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Field-theoretical view of the angular correlation of photons

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We give a field-theoretical derivation of the explicit probability distribution of photon emission by identical N noninteracting atoms (sources), which has been applied to study the tendency of photons to form collimated beams. More precisely, given that each source emits one photon with some given energy, due to the same atomic transition, we derive the conditional probability that the N photons have momenta in arbitrary directions. Each source is assumed separately to emit photons of momenta with uniform distribution for their directions, and *no* assumption is made on the sources being pointlike.

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Much interest has been given in the literature (c.f. Refs. [1,2]) to the tendency of photons to form collimated beams. This result is attributed to the Bose character of photons, giving them the tendency to travel in the same direction. In a recent interesting paper [1], a measure of this latter property was defined by introducing the angular correlation of photons as the average value of the cosine of the angle between the directions of momenta of any two photons. A positive correlation, as shown in Ref. [1], then would give a clear indication of the tendency of photons to travel in the same direction. Here we have N (for simplicity) noninteracting atoms (sources), each of which emits one photon with a given energy assumed to be due to the same atomic transition. To do this an expression for the probability distribution of N photon emissions collectively from the N atoms with arbitrary momentum directions is necessary. The purpose of this note is to give a field-theoretical point of view derivation of this conditional probability distribution based on the vacuum-to-vacuum transition amplitude [3,4] for photons in the presence of the N sources. Each source is assumed to emit separately photons with uniform distribution for the directions of the photons' momenta, and unlike Ref. [1] *no* assumption is made on the sources being pointlike. We choose units such that $\hbar=1=c$.

We work in the radiation (temporal) gauge $A^0=0$ for the vector potential and consider the Lagrangian density

$$L=-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+A^iJ^i \ , \tag{1}$$

where $i=1,2,3$ and J^i is an external source. This leads to Maxwell's equation

$$\partial_\mu F^{\mu\nu}=-J^\nu, \quad J^0=-(\partial_0)^{-1}\partial_i J^i \ , \tag{2}$$

$$(-\Box\delta^{ij}+\partial^i\partial^j)A^j(x)=J^i(x), \quad \Box\equiv\partial^2-\partial^{0^2} \ , \tag{3}$$

and for the Green function $D^{ij}(x-x')=i\langle 0_+|[A^i(x)A^j(x')]_+|0_-\rangle_0/\langle 0_+|0_-\rangle_0$,

$$(-\Box\delta^{ij}+\partial^i\partial^j)D^{jk}(x-x')=\delta^{ik}\delta^4(x-x') \ , \tag{4}$$

which leads to the vacuum-to-vacuum transition amplitude (e.g., [4])

$$\langle 0_+|0_-\rangle_J=\exp\left[\frac{i}{2}\int (dx)(dx')J^i(x)\times D^{ij}(x-x')J^j(x')\right] , \tag{5}$$

where

$$D^{ij}(x-x')=\int\frac{(dQ)}{(2\pi)^4}\frac{e^{iQ(x-x')}}{Q^2-i\epsilon}\left[\delta^{ij}-\frac{Q^iQ^j}{Q^{0^2}}\right] , \quad \epsilon\to 0+ \ . \tag{6}$$

Equation (5) may be also rewritten in a covariant notation from the constraint on J^0 in (2) as

$$\langle 0_+|0_-\rangle_J=\exp\left[\frac{i}{2}\int (dx)(dx')J^\mu(x)D_+(x-x')J_\mu(x')\right], \tag{7}$$

$$D_+(x-x')=\int\frac{(dQ)}{(2\pi)^4}\frac{e^{iQ(x-x')}}{Q^2-i\epsilon}, \quad \epsilon\to 0+ \ . \tag{8}$$

We consider N sources

$$\mathbf{J}(x)=\sum_{i=1}^{N}\mathbf{j}_i(x), \quad \mathbf{j}_i(x)=\mathbf{j}(x^0,\mathbf{x}-\mathbf{R}_i) \ , \tag{9}$$

centered, respectively, at $\mathbf{R}_1,\ldots,\mathbf{R}_N$, and otherwise identical. Each source is assumed to give a uniform distribution for the direction of momentum of each photon emitted. Therefore we will write in general

$$\mathbf{j}(x^0,\mathbf{x}-\mathbf{R})=\int\frac{(dQ)}{(2\pi)^4}e^{i\mathbf{Q}\cdot(\mathbf{x}-\mathbf{R})}e^{-iQ^0x^0}\times\frac{\mathbf{a}(Q)}{|\mathbf{a}_T(Q)|}f(|\mathbf{Q}|) \ , \tag{10}$$

where $Q=(Q^0,\mathbf{Q})$ and $\mathbf{a}(Q), f(|\mathbf{Q}|)$ are arbitrary, and

$$a_T^i(Q)=a^i(Q)-Q^i\frac{\mathbf{Q}\cdot\mathbf{a}(Q)}{Q^{0^2}} \ . \tag{11}$$

The probability that a given source $\mathbf{j}_i$ emits n photons is

$$\frac{\left[\int |f(|\mathbf{Q}|)|^2 d^3\mathbf{Q}/(2\pi)^3 2|\mathbf{Q}|\right]^n}{n!} \times \exp\left[-\int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}|f(|\mathbf{k}|)|^2\right] , \quad (12)$$

and hence provides a uniform distribution for the directions of the momenta of the photons. The situation is not the same when dealing collectively with the N sources. In particular, from (5), (9), and (10) we have for the vacuum persistence probability

