Theory of photorefractive phase-conjugate oscillators. II. Anisotropic four-wave mixing

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This paper is the second in a series describing the theory of photorefractive phase-conjugate oscillators. We apply the formulation developed in Paper I to study the threshold conditions for self-oscillation in the phase-conjugate mirror (PCM), the phase-conjugate resonator (PCR), and the phase-conjugate oscillator (PCO) by considering anisotropic four-wave mixing where the two counterpropagating off-axis pump beams are orthogonally polarized. We consider the two types of situations for orthogonally polarized pump beams corresponding to the two crystal families: one such as the $Bi_{12}SiO_{20}$ crystal family (sillenite family) and the other such as $BaTiO_3$ and the strontium barium niobate crystal family. Degenerate and nondegenerate self-oscillations are shown to occur for both these types for a number of practically important cases. Threshold conditions for oscillations are derived analytically under the assumption of undepleted pumps for most of these cases. It is shown that for the sillenite family of crystals, the coupling coefficient required for degenerate self-oscillation in PCR is one-half of that for PCM when the mirror is perfectly reflecting and when the pump beams have equal intensities. For PCO, the presence of the second mirror, however, increases the threshold value of the coupling coefficient for self-oscillation by a factor $(1+R_1R_2)/(1-R_1R_2)$, where R_1 and R_2 are the power reflection coefficients of the mirrors. For the second type of crystal family, it is shown that PCM also can exhibit self-oscillation. The coupling coefficient required for degenerate self-oscillation in PCR is one-half of that for PCM when $R_2=m$ and $r=m^{-1}$, where m and r are the ratios of the coupling coefficients of the backward and forward gratings and intensities of the backward and forward pumps, respectively.

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I. INTRODUCTION

In Paper I of this series [1], we developed a theory of photorefractive phase-conjugate Fabry-Pérot oscillators assuming that the two counterpropagating pump beams are parallel polarized [2]. This situation is called the isotropic four-wave mixing. In this paper, the case of anisotropic four-wave mixing, where the two pump beams are orthogonally polarized [3], will be considered. Here also we shall consider nondegenerate four-wave mixing. The matrix formulation [1,4–6] will be employed to obtain the threshold conditions for self-oscillations in the phase-conjugate mirror (PCM), phase-conjugate resonator (PCR), and phase-conjugate oscillator (PCO). As described in Paper I, a PCO consists of an optical resonator made up of two plane mirrors (a Fabry-Pérot cavity) containing an intracavity photorefractive phase-conjugate element that is pumped externally by a pair of off-axis counterpropagating orthogonally polarized laser beams of the same frequency. Phase conjugation of an input beam of slightly different frequency occurs because of nondegenerate four-wave mixing. In the absence of the mirrors the PCO becomes a PCM, and in the absence of one of the mirrors it becomes a PCR.

Two types of situations for orthogonally polarized pump beams will be considered in this work. For one type, the amplitudes a_f and a_b and the phases φ_f and φ_b of the coupling coefficients for the forward and backward gratings are related [3] by $a_f=a_b$ and $\varphi_b=\varphi_f+\pi$. For the other type, they are related [3] by $a_f\neq a_b$ and $\varphi_f=\varphi_b$. These two types of situations correspond to two different crystal families. The first type described above applies to crystals such as $Bi_{12}SiO_{20}$, and the second type applies to crystals such as $BaTiO_3$ and strontium barium niobate (SBN). We show that degenerate and nondegenerate self-oscillations are possible in photorefractive PCM, PCR, and PCO for both these types of situations when orthogonally polarized pump beams are used. We present analytical results for the conditions of self-oscillation in a number of practically important cases.

