Frequency and polarization effects in stabilization

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Quantitative results are presented for stabilization of atoms against ionization by strong laser fields over a wide range of frequencies for circular and linear polarization. With circular polarization, stabilization occurs for all frequencies. At high frequency ($\omega > 1$ a.u.), the stabilization intensity is predicted by the occurrence of photon thresholds. This mechanism gives excellent numerical agreement both with present calculations and the results of Pont and Gavrila [Phys. Rev. Lett. **65**, 2362 (1990)]. A different mechanism operates for $\omega < 1$ a.u. Linear polarization differs strongly from circular polarization in many respects.

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I. INTRODUCTION

Investigations using numerical methods [1,2], techniques based on the Kramers-Henneberger transformation [3], Floquet techniques [4], or quantum interference effects [5] have explored the stabilization phenomenon, where beyond a certain field intensity, an atom becomes increasingly resistant to photoionization. The emphasis has been on high-frequency fields, $\omega > 1$. (Atomic units are used throughout.) A classical argument [6] suggests that high frequencies are essential.

The primary results of the present paper are that stabilization with circular polarization occurs at all frequencies, a very successful predictive explanation is found for the stabilization point for $\omega \geq 1$, and important differences are found between linear and circular polarization. This paper represents the first application of Keldysh-like methods to the high-frequency domain.

Stabilization occurs at such high intensity that most techniques encounter significant computational obstacles. The method used here is designed to circumvent most such difficulties. It also has no frequency limitations. The method used is the strong-field approximation (SFA) [7–10] applied to arbitrary frequencies. A brief review of the SFA is given to show the power of the technique and how it allows one to evade the most onerous problems of strong-field calculations.

II. CALCULATIONAL METHOD

The probability amplitude that a state Ψ_i, which has evolved under the combined effects of the atom and laser from an initial undisturbed bound state Φ_i into some other laser-field-free final state Φ_f (corresponding to measurement in a field-free region of the final products of the ionization), is given by the overlap $S_{fi} = \lim_{t\to\infty}(\Phi_f, \Psi_i)$. This leads to the exact expression

$$(S-1)_{fi} = -i\int dt(\Phi_f, H_I\Psi_i) , \tag{1}$$

where H_I is the interaction Hamiltonian of the laser field. Equation (1) poses the severe problem that Ψ_i must fully include both atomic and laser influences. For most methods, the basic problem of strong-field physics is to find an accurate nonperturbative way to describe Ψ_i. An alternative approach—the key to the SFA—is to replace Eq. (1) by the exactly equivalent time-reversed S matrix

$$(S-1)_{fi} = -i\int dt(\Psi_f, H_I\Phi_i) . \tag{2}$$

Now the bulk of the atomic information is in Φ_i, which is accurately known. One need not, and should not, consider how this state is altered by the strong field. The burden of describing the electron in combined laser and atomic fields is shifted to the final state, where an ionized electron has its behavior dominated by the laser field for sufficiently strong fields. When the potential energy of the detached electron in the laser field dominates the atomic binding energy, the electron state can be effectively approximated by a Volkov state. The situation may be likened to the elementary problem of the first-order perturbation calculation of ionization of an atom by a single, very energetic photon. If the ionized electron is energetic as compared to the binding energy, then there is no need to consider the Coulomb influence on the final state—a free-particle assumption for the final electron works well. In the strong-field case, if the laser field dominates the Coulomb field in the final state, then most ionized electrons are very energetic because the above-threshold ionization (ATI) phenomenon is well developed, and the Coulomb influence may be neglected in the final state.

The time-reversed S matrix has no inherent frequency limitations. Applicability of this formalism to a variety of atomic states has been verified by matching [11] high-field photoionization experiments [12] very successfully with no adjustable parameters. This can be regarded as a check of the SFA at low frequencies. [Earlier attempts to match experimental results against the Keldysh-Faisal-Reiss (KFR) [7,13,14] method did not use appropriate atomic information and/or did not refer to experiments at large z_1.] A check of the SFA at high frequencies is provided by the present work, where excellent qualitative and quantitative agreement is obtained with the high-frequency stabilization results of Ref. [3].

