}[(1+\bar{n}_t)(1+e^{-2\zeta^2}\cos\varphi) - 2e^{-\gamma t}\zeta^2 e^{-2\zeta^2}\cos\varphi]-2 \ . \tag{53b}$$

From the above relations we see that at $t=0$ the even CS ($\varphi=0$) is quantum optically squeezed because

$$S_2^{(2)}(t)|_{t=0} = -\frac{4\zeta^2 e^{-2\zeta^2}}{1+e^{-2\zeta^2}} < 0 \ . \tag{54}$$

The maximum squeezing can be obtained for $\zeta \simeq 0.8$ at the initial time $t=0$ [39]. As shown in Refs. [1] if the field mode which is initially prepared in the even CS is coupled to a zero-temperature heat bath then during the time evolution the degree of squeezing becomes smaller [see Eq. (53b) for $\bar{n}=0$ and $\varphi=0$]. Nevertheless, the squeezing is much more robust with respect to damping than the oscillations of the photon-number distribution or the interference term of the Wigner function. For instance, for $\gamma t=0.3$ one can observe a considerable degree of quadrature squeezing, while the Wigner function for this value of γt is almost identical to the Wigner function of the statistical mixture. The rate of decay of quantum coherences and the oscillations of the photon-number distribution is highly sensitive to even a quite small dissipative coupling because the interference part of the Wigner function depends on all moments of the field variables, and higher moments decay more rapidly than lower moments. Quantities such as quadrature squeezing, on the other hand, are more robust against dissipation because they involve only lower moments. The difference between the decay rates of the quantum coherence and the degree of squeezing is also seen in the case of the nonzero-temperature heat bath. Generally, the higher the temperature of the heat bath the more the nonclassical effects will be deteriorated. Nevertheless, the rate of deterioration is not equal for different nonclassical effects.

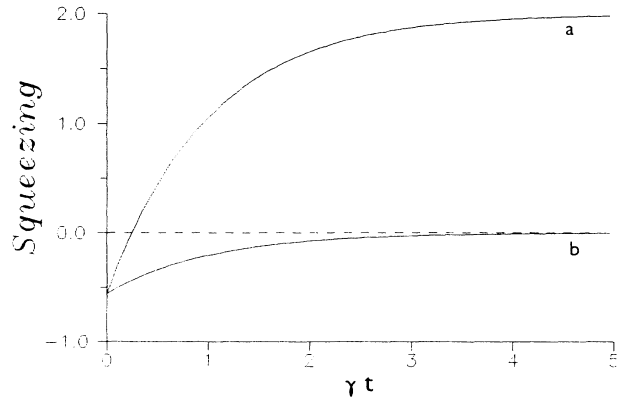

FIG. 4. The time evolution of the squeezing parameter $S_2^{(2)}$ of the field mode initially prepared in the even CS ($\zeta=0.8$). The value of ζ has been chosen to optimize the squeezing at the initial time $t=0$. Line (b) corresponds to the interaction with a zero-temperature heat bath ($\bar{n}=0$) while line (a) describes the time evolution of squeezing when the field mode is coupled to a nonzero-temperature heat bath ($\bar{n}=1$).

Finally we notice that at zero temperature the squeezing factor $S_2^{(2)}(t)$ is less than zero for any $0 \le t < \infty$, which means that the field mode is in a nonclassical state during the whole time evolution. This is reflected by the fact that the Glauber-Sudarshan P function defined by Eq. (11) does not exist for this case. From the integral expression for the P function

$$P(\alpha,t) = \frac{2}{\pi^2}\mathcal{N}\int d^2\xi \exp(-\bar{n}_t|\xi|^2)\exp(-\alpha\xi^* + \alpha^*\xi)\{\cos(2\zeta e^{-\gamma t/2}\xi_i)+\exp(-2\zeta^2)\cosh(2\zeta e^{-\gamma t/2}\xi_r + i\varphi)\} \ , \tag{55}$$

we see that this integral diverges for $\bar{n}=0$. On the other hand, for $\bar{n}>0$ and $t>0$ the integral (55) is a simple Gaussian one which can be evaluated as