II. BASIC EQUATIONS

The geometry of six waves interacting in the photorefractive medium is shown in Fig. 1 of Paper I. Under the undepleted pump approximation the pump waves have constant amplitudes A_5 and A_6. There are four interacting waves with amplitudes obeying the following equations [3]:

$$\frac{dA_1^*}{dz}=\frac{\gamma_{f+}}{I_0}(A_6A_1^*+A_5^*A_2)A_6^* \ , \tag{1a}$$

$$\frac{dA_2}{dz}=\frac{\gamma_{b+}}{I_0}(A_6A_1^*+A_5^*A_2)A_5 \ , \tag{1b}$$

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$$\frac{dA_3^*}{dz}=\frac{\gamma_{f-}}{I_0}(A_6A_3^*+A_5^*A_4)A_6^* \ , \tag{1c}$$

$$\frac{dA_4}{dz}=\frac{\gamma_{b-}}{I_0}(A_6A_3^*+A_5^*A_4)A_5 \ , \tag{1d}$$

where $I_0=|A_5|^2+|A_6|^2$ is the total intensity of the pump waves. The coupling coefficients in Eq. (1) are given by

$$\gamma_{g\pm}=\frac{\gamma_g}{1\pm i\Delta\omega\tau} \ , \tag{2a}$$

$$\gamma_g=ia_g e^{i\varphi_g} \ , \tag{2b}$$

where the subscript g stands for forward (f) and backward (b) gratings and a_g and φ_g are the amplitudes and phases of these gratings. For parallel-polarized bump beams, we have $\gamma_f=\gamma_b$, i.e., $a_f=a_b$ and $\varphi_f=\varphi_b$. This situation was considered in Paper I. If orthogonally polarized pumps are used [3] in crystals of the sillenite family such as $Bi_{12}SiO_{20}$ (case 1), we have $\gamma_f=-\gamma_b$, i.e., $a_f=a_b$ and $\varphi_b=\varphi_f+\pi$. On the other hand, if orthogonally polarized pumps are used in ferroelectric crystals such as $BaTiO_3$ and SBN, we have $a_f\neq a_b$ and $\varphi_f=\varphi_b$ (case 2).

In ferroelectrics of case 2, birefringence creates complications for phase matching. This is because the wave vector k is different for the extraordinary and ordinary polarizations of the birefringent crystals. This introduces the Bragg angle detuning. In writing Eq. (1), we have neglected this detuning. This is justified [3] for the crystals with large r_{33}, such as SBN. We shall now consider the two cases described above separately.

III. CASE 1

Here we have $a_f=a_b$ and $\varphi_b=\varphi_f+\pi$. The matrix formulation of Paper I will be applied. The 4×4 matrix $\mathbf{K}$, which relates the four complex amplitudes $\{A_j(L)\}$ to the four complex amplitudes $\{A_j(0)\}$ is given by Eq. (8) of Paper I. The coefficients $M_\pm$, $N_\pm$, $P_\pm$, and $Q_\pm$ appearing in the matrix $\mathbf{K}$, listed in Appendix A, are different from those in Paper I. With the matrix $\mathbf{K}$ known, the phase-conjugate reflection and transmission coefficients are given by Eqs. (10) to (13) and the self-oscillation condition is given by Eq. (14) of Paper I. The coefficients F_{ij} are listed in Appendix B of Paper I. We are now ready to investigate oscillation in PCM, PCR, and PCO for this case.

A. Phase-conjugate mirror (PCM)

For the PCM, the phase-conjugate power reflection coefficient R_p is given by [3]

$$R_p=\left|\frac{\sinh\left[\frac{S_+L}{2}\right]}{\sinh\left[\frac{S_+L}{2}-\frac{\ln r}{2}\right]}\right|^2 \ , \tag{3}$$

where

$$S_+=\gamma_{b+}\left[\frac{1-r}{1+r}\right]=-\gamma_{f+}\left[\frac{1-r}{1+r}\right] \tag{4a}$$

and

$$r=|A_5|^2/|A_6|^2 \ . \tag{4b}$$

The self-oscillation condition can be obtained by equating the denominator of Eq. (3) to zero. In the special case of equal pumps ($r=1$),