In all cases considered here, stabilization is found

within the intensity domain $z_1 > 1$ and $z_f < 1$, where $z_1 \equiv 2U_p/E_B$ (U_p is the ponderomotive potential, E_B is the binding energy), and $z_f \equiv 2U_p/mc^2$. The condition $z_1 > 1$ means that the ionized electron behavior is dominated by the laser field. It is the only condition required for applicability of the SFA. (Note that $z_1 = 1/\gamma^2$, where γ is the Keldysh parameter.) The condition $z_f < 1$ means that stabilization occurs in the nonrelativistic domain, in which case the SFA is analytically the same as what has been called the KFR [7,13,14] method. The designation SFA is preferred here, because it bespeaks extension to the relativistic domain [9,10], it is not limited to the hydrogen atom nor to tunneling (as is the Keldysh method in its usual application), and its formal basis employs the time-reversed S matrix, which carries implications for applicability to all frequencies (not present in the direct-time S matrix, high-frequency-limited basis of Ref. [14]).

The SFA is an S-matrix method that concentrates on asymptotic conditions, so it provides physical interpretations rather different from other techniques. Despite the analytical simplicity of the SFA, a great deal of physics is subsumed in the S matrix. A consequence is that the SFA is computer intensive when large numbers of photons are involved in the ionization. The tunneling approximation as employed in the Keldysh method [13] affords great simplification, but is not used here for reasons given below.

III. STABILIZATION WITH CIRCULAR POLARIZATION

The KFR result for the photoionization transition rate for $1S$ hydrogen is

$$W = \frac{4}{\pi}\left[\frac{E_B}{\omega}\right]^{3/2} \int d\Omega \sum_{n_0}^{\infty} \frac{(n - z - E_B/\omega)^{1/2}}{(n-z)^2} (J_n)^2 , \quad (3)$$

where $z \equiv U_p/\omega$; $n_0 = \{z + E_B/\omega\}$, and the curly brackets indicate the smallest integer containing the number inside; for circular polarization J_n is the ordinary Bessel function $J_n(z^{1/2}\beta)$, where $\beta = 2(n - z - E_B/\omega)^{1/2}\sin\theta$ (θ is the angle from the propagation direction of the laser); and for linear polarization J_n is the generalized Bessel function [7] $J_n(z^{1/2}\alpha, -z/2)$, where $\alpha = 8^{1/2}(n - z - E_B/\omega)^{1/2}\cos\theta$ (θ is the angle from the polarization direction of the laser).

Figure 1 shows the outcome of Eq. (3) in a circularly polarized laser field for frequencies from $\omega = 1/32$ to 8. Stabilization occurs for the entire frequency range. At the higher frequencies, $\omega = 2$ and 8, good numerical correspondence is found with the high-frequency method of Pont and Gavrila [3] at low intensity, at high intensity, and for the intensity at which stabilization occurs. (See Fig. 1 of Ref. [3].) The primary difference is that the present results show structure in the rate just beyond the stabilization point, not found in Ref. [3].

Important properties of stabilization with circular polarization are exhibited in Fig. 2. Figure 2 shows the intensity at which stabilization occurs plotted against frequency, with the intensity measured by the z_1 parameter. The stabilization intensity itself is always at $z_1 > 1$, just

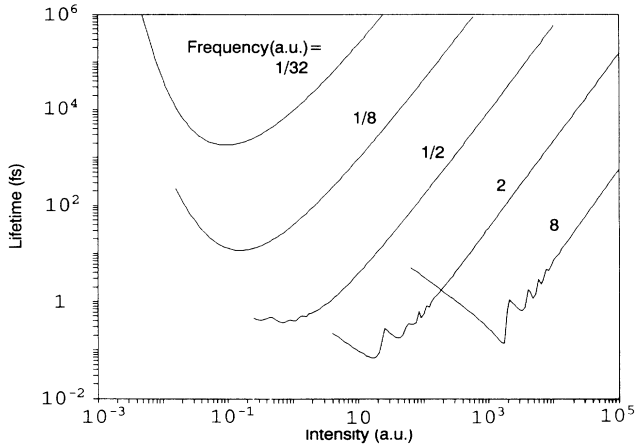

FIG. 1. Lifetime (in fs) as a function of intensity (a.u.) for the photoionization of ground-state hydrogen by a circularly polarized field. The labels on the curves give the frequency in atomic units.