$$P(\alpha,t) = \frac{\mathcal{N}e^{-\zeta^2}}{\pi\bar{n}_t}\exp\left[-\frac{1}{\bar{n}_t}|\alpha|^2\right]\left\{\exp\left[\left[1-\frac{e^{-\gamma t}}{\bar{n}_t}\right]\zeta^2\right]\left[\exp\left[\frac{2\zeta e^{-\gamma t/2}}{\bar{n}_t}\alpha_r\right]+\exp\left[-\frac{2\zeta e^{-\gamma t/2}}{\bar{n}_t}\alpha_r\right]\right] + 2\exp\left[-\left[1-\frac{e^{-\gamma t}}{\bar{n}_t}\right]\zeta^2\right]\cos\left[\frac{2\zeta e^{-\gamma t/2}}{\bar{n}_t}\alpha_i\right]\right\} \ . \tag{56}$$

This P function can have negative values for some values of α which indicates that the field is still in a nonclassical state, which explains why for some values of the interaction time, even for $\bar{n}>0$, the quadrature squeezing can still be observed [i.e., $S_2<0$ as shown in Fig. 4(a)]. Nevertheless, due to the interaction with the *nonzero*-temperature heat bath, the P function eventually becomes positive as well as do both squeezing parameters $S_i^{(2)}$, which means that the field mode is in a classical state. In the stationary limit $S_1^{(2)} = S_2^{(2)} = 2\bar{n}$ and the P function takes the form

$$P(\alpha,t)|_{t\to\infty} = \frac{1}{\pi\bar{n}}\exp\left[-\frac{|\alpha|^2}{\bar{n}}\right] \ , \tag{57}$$

corresponding to a thermal state.

We conclude that while the superposition state (15a) is interacting with the zero-temperature heat bath the field state stays as a nonclassical for any $t \geq 0$. On the other hand, if the temperature of the heat bath is larger than zero, then the field mode will be transformed from the nonclassical state to the classical one at some finite time t, for which $P(\alpha, t) > 0$.

VII. DISCUSSION AND CONCLUSIONS

In this paper we have studied the influence of a thermal heat bath on the nonclassical properties of the quantum superposition states. We have shown that at nonzero temperature the loss of coherences is much faster than at zero temperature. The decay rate depends on the separation between the component states and on the temperature of the heat bath [see Eq. (34)]. Moreover, we have shown that the interaction with nonzero-temperature heat bath leads to a transformation of a nonclassical state to a classical state.

The sensitivity of the quantum coherence to the presence of the nonzero-temperature heat bath can lead to some difficulties in the preparation of Schrödinger-cat states in microwave cavities, which operate at low, but not zero temperature. In particular, if we adopt the values of the decay rate γ, the amplitude ζ, and the time of measurement t as described in a recent paper by Haroche and co-workers [19] (i.e., $\gamma = 10$ s^{-1}, $\zeta = 5$, and $t = 10^{-3}$ s) and if we suppose $\bar{n} = 1$ (which is actually substantially higher than the number of thermal photons in the experiment proposed by Haroche and co-workers [19]), we find that the quantum-interference effects can be significantly suppressed. If we compare the photon-number distribution in the case of the zero-temperature heat bath [see Fig. 5(a)] and in the case of the nonzero-temperature heat bath [Fig. 5(b)] we see that the oscillations in the latter are much smaller. To measure the reduction of the amplitude of the oscillations more precisely, we introduce a new parameter (visibility) V, which we define as