$$R_p=\left|\frac{\gamma_{b+}L}{\gamma_{b+}L+2}\right|^2 \ . \tag{5}$$

Thus the self-oscillation condition is $\gamma_{b+}L=-2$, from which

$$a_bL\sin\varphi_b=2 \ , \tag{6a}$$

$$\Delta\omega\tau=-\frac{1}{\tan\varphi_b} \ . \tag{6b}$$

In the general case $r\neq1$ the self-oscillation condition is

$$a_bL\sin\varphi_b=\frac{r+1}{r-1}\ln r \ , \tag{7a}$$

$$\Delta\omega\tau=-\frac{1}{\tan\varphi_b} \ . \tag{7b}$$

The conditions (6) and (7) are the generalizations of the results in Ref. [3]. We have seen in Paper I that the PCM cannot exhibit degenerate self-oscillation for $\varphi=\pi/2$ when the pump beams are parallel polarized. The situation, however, changes when the pumps are orthogonally polarized for the sillenite family of crystals. Degenerate self-oscillation is now possible for $\varphi_b=\pi/2$ but impossible for $\varphi_b=0$. For any other value of φ_b, nondegenerate self-oscillation is, of course, possible.

B. Phase-conjugate resonator (PCR)

For the PCR, the threshold condition for self-oscillation is

$$\left[\frac{\sinh(S_+L/2)}{\sinh\left[\frac{S_+L}{2}-\frac{\ln r}{2}\right]}\right]\left[\frac{\sinh(S_-^*L/2)}{\sinh\left[\frac{S_-^*L}{2}-\frac{\ln r}{2}\right]}\right]=\frac{1}{R_2} \ , \tag{8}$$

where

$$S_-=\gamma_{b-}\left[\frac{1-r}{1+r}\right]=-\gamma_{f-}\left[\frac{1-r}{1+r}\right] \ . \tag{9}$$

We first consider the special case $r=1$. The condition (8) reduces to

$$\frac{\gamma_{b+}L\gamma_{b-}^*L}{(\gamma_{b+}L+2)(\gamma_{b-}^*L+2)}=\frac{1}{R_2} \ . \tag{10}$$

For $R_2=1$, Eq. (10) gives $(\gamma_{b+}+\gamma_{b-}^*)L=-2$, i.e.,

$a_b L \sin\varphi_b = 1$ and $\Delta\omega\tau = 0$. This shows that the coupling coefficient $a_b L$ required for degenerate self-oscillation is one-half of that for PCM when the plane mirror is perfectly reflecting. *However, nondegenerate self-oscillation is not possible in this case.* For $R_2 < 1$ and $\Delta\omega\tau = 0$ (degenerate case), we have

$$a_b L = \frac{2\sin\varphi_b \pm 2(R_2 - \cos^2\varphi_b)^{1/2}}{1 - R_2}, \quad R_2 \geq \cos^2\varphi_b \ . \tag{11}$$

For $\varphi_b = \pi/2$, $a_b L = 2(1 \pm \sqrt{R_2})/(1 - R_2)$ and for $R_1 = 1$, the solution with minus sign gives $a_b L = 1$. Equation (11) reduces to $a_b L \sin\varphi_b = 2$ when $R_2 = \cos^2\varphi_b$. Thus the presence of a mirror makes it possible to obtain degenerate self-oscillation at φ_b other than $\pi/2$. For $R_2 < 1$ and $\Delta\omega\tau \neq 0$ (nondegenerate case), we have

$$a_b L \sin\varphi_b = 2 \ , \tag{12a}$$

$$(\Delta\omega\tau)^2 = \frac{\cos^2\varphi_b - R_2}{\sin^2\varphi_b}, \quad R_2 < \cos^2\varphi_b \ . \tag{12b}$$

In the general case $r \neq 1$ the condition (8) can be separated into real and imaginary parts and can be expressed as follows:

$$\cos v_0(\cosh u_0' - R_2 \cosh u_0) = (1 - R_2)\cos v_1 \cosh u_1 \ , \tag{13a}$$

$$\sin v_0(\sinh u_0' - R_2 \sinh u_0) = (1 - R_2)\sin v_1 \sinh u_1 \ , \tag{13b}$$

where

$$u_0' = u_0 - \ln r \ , \tag{14a}$$

$$u_0 = -\frac{a_b L \sin\varphi_b}{1 + (\Delta\omega\tau)^2}\left[\frac{1-r}{1+r}\right], \tag{14b}$$

$$v_0 = \frac{a_b L \sin\varphi_b \Delta\omega\tau}{1 + (\Delta\omega\tau)^2}\left[\frac{1-r}{1+r}\right], \tag{14c}$$

$$u_1 = \frac{a_b L \cos\varphi_b \Delta\omega\tau}{1 + (\Delta\omega\tau)^2}\left[\frac{1-r}{1+r}\right], \tag{14d}$$

$$v_1 = \frac{a_b L \cos\varphi_b}{1 + (\Delta\omega\tau)^2}\left[\frac{1-r}{1+r}\right]. \tag{14e}$$