$$|\langle 0_+|0_-\rangle|^2 = \exp - \sum_{i=1}^{N}\sum_{j=1}^{N}\int \frac{d^3\mathbf{Q}}{(2\pi)^3 2|\mathbf{Q}|}|f(|\mathbf{Q}|)|^2 \times \exp[i|\mathbf{Q}|\mathbf{n}\cdot(\mathbf{R}_i-\mathbf{R}_j)] , \quad (13)$$

where $\mathbf{Q}=|\mathbf{Q}|\mathbf{n}$, $d^3\mathbf{Q}=|\mathbf{Q}|^2 d|\mathbf{Q}|d\Omega$, and the integrand depends on the direction $\mathbf{n}$ of the momentum. The amplitude that the sources collectively create N photons each with energy k, M_1 of which have momenta in direction $\mathbf{n}_1$, M_2 of which have momenta in direction $\mathbf{n}_2$, and so on, $M_1+M_2+\cdots=N$, is then [4]

$$(i)^N[f(|\mathbf{k}|)]^N \frac{\left[\sum_{i=1}^{N} e^{-ik\mathbf{n}_1\cdot\mathbf{R}_i}\right]^{M_1}}{\sqrt{M_1!}} \frac{\left[\sum_{i=1}^{N} e^{-ik\mathbf{n}_2\cdot\mathbf{R}_i}\right]^{M_2}}{\sqrt{M_2!}} \cdots \langle 0_+|0_-\rangle_J . \quad (14)$$

Upon using the multinomial expansion

$$\frac{\left[\sum_{i=1}^{N} e^{-ik\mathbf{n}_1\cdot\mathbf{R}_i}\right]^{M_1}}{\sqrt{M_1!}} = \sqrt{M_1!} \sum_{m_{1i_1}+\cdots+m_{1i_N}=M_1} \frac{[e^{-ikm_{1i_1}\mathbf{n}_1\cdot\mathbf{R}_1}]\cdots[e^{-ikm_{1i_N}\mathbf{n}_1\cdot\mathbf{R}_N}]}{m_{1i_1}!\cdots m_{1i_N}!} ,$$

we have, for the amplitude that each source creates exactly one photon from (14) and (15), with $M_1=1,\ldots,M_N=1$:

$$(i)^N[f(|\mathbf{k}|)]^N\langle 0_+|0_-\rangle_J \prod_{j=1}^{N} \sum_{m_{ji_1}+\cdots+m_{ji_N}=1}{}' e^{-ikm_{ji_1}\mathbf{n}_j\cdot\mathbf{R}_1}\cdots e^{-ikm_{ji_N}\mathbf{n}_j\cdot\mathbf{R}_N} , \quad (16)$$

where the summations are over all non-negative integers $m_{ji_1},\ldots,m_{ji_N}$ such that $m_{ji_1}+\cdots+m_{ji_N}=1$ for all $j=1,\ldots,N$, with the additional restriction that $m_{1i_1}+m_{2i_1}+\cdots+m_{Ni_1}=1,\ldots,m_{1i_N}+\cdots+m_{Ni_N}=1$ in each term in the summand in Eq. (16), the expression

$$(i)^N[f(|\mathbf{k}|)]^N\langle 0_+|0_-\rangle_J \sum_P \exp\{-ik[\mathbf{n}_{i_1}\cdot\mathbf{R}_1+\cdots+\mathbf{n}_{i_N}\cdot\mathbf{R}_N]\} , \quad (17)$$

where the sum is over all permutations of $\{i_1,\ldots,i_N\}$ of the set $\{1,\ldots,N\}$. Therefore *given* that each of the N sources creates exactly one photon each with energy $|\mathbf{k}|=k$, then the conditional probability density that the N photons have momenta in the directions $\mathbf{n}_1,\ldots,\mathbf{n}_N$ is from (17)

$$p(\mathbf{n}_1,\ldots,\mathbf{n}_N)=\frac{1}{C}\left|\sum_P \exp\{-ik[\mathbf{n}_{i_1}\cdot\mathbf{R}_1+\cdots+\mathbf{n}_{i_N}\cdot\mathbf{R}_N]\}\right|^2 , \quad (18)$$

where

$$C=\int d\Omega_1\int\ldots\int d\Omega_N\left|\sum_P \exp\{-ik[\mathbf{n}_{i_1}\cdot\mathbf{R}_1+\cdots+\mathbf{n}_{i_N}\cdot\mathbf{R}_N]\}\right|^2 , \quad (19)$$

and no assumption is made on a pointlike nature for the sources. The angular correlation of two photons is defined by

$$\langle c\rangle=\int d\Omega_1\int\cdots\int d\Omega_N \mathbf{n}_i\cdot\mathbf{n}_j p(\mathbf{n}_1,\ldots,\mathbf{n}_N) . \quad (20)$$

For the application of (18) for various configurations of the N atoms, see Ref. [1], where the universal positivity character of $\langle c\rangle$ is also studied.

This paper treats the angular correlation of the *momenta* of a pair of photons. Our recent theoretical development of localized photon excitations [5] in spacetime opens the possibility for a study of correlation and interference effects of such a pair in configuration space and is planned to be considered elsewhere. This in turn will bring us in contact with an earlier study [6] in configuration space, and a comparison with some of the conclusions reached there with the ones from our analysis will then be made.

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