Theory of free-wave acceleration

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(Received 31 March 1992)

Laser acceleration of electrons is considered. A general formulation of the problem is developed that would allow the recently proposed accelerator based on nonlinear amplification of inverse bremsstrahlung electron acceleration (NAIBEA) to function with any state of polarization and shape of the laser pulse, and any kind of applied field. Analytical estimates as well as numerical analysis for the case of an applied magnetic field are made. It is found that electrons, injected with few tens of mega-electron-volts, can be accelerated to giga-electron-volt energies using a NAIBEA accelerator of a length of few meters.

PACS number(s): 41.75.Fr

Laser acceleration of particles has been the focus of significant attention in the past few years. Recently, two of us [1] have proposed a method of linearly accelerating electrons with the aid of a powerful laser coupled to an optimally determined static electric field that is made to change sign at appropriate places along the accelerator. With initial electron energy of 35 MeV, a laser power of 10^{15} W/cm^2 for a wavelength of 10 μm, and an applied field intensity of $\sim 10^{-5}$, that of the laser intensity, we predicted a final electron energy of, say, $\sim$400 MeV if three inversions of the static field are made along an accelerator tube of a length of about 3 m. The advantages of such a machine obviously call for a more general discussion concerning how optimal the setup is and the dependence on the laser parameters as well as the nature of the applied field.

In the present paper we develop a very general formulation of the problem that allows both semianalytical and numerical answers to the above questions. Before entering into detail, we first give some general remarks concerning the acceleration principle, which we called nonlinear amplification of inverse bremsstrahlung electron acceleration (NAIBEA) in Ref. [1]. First, the new machine is meant to be a very high-energy booster; the initial particle energy should already be highly relativistic. The reason is very simple. What sets the macroscopic scale of the machine is basically the Doppler shift of the laser wavelength as seen in the particle rest frame. Calling the laser wavelength λ_0 and the velocity of the particle v_0, then the shifted wavelength obtained with the appropriate Lorentz transformation is

$$\lambda=\lambda_0\left[\frac{1+\beta_0}{1-\beta_0}\right]^{1/2}, \quad \beta_0=v_0/c \ , \tag{1a}$$

whereas λ_0 is microscopic (measured in micrometers), λ becomes macroscopic (measured in centimeters) if β_0 is close to unity. The distance Δz traveled by the electron in the laboratory during a one-cycle encounter with the laser is then

$$\Delta z=v_0(\lambda/c)\gamma=\lambda_0\frac{\beta_0}{1-\beta_0} \ . \tag{1b}$$

Clearly Δz is one of the quantities that would decide the size of the accelerator.

Second, the rate of change of the particle energy is determined by $\mathbf{v}\cdot\mathbf{E}$ and thus to keep the particle gaining energy from the laser, $\mathbf{v}\cdot\mathbf{E}$ should be made always positive. This calls for another degree of freedom to couple to the system. A possible simple form for this extra degree of freedom was found in Ref. [1] to be an array of applied static electric field with interchanging signs placed at appropriate positions along the accelerator. Clearly a more general form for this degree of freedom can be found that would make the machine more efficient. A combination of electric and magnetic fields, not necessarily static, could be one of the choices. To better determine the nature of the applied field, one needs an equation, and we now turn our attention to its derivation.

To simplify the presentation, we consider the units of mass in mc^2, the vector potential A in mc/e, the distance x in $1/k$, where k is the wave number, and time t in $1/\omega$, ω being the frequency. We take the direction of wave propagation and particle acceleration to be along z. The Hamiltonian of the system, particle plus laser plus applied field, is (note that we are working in the temporal gauge)

$$H=[1+P_z^2+(P_x-A_x)^2+(P_y-A_y)^2]^{1/2}\equiv\gamma \tag{2}$$

in our units. In Eq. (2) $\mathbf{A}=\mathbf{A}^{(0)}(t-z)+\mathbf{A}_{\rm app}(t,z)$. Then $\mathbf{E}=-\dot{\mathbf{A}}_{\rm app}$ and $\mathbf{B}_{\rm app}=\nabla\times\mathbf{A}_{\rm app}$ are the external applied fields that add to those of the traveling laser pulse $\mathbf{A}^{(0)}(t-z)$. Hamilton's equations follow from Eq. (1), i.e.,