It can be verified that for $R_2 = 0$, Eqs. (13a) and (13b) reproduce Eqs. (7a) and (7b). Let us now consider degenerate and nondegenerate cases separately.

(a) *Degenerate case.* Here we have $\Delta\omega\tau = 0$. The condition (13b) is automatically satisfied. The condition (13a) gives

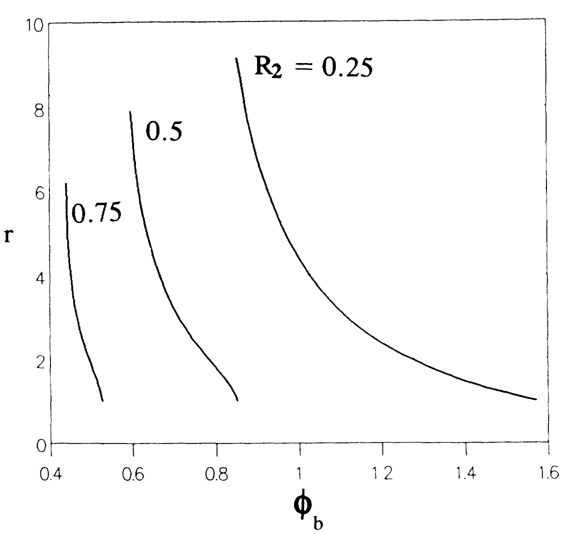

FIG. 1. Plots of r vs φ_b for the case of a PCR using Eq. (15) when degenerate self-oscillation takes place. The values of R_2 taken are $R_2 = 0.25$, 0.5, and 0.75. The value of $a_b L = 4.0$. Results apply to the sillenite family of crystals.

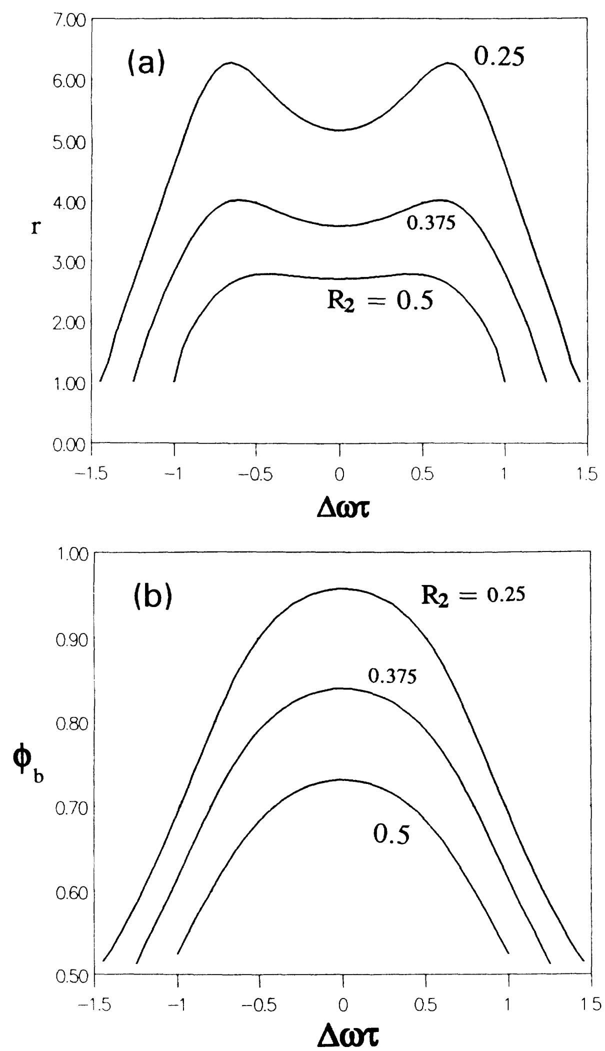

FIG. 2. (a) Plots of r vs $\Delta\omega\tau$ for the case of a PCR when nondegenerate self-oscillation takes place. The values of R_2 taken are $R_2 = 0.25$, 0.375, and 0.5. The value of $a_b L = 4$. Equations (13a) and (13b) are used for calculations. The results apply to the sillenite family of crystals. (b) Plots of φ_b vs $\Delta\omega\tau$. Everything is same as in (a).