$$V = \frac{p(n_{\max}) - p(n_{\max} - 1)}{p(n_{\max})} , \tag{58}$$

where $n_{\max}$ is the photon number for which $p(n)$ reaches

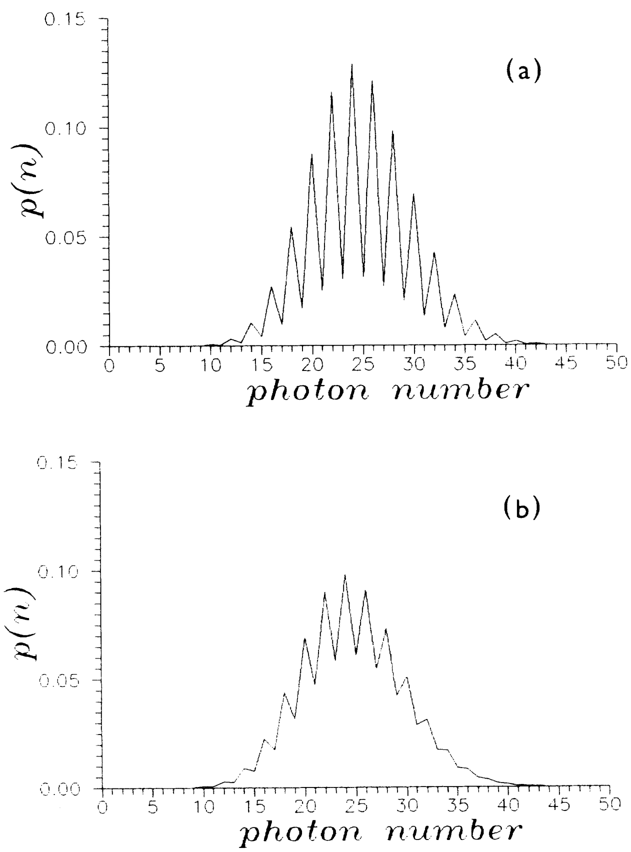

FIG. 5. The photon-number distribution $p(l,t)$ at $\gamma t = 0.01$ of the field mode initially prepared in the even CS ($\zeta = 5$). The field mode interacts with a zero-temperature heat bath (a) and with a nonzero-temperature heat bath with $\bar{n} = 1$ (b). We see that the oscillations of the PND are much more visible when the field mode interacts with the zero-temperature heat bath.

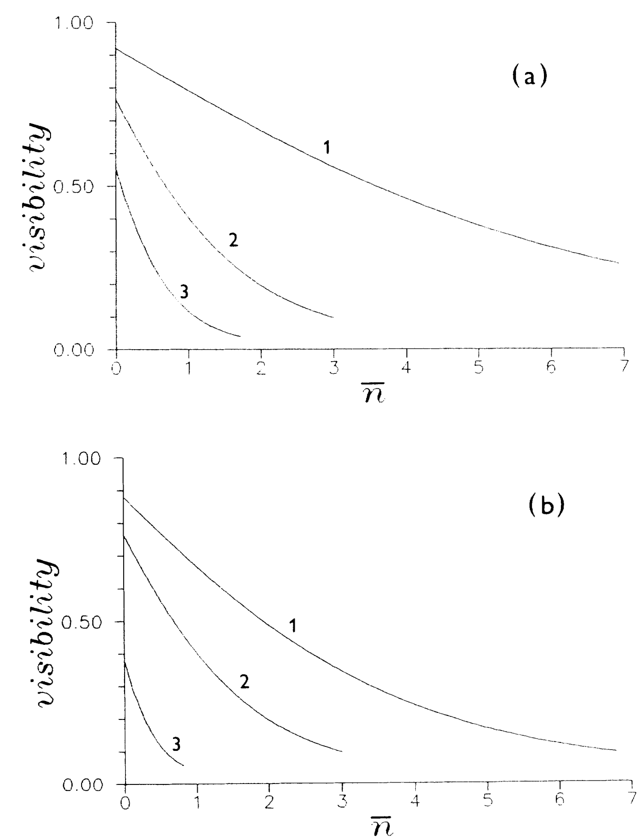

FIG. 6. The visibility of the field mode initially prepared in the even CS is plotted as a function of $\bar{n}$. In (a) we plot the visibility at $\gamma t = 0.01$ and $\zeta = 3$ (line 1); $\zeta = 5$ (line 2), and $\zeta = 7$ (line 3). We can easily see the decrease of the visibility with increasing of ζ. In (b) we plot the visibility for a fixed value of $\zeta = 5$ and various values of γt. In particular, line 1 corresponds to $\gamma t = 0.005$, line 2 is for $\gamma t = 0.01$, and line 3 is for $\gamma t = 0.03$. The suppression of the visibility with increasing of the interaction (measurement) time is obvious.