barely satisfied at $\omega \approx 1$, and amply satisfied at other frequencies. This assures applicability of the SFA. The frequency $\omega \approx 1$ represents a division between sharply different types of behavior. The essential element appears to be that photoionization for $\omega > 1$ is always a single-photon process at low intensity, and does not require two or more photons until relatively high intensities are reached. (Note that $\omega = 1$ means that the photon energy is twice the binding energy of ground-state hydrogen.)

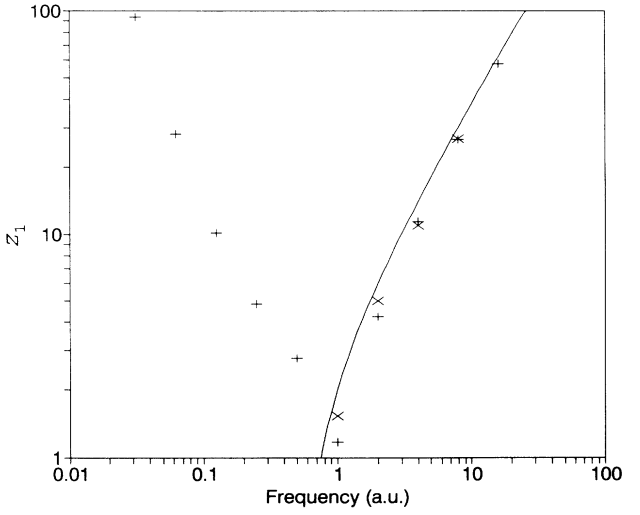

FIG. 2. Location of the stabilization point, i.e., the intensity for which maximum rate or minimum lifetime occurs, expressed in terms of the intensity parameter z_1 plotted against the frequency. (The parameter z_1 is defined as twice the ponderomotive potential divided by the binding energy.) Results are for ionization of ground-state hydrogen by circularly polarized radiation, as in Fig. 1, with additional frequencies. Calculations by the SFA method are shown by the plus signs, results of Pont and Gavrila [3] are shown by the crosses, and the smooth curve is the approximation of Eq. (4).