$$r=\exp\left[-\cosh^{-1}\left\{R_2\cosh\left[a_bL\sin\varphi_b\frac{1-r}{1+r}\right]+(1-R_2)\cos\left[a_bL\cos\varphi_b\frac{1-r}{1+r}\right]\right\}-a_bL\sin\varphi_b\left[\frac{1-r}{1+r}\right]\right]. \quad (15)$$

This is a transcendental equation for r that requires a numerical solution. It may be noted that Eq. (15) can be satisfied only when $r\geq 1$. The solution $r=1$ is a trivial solution of Eq. (15). Self-oscillation conditions for $r=1$ have been already obtained earlier. Here we are interested in the solution of Eq. (15) when $r\neq 1$. In Figure 1, we have plotted r versus φ_b for $R_2=0.25$, 0.5, and 0.75, keeping $a_bL=4.0$. It can be seen that r decreases when φ_b is increased. There is a finite range of φ_b for which the solution $r\neq 1$ exists, and this range is reduced when R_2 is increased. For $R_2=1$, this range reduces to zero, and we get $r=1$ for all φ_b.

(b) *Nondegenerate case.* Here we have $\Delta\omega\tau\neq 0$. The conditions (13a) and (13b) are to be simultaneously considered. Here also we are interested in $r\neq 1$. The Eqs. (13a) and (13b) are solved numerically. In Figs. 2(a) and 2(b) we have plotted r and φ_b versus $\Delta\omega\tau$, keeping $a_bL=4$ and $R_2=0.25$, 0.375, and 0.5. The plots in Fig. 2(a) show a minimum for $\Delta\omega\tau=0$ and two maxima equally spaced from $\Delta\omega\tau=0$. The plots in Fig. 2(b) show the central maximum at $\Delta\omega\tau=0$. Here also, r values are different from 1 only for a certain range of $\Delta\omega\tau$ values.

C. Phase-conjugate oscillator (PCO)

The self-oscillation conditions in this most general case are complicated and can only be determined numerically. The situation is considerably simplified in the special case of equal intensity pumps $r=1$. The self-oscillation condition is then

$$2\sqrt{R_1R_2}\cos\Psi(\gamma_{b+}L+2)^2(\gamma_{b-}^*L+2)^2+4[(\gamma_{b+}L+2)(\gamma_{b-}^*L+2)+R_1R_2(\gamma_{b+}L-2)(\gamma_{b-}^*L-2)-\gamma_{b+}L\gamma_{b-}^*L(R_1+R_2)]=0\;, \quad (16)$$

where $\Psi=2kd$, $k=\frac{1}{2}(k_++k_-)$, and d is the total length of the resonator. For $\Psi=(2p+1)\pi/2$ where p is an integer, the condition (16) can reduce to

$$R_a\gamma_{b+}L\gamma_{b-}^*L+2R_b(\gamma_{b+}L+\gamma_{b-}^*L)+4R_c=0\;, \quad (17)$$

where

$$R_a=(1-R_1)(1-R_2)\;, \quad (18a)$$

$$R_b=1-R_1R_2\;, \quad (18b)$$

$$R_c=1+R_1R_2\;. \quad (18c)$$

For $R_1=0$, condition (17) coincides with condition (10). For $R_2=1$, Eq. (17) gives $a_bL\sin\varphi_b=(1+R_1)/(1-R_1)$ and $\Delta\omega\tau=0$. For $R_1,R_2<1$ and $\Delta\omega\tau=0$ (degenerate case), we have

$$a_bL=\frac{2R_b}{R_a}\left[\sin\varphi_b\pm\left[\sin^2\varphi_b-\frac{R_aR_c}{R_b^2}\right]^{1/2}\right]. \quad (19)$$