its maximum. Obviously in the case of the even CS at $t=0$ the visibility is equal to unity and during the time evolution it becomes less than unity. In Fig. 6(a) we plot the visibility as a function of $\bar{n}$ at $\gamma t=0.01$ for various values of ζ. From this picture it follows that the higher the $\bar{n}$ or ζ, the smaller the visibility is. In particular, for $\zeta=7$ the visibility for $\bar{n}=0$ is approximately five times larger than for $\bar{n}=1$. In Fig. 6(b) we plot the visibility as a function of $\bar{n}$ at the fixed value of ζ (equal to 5) and for various values of γt. It is seen that for $\gamma t=0.03$ and $\bar{n}=1$ the visibility is almost negligible, while for $\gamma t=0.005$ and $\bar{n}=1$ the visibility is approximately equal to 0.7.

From above we can conclude that in order to observe nonclassical effects such as the oscillations of the photon-number distribution, one has either to cool the microwave cavity well below 1 K or to operate the cavity at high frequencies, and simultaneously to perform the measurements in a shorter time than $\gamma t \simeq 0.01$.

ACKNOWLEDGMENTS

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APPENDIX

We use the method proposed by Peřinova and Lukš [37] (see also Ref. [31]) to solve the Fokker-Planck equation (12) and introduce a new function

$$R(\alpha,t)=\exp(|\alpha|^2)Q(\alpha,t)\,, \tag{A1}$$

and rewrite the Fokker-Planck equation (12) in the form

$$\frac{\partial R(\alpha,t)}{\partial t}=\left\{-\gamma(\bar{n}+\tfrac{1}{2})\left[\alpha\frac{\partial}{\partial\alpha}+\alpha^*\frac{\partial}{\partial\alpha^*}\right]+\gamma(\bar{n}+1)\frac{\partial^2}{\partial\alpha\partial\alpha^*}+\gamma\bar{n}(|\alpha|^2-1)\right\}R(\alpha,t)\,. \tag{A2}$$

Substituting

$$R(\alpha,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\alpha^m\alpha^{*n}\exp[|\alpha|^2g(t)]h_{mn}(t) \tag{A3}$$

into Eq. (A2) we find two differential equations for the functions $g(t)$ and $h_{mn}(t)$

$$\frac{dg(t)}{dt}=\gamma(\bar{n}+1)g^2(t)-\gamma(2\bar{n}+1)g(t)+\gamma\bar{n}\,, \tag{A4}$$

$$\frac{h_{mn}(t)}{dt}=[\Theta_{mn}+\Upsilon_{mn}g(t)]h_{mn}(t)+\Xi_{mn}h_{m+1,n+1}(t)\,, \tag{A5}$$

where

$$\Theta_{mn}=-\frac{\gamma}{2}(m+n)-\gamma\bar{n}(m+n+1)\,,$$

$$\Upsilon_{mn}=\gamma(\bar{n}+1)(m+n+1)\,,$$

$$\Xi_{mn}=\gamma(\bar{n}+1)(m+1)(n+1)\,.$$

The solution of Eq. (A4) is

$$g(t)=\frac{\bar{n}_t}{1+\bar{n}_t}\,, \tag{A6}$$

where $\bar{n}_t$ is defined in Eq. (18). Using the substitution

$$h_{mn}(t)=\exp\left[\Theta_{mn}t+\Upsilon_{mn}\int_0^t g(t')dt'\right]r_{mn}(t)\,, \tag{A7}$$

we simplify Eq. (A5) in the following way:

$$\frac{dr_{mn}(t)}{dt}=\Xi_{mn}I(t)r_{m+1,n+1}(t)\,, \tag{A8}$$

where

$$I(t)=\exp\left[-\gamma(1+2\bar{n})t+2\gamma(\bar{n}+1)\int_0^t g(t')dt'\right]\,, \tag{A9}$$

with

$$\int_0^t I(t')dt'=\frac{1}{\gamma\bar{n}}g(t)\,. \tag{A10}$$

The general solution of Eq. (A8) is

$$r_{mn}(t)=\sum_{l=0}^{\infty}\frac{1}{l!}\left[\frac{\bar{n}+1}{\bar{n}}g(t)\right]^l\frac{(m+l)!(n+l)!}{m!n!}\times h_{m+l,n+l}(0)\,, \tag{A11}$$