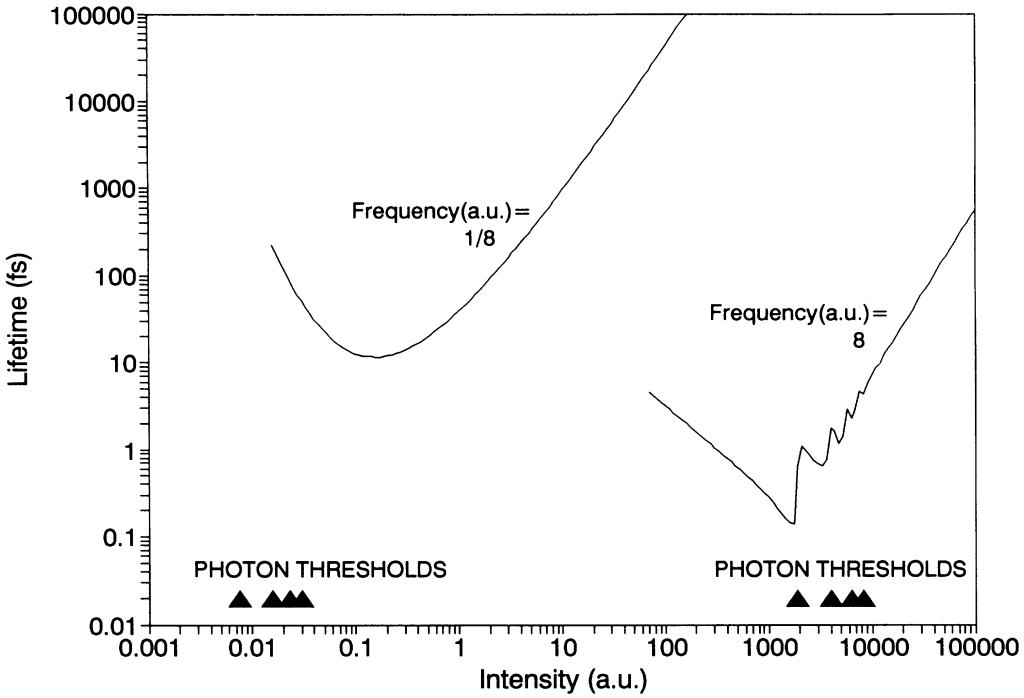

FIG. 3. Location of photon thresholds as they relate to a high-frequency ($\omega=8$) and a low-frequency ($\omega=1/8$) case from Fig. 1. The apexes of the triangles point to the thresholds. The thresholds refer to the intensity at which the minimum required number of photons indexes upward by unity due to an increase in the ponderomotive potential of the detached electron.

Above $\omega\approx 1$, stabilization occurs at about the same intensity as suppression of the lowest-order process due to the necessity for supplying "jitter" energy to the free electron (i.e., the interaction energy of the detached electron with the laser field). This threshold is at

$$z_1=2(\omega/E_B-1) \tag{4}$$

in general, or $z_1=2(2\omega-1)$ for ground-state hydrogen. Included in Fig. 2 are results from Pont and Gavrila [3]. Equation (4) does very well for $\omega\geq 1$, both for the results of Pont and Gavrila as well as the present SFA calculations, as shown in Fig. 2. For this high-frequency domain, the two sets of computations are nearly identical.

Figure 3 shows one high-frequency and one low-frequency example from Fig. 1. Indicated near the bottom of the figure are the thresholds for indexing upward of the minimum photon numbers for the process. For high frequency, this indexing from 1 to 2 to 3 to . . . is accompanied each time by sharp structure in the lifetime curve, of which the first feature lies close to the minimum lifetime. For low frequencies, the minimum low-intensity order is greater than 1, and no observable structure accompanies successive photon thresholds. This behavior reinforces the conclusion from Fig. 2 that $\omega>1$ and $\omega<1$ are fundamentally different for stabilization. Equation (4) has no predictive value for stabilization for $\omega<1$.

Insight into this major change in stabilization behavior at $\omega\approx 1$ comes from the following. The number of photons of a laser beam available in a volume the size of the undisturbed atom is $n=4\pi\rho/3$, where ρ is photon density (the atomic radius is taken to be the Bohr radius). The minimum number of photons required for ionization is $n_0=\{z+E_B/\omega\}>z$. The condition (photons available) > (photons required) gives $4\pi\rho/3>z$. But z can be written in terms of photon density as $z=\rho/\omega^2$, so the "photon availability" condition becomes $\omega>(3/4\pi)^{1/2}\approx 0.5$. This is of the order of magnitude $\omega=1$, so that when ω is smaller than this value, the photons required for ionization of the atom are not to be found in the immediate atomic volume, but must be obtained from more distant parts of the laser beam. This is one way to view the rising z_1 requirement as ω diminishes, so evident in Fig. 2.

IV. STABILIZATION WITH LINEAR POLARIZATION

Linear- and circular-polarization lifetimes are compared in Fig. 4(a) for $\omega=1$ and in Fig. 4(b) for $\omega=8$. The circular results correspond to Fig. 1. Both of these frequencies may be viewed as high, since they represent single-photon processes at low intensity. Circular and linear lifetimes are coincident at low intensity (a well-known situation), are similar near the stabilization point, and become contrasting at higher intensities. The most prominent difference is the sharp oscillation of the linear polarization case, especially for $\omega=8$. The first few oscillations after the stabilization point (maximum rate) correspond to photon thresholds, where the lowest photon order becomes kinematically forbidden because of the increasing demands of the ponderomotive potential. This is no longer true for subsequent oscillations. Averaged over oscillations, both circular and linear lifetimes show an asymptotic linear increase on this log-log plot, but