$$\dot{P}_z=-\frac{\partial\gamma}{\partial z},\quad \dot{P}_x=0\ ,\quad \dot{P}_y=0\ ,$$
$$\dot{\gamma}=\frac{\partial\gamma}{\partial t}=-\frac{\mathbf{P}-\mathbf{A}}{\gamma}\cdot\left[\frac{\partial\mathbf{A}^{(0)}}{\partial t}+\frac{\partial\mathbf{A}_{\rm app}}{\partial t}\right]\ . \tag{3}$$

Further reduction of $\dot{P}_z$ gives

$$\dot{P}_z=\frac{\mathbf{P}-\mathbf{A}}{\gamma}\cdot\left[\frac{\partial\mathbf{A}^{(0)}}{\partial z}+\frac{\partial\mathbf{A}_{\rm app}}{\partial z}\right]\ . \tag{4}$$

Combining $\dot{\gamma}$ and $\dot{P}_z$ we obtain

$$\dot{\gamma}-\dot{P}_z=\frac{1}{\gamma}[(\mathbf{A}\times\mathbf{B}_{\rm app})_z-\mathbf{A}\cdot\mathbf{E}_{\rm app}]\ , \tag{5}$$

which, when integrated, yields

$$\gamma=P_z+u\ ,\quad u=u_0+\int_{-\infty}^{t}\frac{[(\mathbf{A}\times\mathbf{B}_{\rm app})_z-\mathbf{A}\cdot\mathbf{E}_{\rm app}]}{\gamma}dt'\ ,$$
$$u_0=\frac{1}{\gamma_0}(1+\beta_0)^{-1}=\left[\frac{1-\beta_0}{1+\beta_0}\right]^{1/2}\ . \tag{6}$$

At this point, we remark, as was done in Ref. [1], that since $\mathbf{A}^{(0)}$ will be the dominant field, it is more convenient to use the phase $\varphi-t-z$ as an integration variable. Then since $\dot{\varphi}=1-\dot{z}$ and $\dot{z}=\partial\gamma/\partial P_z=P_z/\gamma$ and from (6), $1-\dot{z}=u/\gamma$, we have $d\varphi/u=dt/\gamma$. Further, since $\dot{u}=(1-\dot{z})du/d\varphi=(u/\gamma)du/d\varphi$, we obtain from Eq. (6) the following:

$$u^2(\varphi)=u_0^2+2\int_{-\infty}^{\varphi}[(\mathbf{A}\times\mathbf{B}_{\rm app})_z-\mathbf{A}\cdot\mathbf{E}_{\rm app}]d\varphi'\ , \tag{7}$$

and from $\gamma^2=(P_z+u)^2=1+P_z^2+\mathbf{A}^2$ [Eq. (2)],

$$\gamma(\varphi)=\frac{1+\mathbf{A}^2+u^2}{2u}\ ,\quad P_z=u\frac{dz}{d\varphi}=\frac{1+\mathbf{A}^2-u^2}{2u}\ , \tag{8}$$

$$z(\varphi)=\int_{-\infty}^{\varphi}\left[\frac{1+\mathbf{A}^2-u^2}{2u^2}\right]d\varphi'=\int_{-\infty}^{\varphi}\left[\frac{\gamma(\varphi')}{u(\varphi')}-1\right]d\varphi'\ . \tag{9}$$

The x and y coordinates of the particle are determined from the equations $\dot{P}_x=0$ and $\dot{P}_y=0$, which yield for the canonical momenta $P_x=0$ and $P_y=0$, and thus the physical momenta are given by

$$p_x=\gamma\dot{x}=-A_x\ ,\quad p_y=\gamma\dot{y}=-A_y \tag{10}$$

or

$$x(\varphi)=-\int_{-\infty}^{\varphi}\frac{A_x(\varphi')d\varphi'}{u(\varphi')}\ , \tag{11}$$

$$y(\varphi)=-\int_{-\infty}^{\varphi}\frac{A_y(\varphi')d\varphi'}{u(\varphi')}\ . \tag{12}$$