where $h_{m+l,n+l}(0)$ is given by the initial condition. With the use of Eqs. (A6), (A7), and (A11) we find for the function $R(\alpha,t)$ the expression

$$R(\alpha,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\alpha^m\alpha^{*n}\exp\left[\frac{\bar{n}_r|\alpha|^2}{1+\bar{n}_r}+\frac{\gamma}{2}t\right]\left[\frac{e^{-(\gamma/2)t}}{1+\bar{n}_r}\right]^{m+n+1}\sum_{l=0}^{\infty}\frac{1}{l!}\left[1-\frac{e^{-\gamma t}}{\bar{n}_r+1}\right]^l\frac{(m+l)!(n+l)!}{m!n!}h_{m+l,n+l}(0)\,. \tag{A12}$$

Substituting this result into Eq. (A1) and using the hypergeometric function $F(x,y,z;h)$, we write the Q function for the general initial state as

$$Q(\alpha,t)=\frac{e^{-|\alpha|^2}}{\bar{n}(1-e^{\gamma t})+1}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\alpha^m\alpha^{*n}\left[\frac{e^{-\gamma t/2}}{\bar{n}(1-e^{-\gamma t})+1}\right]^{m+n}\times\sum_{l=0}^{\infty}\left[1-\frac{e^{-\gamma t}}{\bar{n}(1-e^{-\gamma t})+1}\right]^l\frac{(m+l)!(n+l)!}{m!n!l!}h_{m+l,n+l}(0)\times F(-m,-n,l+1;4\bar{n}(\bar{n}+1)\sinh^2(\gamma/2)t)\ . \tag{A13}$$

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[1] V. Bužek and P. L. Knight, Opt. Commun. **81**, 331 (1991); A. Vidiella-Barranco, V. Bužek, P. L. Knight, and W. K. Lai, in *Quantum Measurement in Optics, NATO Advanced Study Institute Series B: Physics*, edited by P. Tombesi and D. F. Walls (Plenum, New York, 1992); V. Bužek, P. L. Knight, and A. Vidiella-Barranco, in *Squeezing and Uncertainty Relations*, edited by D. Han and Y. S. Kim (NASA, Washington, DC, 1991); V. Bužek, A. Vidiella-Barranco, and P. L. Knight, Phys. Rev. A **45**, 6570 (1992).

[2] W. Schleich, M. Pernigo, and Fam Le Kien, Phys. Rev. A **44**, 2174 (1991); W. Schleich, J. P. Dowling, R. J. Horowicz, and S. Varro, in *New Frontiers in Quantum Optics and Quantum Electrodynamics*, edited by A. Barut (Plenum, New York, 1990), p. 31; W. P. Schleich, D. F. Walls, and J. A. Wheeler, Phys. Rev. A **38**, 1177 (1988); W. P. Schleich and J. A. Wheeler, J. Opt. Soc. Am. B **4**, 1715 (1987); W. P. Schleich, R. J. Horowicz, and S. Varro, in *Quantum Optics V*, edited by J. D. Harvey and D. F. Walls (Springer-Verlag, Berlin, 1989), p. 134; J. P. Dowling, W. P. Schleich, and J. A. Wheeler, Ann. Phys. (Leipzig) **48**, 423 (1991).

[3] K. Wódkiewicz, P. L. Knight, S. J. Buckle, and S. M. Barnett, Phys. Rev. A **35**, 2567 (1987); J. Janszky and A. V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990); P. Adam and J. Janszky, Phys. Lett. A **149**, 67 (1990).

[4] For a review of light squeezing see, for instance, R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987); or K. Zaheer and M. S. Zubairy, in *Advances in Molecular and Optical Physics*, edited by D. Bates and B. Bederson (Academic, New York, 1990), Vol. 28, p. 143.

[5] V. Bužek, I. Jex, and T. Quang, J. Mod. Opt. **37**, 159 (1990). For the definition of higher-order squeezing see C. K. Hong and L. Mandel, Phys. Rev. Lett. **54**, 323 (1985).