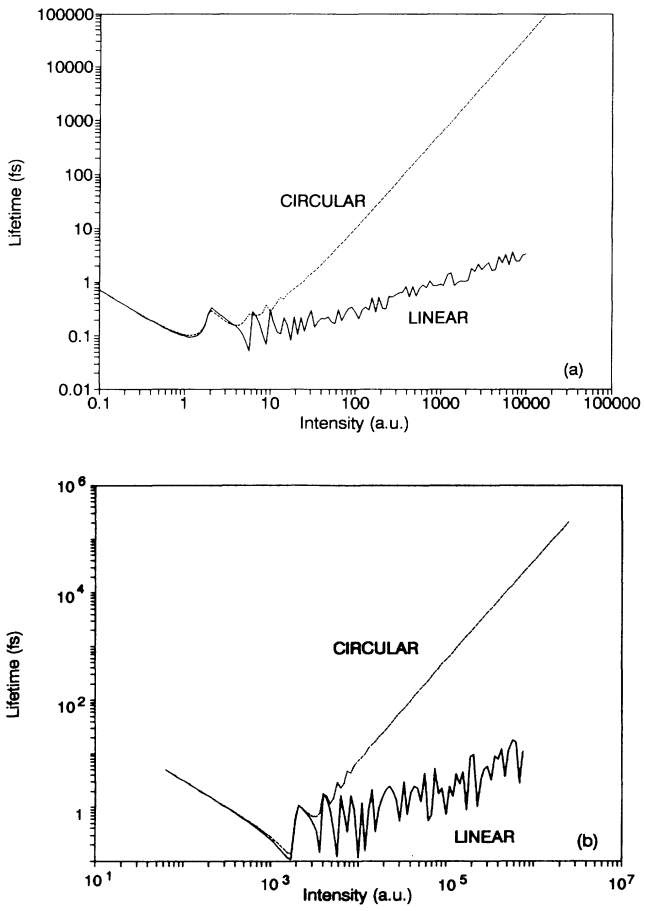

FIG. 4. Comparison of ionization lifetimes due to circularly and linearly polarized fields. Circular polarization results are from Fig. 1. (a) Frequency $=1$, (b) frequency $=8$.

with a much stronger increase for circular than for linear polarization. The frequency-independent asymptotic slope is about 1.8 for circular polarization and 0.5 for linear polarization.

The absence of certain linear-polarization results from this paper is instructive. Figure 4 does not carry the linear-polarization results as far as the circular polarization case, which goes to an intensity corresponding to $z_f = 1$, where relativistic effects occur [9,10]. The linear calculation, beyond the terminus shown, exceeds the capabilities of the modern mainframe used. For $\omega \geq 1$, no tunneling or low-frequency approximation is applicable. However, linear polarization results are omitted for the $\omega = \frac{1}{32}$ and $\frac{1}{8}$ cases given in Fig. 1 for circular polarization. The reason is that the pretunneling asymptotic generalized Bessel function results of Ref. [7], while useful and accurate from $z_1 \geq 1$ to beyond the stabilization intensity, begin to fail at higher intensities. The essential point is that the high-intensity failure of these asymptotic forms means that the tunneling approximation fails as well, since the asymptotic generalized functions give rise to tunneling forms as a limiting case [7].

V. REMARKS

Experiments to observe the stabilization phenomenon are difficult to design. One problem is the depletion of un-ionized atoms in the target gas as they are subjected to the large ionization rates in the prestabilization part of a laser pulse. Another problem is that when the stabilization regime is reached in the core of a laser pulse, it is surrounded by a very large volume of the pulse characterized by prestabilization intensities, where the ionization yield can swamp the output from the poststabilization core. Subtle solutions to this problem may be found, but the present results suggest a straightforward possible approach. The problem of premature depletion can be addressed by using an environment with low overall rates. This means low frequency, as Fig. 1 shows. Dominance of the poststabilization-core region of a laser pulse by the more voluminous prestabilization outer region can be relieved by using circular polarization, which has more prominent stabilization behavior than linear polarization, as shown by Fig. 4. Further investigation using realistic laser pulses is required.

ACKNOWLEDGMENTS

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