For $\sin^2\varphi_b=R_aR_c/R_b^2$, Eq. (19) reduces to $a_bL\sin\varphi_b=2R_c/R_b$. For R_1, $R_2<1$ and $\Delta\omega\tau\neq 0$ (nondegenerate case), we have

$$a_bL\sin\varphi_b=\frac{2R_c}{R_b}\;, \quad (20a)$$

$$(\Delta\omega\tau)^2=\frac{R_aR_c-R_b^2\sin^2\varphi_b}{R_b^2\sin^2\varphi_b}\;. \quad (20b)$$

IV. CASE 2

In this case $a_f\neq a_b$ and $\varphi_f=\varphi_b$. This situation occurs in crystals such as $BaTiO_3$ and SBN. Here also, the matrix formulation of Paper I has been applied. The coefficients $M_\pm$, $N_\pm$, $P_\pm$, and $Q_\pm$ appearing in the 4×4 matrix **K** are again different and are now listed in Appendix B. The phase-conjugate reflection and transmission coefficients can be obtained by using Eqs. (10) to (13) of Paper I and Appendix B of this paper. The self-oscillation condition is given by Eq. (14) of Paper I. We now investigate self-oscillation in PCM, PCR, and PCO under these conditions for this case.

A. Phase-conjugate mirror (PCM)

For PCM, power reflection coefficient R_p is

$$R_p=m\left|\frac{\sinh\left[\frac{S_+L}{2}\right]}{\cosh\left[\frac{S_+L}{2}+\frac{\ln mr}{2}\right]}\right|^2\;, \quad (21)$$

where

$$m=\frac{a_b}{a_f}\;, \quad (22a)$$

$$S_+=\gamma_{b+}T\;, \quad (22b)$$

$$T=\frac{m^{-1}+r}{1+r}\;. \quad (22c)$$

The oscillation condition is obtained by equating the denominator of Eq. (21) to zero. Comparison of Eq. (21) with Eq. (15a) of Paper I for R_p suggests that to obtain the oscillation condition one may replace γ_+, a, and r of Paper I by S_+, a_bT, and mr, respectively. One obtains the following self-oscillation conditions:

$$a_b L = \frac{\pi}{F_1}\left[\frac{1+r}{r}\right]\left[1+(\Delta\omega\tau)^2\right]\frac{\exp(\pi F_2/F_1)}{1+\exp(\pi F_2/F_1)} , \tag{23a}$$

$$a_f L = \frac{\pi}{F_1}(1+r)[1+(\Delta\omega\tau)^2]\frac{1}{1+\exp(\pi F_2/F_1)} , \tag{23b}$$

where

$$F_1 = |\cos\varphi + (\Delta\omega\tau)\sin\varphi| \tag{23c}$$

and

$$F_2 = \sin\varphi - (\Delta\omega\tau)\cos\varphi . \tag{23d}$$

Here $\varphi = \varphi_b = \varphi_f$. The conditions (23) can be also expressed in terms of m. For example, for $\Delta\omega\tau = 0$ (degenerate case), we obtain

$$a_b L \cos\varphi = m\pi\left[\frac{1+r}{1+mr}\right] , \tag{24a}$$

$$r = \frac{1}{m}\exp(\pi\tan\varphi) . \tag{24b}$$

For $r=1$, one obtains

$$a_b L = \left[\frac{2\pi}{\cos\varphi}\right]\frac{\exp(\pi\tan\varphi)}{1+\exp(\pi\tan\varphi)} . \tag{25}$$

In Ref. [3], it is stated that self-oscillation is not possible for the case of PCM under consideration. Our results are in contradiction with this assertion.