[6] L. Mandel, Phys. Scr. **T12**, 34 (1986); M. C. Teich and B. E. A. Saleh, in *Progress in Optics XXVI*, edited by E. Wolf (Elsevier, Amsterdam, 1988), p. 2.; M. C. Teich and B. E. A. Saleh, Quantum Opt. **1**, 153 (1989).

[7] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986). For the generation of superposition states in a nonlinear medium modeled as an anharmonic oscillator, see also A. Mecozzi and P. Tombesi, Phys. Rev. Lett. **58**, 1055 (1987); P. Tombesi and A. Mecozzi, J. Opt. Soc. Am. B **4**, 1700 (1987); M. Wolinsky and H. J. Carmichael, Phys. Rev. Lett. **60**, 1836 (1988).

[8] P. D. Drummond and D. F. Walls, J. Phys. A **13**, 725 (1980); R. Tanas, in *Coherence and Quantum Optics V*, edited by L. Mandel and E. Wolf (Plenum, New York, 1984), p. 645; P. Tombesi and H. P. Yuen, in *ibid*., p. 751; M. Kitagawa and Y. Yamamoto, Phys. Rev. A **34**, 3974 (1986); M. Kitagawa, N. Imoto, and Y. Yamamoto, *ibid.* **35**, 5270 (1987); A. Miranowicz, R. Tanas, and S. Kielich, Quantum Opt. **2**, 253 (1990).

[9] U. M. Titulaer and R. J. Glauber, Phys. Rev. **145**, 1041 (1965).

[10] D. Stoler, Phys. Rev. D **4**, 2309 (1971); Z. Bialynicka-Birula, Phys. Rev. **173**, 1207 (1968).

[11] C. M. Savage and W. A. Cheng, Opt. Commun. **70**, 439 (1989).

[12] C. M. Savage, S. L. Braunstein, and D. F. Walls, Opt. Lett. **15**, 628 (1990).

[13] S. Song, C. M. Caves, and B. Yurke, Phys. Rev. A **41**, 5261 (1990); B. Yurke, W. Schleich, and D. F. Walls, *ibid.* **42**, 1703 (1990); A. La Porta, R. E. Slusher, and B. Yurke, Phys. Rev. Lett. **62**, 26 (1989); B. Yurke, J. Opt. Soc. Am. B **2**, 732 (1986); R. M. Shelby and M. D. Levenson, Opt. Commun. **64**, 553 (1987); M. Ueda, N. Imoto, and T. Ogawa, Phys. Rev. A **41**, 3891 (1990); T. Ogawa, M. Ueda, and N. Imoto, *ibid.* **43**, 6458 (1991); Phys. Rev. Lett. **66**, 1046 (1991).

[14] S. J. D. Phoenix and P. L. Knight, Ann. Phys. (N. Y.) **186**, 381 (1988); S. J. D. Phoenix and P. L. Knight, J. Opt. Soc. Am. B **7**, 116 (1990); Phys. Rev. Lett. **66**, 2833 (1991); Phys. Rev. A **44**, 6023 (1991).

[15] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963). For reviews, see L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975); S. Haroche and J. M. Raimond, in *Advances in Atomic and Molecular Physics*, edited by D. Bates and B. Bederson (Academic, New York, 1985), Vol. 20, p. 347; S. M. Barnett, P. Filipowicz, J. Javanainen, P. L. Knight, and P. Meystre, in *Frontiers in Quantum Optics*, edited by E. R. Pike and S. Sarkar (Hilger, Bristol, 1986), p. 485.

[16] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990); Phys. Rev. A **44**, 5913 (1991).

[17] V. Bužek, H. Moya-Cessa, P. L. Knight, and S. J. D. Phoenix, Phys. Rev. A **45**, 8190 (1992).

[18] J. J. Slosser, P. Meystre, and E. M. Wright, Opt. Lett. **15**, 233 (1990); J. J. Slosser and P. Meystre, Phys. Rev. A **41**, 3867 (1990); M. Wilkens and P. Meystre, *ibid.* **43**, 3832 (1991); P. Meystre, J. Slosser, and M. Wilkens, *ibid.* **43**, 4959 (1991); P. Meystre, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1992), Vol. 30.