The set of equations (7–11) constitutes the free-wave accelerator (FWA) equations. These equations are important generalizations of the NAIBEA equations of Ref. [1] in that (i) the laser could be in any state of polarization and have any pulse shape, and (ii) the applied electromagnetic (em) field $E_{\rm app}$ and $B_{\rm app}$ is quite general. Note that since $\mathbf{A}$ is defined to within an arbitrary constant, we have here full freedom in giving the electron a nonzero initial value of p_x and/or p_y [Eq. (10)]. The trajectory parameter Q, which was introduced in Ref. [1], is here generalized to be a vector in the x-y plane and is defined by the equation

$$\frac{d\mathbf{Q}}{d\varphi}=-\mathbf{A}\ . \tag{13}$$

It is a simple matter to show that the second derivative of $\mathbf{Q}$ can be written as

$$\frac{d^2\mathbf{Q}}{d\varphi^2}=\frac{-d\,\mathbf{A}^{(0)}(\varphi)}{d\varphi}+\frac{\gamma}{u}\mathbf{E}_{\rm app}+\left[\frac{\gamma}{u}-1\right]\tilde{\mathbf{B}}_{\rm app}\ ,$$
$$\tilde{\mathbf{B}}_{\rm app}\equiv\hat{\mathbf{k}}\times\mathbf{B}_{\rm app} \tag{14}$$

and the rate change of γ with respect to φ

$$\frac{d\gamma}{d\varphi}=(p\cdot\mathbf{E})/u=-\frac{1}{u}\mathbf{A}\cdot\mathbf{E}=\frac{1}{u}\frac{d\mathbf{Q}}{d\varphi}\cdot\mathbf{E}(\varphi)\ . \tag{15}$$

Note that Eq. (14) is a nonlinear second-order differential equation for $\mathbf{Q}$, since u and γ depend on $d\mathbf{Q}/d\varphi$ and $d\mathbf{Q}^2/d\varphi^2$. The solution of this equation completely determines the trajectory of the particle.

For γ to increase with φ, $(d\mathbf{Q}/d\varphi)\cdot\mathbf{E}(\varphi)$ must always be positive [notice that $u(\varphi)>0$ instead of a maximum] guarantees that γ keeps increasing for $\varphi>\varphi_j$. If $B_{\rm app}$ is taken to be zero as Ref. [1], then $\mathbf{p}\cdot\mathbf{E}=\mathbf{p}\cdot\mathbf{E}^{(0)}+\mathbf{p}\cdot\mathbf{E}_{\rm app}$. If $\mathbf{E}$ is taken in the y direction, then we have $p_yE_y^{(o)}+p_yE_{\rm app}>0$. Since $E_y^{(0)}$ is the dominant field, except when passing through zero, the above condition says that we use $E_{\rm app}$ to "fine tune" the sign of p_y so that it is always the same as that of $E_y^{(0)}$. The fundamental role of the applied field is to guarantee the validity of Eq. (16). This can happen even if $E_{\rm app}/E^{(0)}$ or $B_{\rm app}/B^{(0)}$ or both are much smaller than unity. The injection of electrons with a nonzero $p_x(0)$ or $p_y(0)$, albeit very small, is very important to set the machine to work. This is so since in the x-y plane the motion of the electron must be oscillatory in accordance with Eq. (16).