[19] M. Brune, S. Haroche, V. Lefevre, J. M. Raimond, and N. Zagury, Phys. Rev. Lett. **65**, 976 (1990); S. Haroche, in *Fundamental Systems in Quantum Optics*, edited by J. Dalibard, J. M. Raimond, and J. Zinn-Justin (Elsevier, Amsterdam, 1991); M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45**, 5193 (1992); see also H. Paul, Quantum Opt. **3**, 179 (1991).

[20] N. F. Ramsey, *Molecular Beams* (Oxford University Press, New York, 1985).

[21] D. F. Walls and G. J. Milburn, Phys. Rev. A **31**, 2403 (1985); G. J. Milburn and D. F. Walls, *ibid.* **38**, 1087 (1988).

[22] G. J. Milburn and C. A. Holmes, Phys. Rev. Lett. **56**, 2237 (1986); G. J. Milburn, Phys. Rev. A **33**, 674 (1986).

[23] T. A. B. Kennedy and P. D. Drummond, Phys. Rev. A **38**, 1319 (1988).

[24] G. S. Agarwal and P. Adam, Phys. Rev. A **39**, 6259 (1989).

[25] A. Vourdas and R. H. Wiener, Phys. Rev. A **36**, 5866 (1987).

[26] S. J. D. Phoenix, Phys. Rev. A **41**, 5132 (1990).

[27] A. J. Leggett, S. Chavkravorky, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987); A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983); Physica A **121**, 587 (1983); Phys. Rev. A **31**, 1057 (1985); A. J. Leggett, Contemp. Phys. **25**, 583 (1984).

[28] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981); **26**, 1862 (1982); in *Frontiers of Nonequilibrium Statistical Physics*, edited by P. Meystre and M. O. Scully (Plenum, New York, 1986), p. 145; W. G. Unruh and W. H. Zurek, Phys. Rev. D **40**, 4056 (1989); W. H. Zurek, Phys. Today **44**, (10), 36 (1991).

[29] E. Joos and H. D. Zeh, Z. Phys. B **59**, 223 (1985).

[30] G. S. Agarwal, Phys. Rev. A **4**, 739 (1971).

[31] D. J. Daniel and G. J. Milburn, Phys. Rev. A **39**, 4628 (1989).

[32] C. W. Gardiner, *Handbook of Stochastic Methods* (Springer-Verlag, Berlin, 1983).

[33] K. E. Cahill and R. J. Glauber, Phys. Rev. **177**, 1857 (1969); **177**, 1882 (1969).

[34] R. J. Glauber, in *Quantum Optics,* edited by S. M. Kay and A. Maitland (Academic, London, 1970), p. 53; E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).

[35] E. Wigner, Phys. Rev. **40**, 749 (1932); Z. Phys. Chem. B **19**, 203 (1932); M. Hillery, R. F. O'Connell, M. O. Scully, and E. P. Wigner, Phys. Rep. **106**, 121 (1984).

[36] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[37] A. Lukš and V. Peřinová, Czech, J. Phys. B **37**, 1224 (1987); V. Peřinová and A. Lukš, Phys. Rev. A **41**, 414 (1990); V. Peřinová, A. Lukš, and M. Kárská, J. Mod. Opt. **37**, 1055 (1990).

[38] *Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1972).

[39] We should note here that the even CS is not only second-order squeezed, but it also exhibits fourth-order squeezing, for weak fields with $\zeta^2 < \frac{3}{2}$. Moreover, the degree of fourth-order squeezing is even larger than the degree of second-order squeezing (see Ref. [1]). We should underline here that in the case of the even CS there is a close relation between the presence of squeezing and the shape of the Wigner function. As seen from Fig. 1(b) the Wigner function itself is "squeezed" in phase space in the y direction corresponding to the reduction of fluctuations in the $\hat{a}_2$ quadrature. Moreover, the even CS has super-Poissonian photon statistics for any value of ζ^2, that is, the Mandel Q parameter $Q=4\zeta^2\exp(-2\zeta^2)/[1-\exp(-4\zeta^2)] > 0$ is positive for any value of ζ^2.