We turn now to specific choices of the accelerator. We consider a linearly polarized pulse with $\mathbf{A}$ taken to be along the y direction. We first consider a constant applied electric field $E_{\rm app}$ then

$$\mathbf{A}=[A_y^{(0)}(\varphi)-E_{\rm app}t-p_y(0)]\hat{\mathbf{j}}\ . \tag{16}$$

With the $\mathbf{A}$ above used in the FWA equations we recover the NAIBEA equation of Ref. [1]. Inversions of $E_{\rm app}$ are made at appropriate values of z to assure the validity of Eq. (16), namely $p_y(n\pi)=0$. This means that the applied field is inverted at φ_j's such that $[dp_y(\varphi)/d\varphi]|_{\varphi_j=n\pi/2}=0>$.

We now replace $E_{\rm app}$ by a constant magnetic field along x. Then

$$\mathbf{A}=[A_y^{(0)}(\varphi)-B_{\rm app}z-p_y(0)]\hat{\mathbf{j}}\ . \tag{17}$$

The resulting NAIBEA equations are almost identical to those of Ref. [1] except for a change in sign of the second term in Eq. (8) of that reference (with $E_{\rm app}$ replaced by $B_{\rm app}$). Further, Eq. (7) reduces to

$$u^2=u_0^2-2B_{\rm app}Q\ . \tag{18}$$

The y component of the momentum is given by [Eq. (10)]

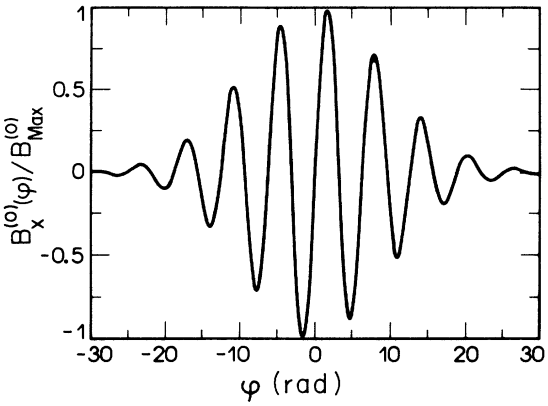

FIG. 1. The laser pulse vs φ used in our calculation of the width $\Delta=3\pi$.

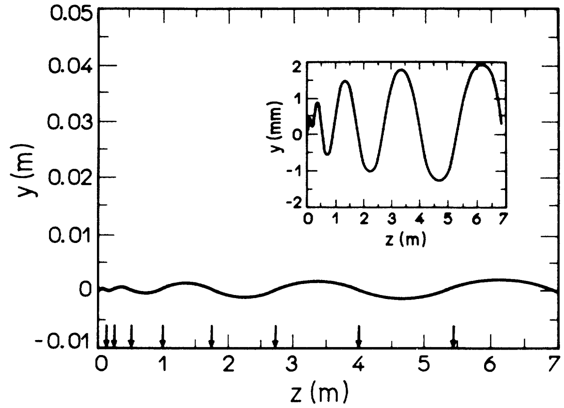

FIG. 3. The trajectory of the particle in the y-z plane. Inset: scale of y in mm. The arrows indicate the positions where the applied magnetic field is reversed.

$$p_y(\varphi)=p_y(0)-A_y^{(0)}(\varphi)+B_{\text{app}}z \ . \tag{19}$$

The continuous acceleration of the particle results if B_{app} is chosen so that $p_y(n\pi)=0$. This requires changing the sign of B_{app} at appropriate places $[\varphi\simeq(n/2)\pi]$.

We consider the following numerical example. The initial value of $\gamma,\gamma_0=70$, $P=\frac{3}{2}\nu_0^2 10^{+15}$ W/cm^2 for $\lambda_0=10^{-3}$ cm. The parameter $\nu_0=1$ refers to the maximum value of electric field of the pulse in our units. We also take nine cycles within the pulse. The shape of the pulse is taken to be a Gaussian, $\overline{A}_0^2=\exp(-\varphi^2/\Delta^2)$, Fig. 1, with $\Delta=3\pi$. The applied field intensity is taken to be 2.34 T, which corresponds to $\sim 5\times 10^{-5}$, that of the laser. The electrons are injected at an angle of 0.6° with respect to the z axis $[p_y(0)\approx(o.6/180)\pi\gamma_0]$. We consider as an example nine changes in the sign of the modulated applied magnetic field. Since we have not self-consistently chosen the position of the field reversal, we have not actually optimized the decrease of u in Eq. (6). In Fig. 2 we show the change of γ versus z obtained by solving Eqs. (14), (8), and (9). The gain in energy is a factor of 35 over a distance (accelerator length) of 7 m. The accelerator length could be made smaller if full optimization is accomplished. The corresponding trajectory of the particle confined in the (z,y) plane is shown in Fig. 3.

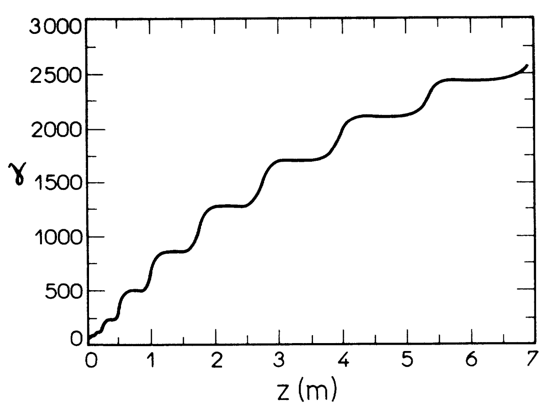

FIG. 2. The energy of the particle in units of mc^2 vs the traveled distance z.

Note that

$$y=-\int_{-\infty}^{\varphi}\frac{A_y^{(0)}(\varphi')d\varphi'}{u(\varphi')} \tag{20}$$

is contained within the laser transversal extension. As we see from Fig. 2, at the end of the acceleration the dispersion in y is ~ 2 mm, which could be within available laser transversal dimensions [2].

Before ending we comment briefly on the case of the circularly polarized laser pulse,

$$\mathbf{A}^{(0)}=\overline{A}(\varphi)[\mathbf{i}\cos\varphi+\hat{\mathbf{j}}\sin\varphi] \ ,$$
$$\overline{A}(\varphi)=\nu_0\exp(-\varphi^2/2\Delta^2) \ . \tag{21}$$

Clearly $\mathbf{A}^2$ is just $\mathbf{A}^2(\varphi)$, which does not oscillate. Then we get approximately for the accelerator length

$$\Delta z=\frac{1-u_0^2}{2u_0^2}T+\frac{\nu_0^2}{2u_0^2}T\approx(2_0^2)(1+\nu_0^2)T \ , \tag{22}$$

where T is $\Delta\varphi$ of the pulse defined as $\int_{-\infty}^{\infty}A^2 d\varphi'=\nu_0^2 T\equiv 2\pi\nu_0^2 n$, where n is the number of oscillations in the pulse. Thus Δz is determined by three large factors (γ_0^2,ν_0^2,n) which can easily make it macroscopic even though it is in units of $\lambda/2\pi$. This generalizes the simple Doppler-shift argument concerning the size of the accelerator.

In conclusion, we have developed here a general formalism for free-wave acceleration. A combined powerful laser pulse plus an optimally determined em field, which could be time dependent, was found to be able to accelerate electrons to very high energies with relatively small accelerator dimensions. Radiation damping has not been included in our theory and it may modify some of our conclusions, even in setting a limit to the final energy of the particle [3,4]. Recently we have also learned of related work done in Refs. [5] and [6].

Finally we remark that the acceleration principle discussed above can also be applied to accelerate protons to tesla-electron-volt energies with the appropriate laser and applied fields. In fact with $\gamma_0=500$ (initial proton energy

0.5 TeV), laser power of $P \sim 6\times10^{20}$ W/cm^2 for $\lambda_0=50$ μm and ten alternating sign magnets of 10 T each placed at optimally determined positions along a length of 3 km, one calculates a final proton energy of 20 TeV.

M. S. H. and M. P. P. acknowledge partial support by the CNPq. A.K.K. acknowledges partial support by funds provided by the U.S. Department of Energy under Contract No. DE-ACO2-7-ERO3069.

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