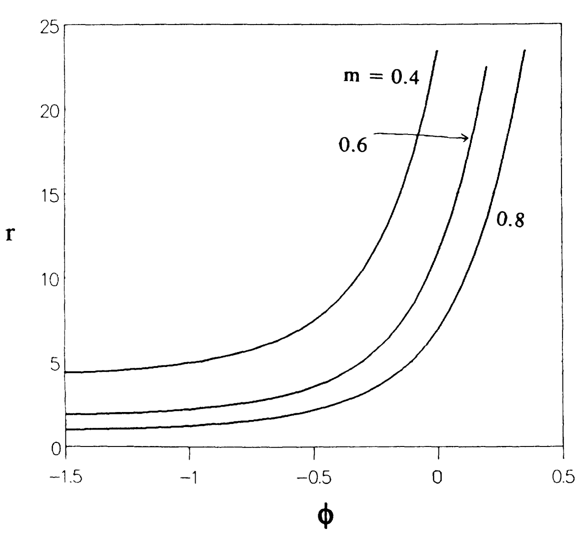

FIG. 3. Plots of r vs φ for the case of a PCR using Eq. (28) when degenerate self-oscillation takes place. The values of m taken are $m=0.4$, 0.6, and 0.8. The values of R_2 and $a_b L$ are $R_2=0.75$ and $a_b L = 3.0$. Results apply to the family of crystals such as $BaTiO_3$ and SBN.

B. Phase conjugate resonator (PCR)

The self-oscillation condition for the PCR is

$$\left[\frac{\sinh(S_+L/2)}{\cosh\left[\frac{S_+L}{2}+\frac{\ln mr}{2}\right]}\right]\left[\frac{\sinh(S_-^*L/2)}{\cosh\left[\frac{S_-^*L}{2}+\frac{\ln mr}{2}\right]}\right] = \frac{m}{R_2} , \tag{26}$$

where

$$S_- = \frac{\gamma_{b-}(m^{-1}+r)}{1+r} = \gamma_{b-}T . \tag{27}$$

Comparison of Eq. (26) with Eq. (21) of Paper I suggests that one may replace γ_+, γ_-, a, r, and R_2 of Paper I by S_+, S_-, $a_b T$, mr, and R_2/m, respectively, to obtain condition (26). Now, we shall consider the degenerate and nondegenerate cases separately.

(a) *Degenerate case.* Here $\Delta\omega\tau=0$. The self-oscillation condition becomes

$$r = \tfrac{1}{m}\exp\left[\cosh^{-1}\left\{\frac{R_2}{m}\cosh(a_b LT\sin\varphi) - \left[1+\frac{R_2}{m}\right]\cos(a_b LT\cos\varphi)\right\} + a_b LT\sin\varphi\right] . \tag{28}$$

This is a transcendental equation for r requiring numerical solution. Let us consider some special cases.

(i) $\varphi=0$. From Eq. (28), we have $\cos(a_b LT) \le -(m-R_2)/(m+R_2)$. This suggests that $\pi/2 \le a_b LT \le \pi$. For $R_2=0$, $a_b LT=\pi$ and for $R_2=m$, $a_b LT \ge \pi/2$. Thus the smallest value of $a_b LT$ is one-half of the value for PCM with $mr=1$. For $a_b LT = \pi - \cos^{-1}[(m-R_2)/(m+R_2)]$, we have $mr=1$. For large m, $a_b LT \approx \pi$.

(ii) $\varphi=\pi/2$. We must have $a_b LT \ge \cosh^{-1}[(R_2+2m)/R_2]$. For $R_2=0$, self-oscillation is impossible. For $R_2=m$, we have $a_b LT \ge 1.763$ and $mr \ge 5.828$. Thus for $\varphi=\pi/2$, the presence of a mirror ($R_2 \ne 0$) makes self-oscillation possible.

(iii) $\varphi=-\pi/2$. For $R_2=0$, the situation is the same as that for the case with $\varphi=\pi/2$. For $R_2=m$, we have $a_b LT \ge 1.763$ and $mr \ge 0.172$.

In Fig. (3), we show the plots of r versus φ for various values of m at $R_2=0.75$ and $a_b L=3$ using Eq. (28). For $m>1$, the solution for r does not exist.

(b) *Nondegenerate case.* Here $\Delta\omega\tau \ne 0$. Let us consider some special cases.

(i) $\varphi=0$. The self-oscillation conditions are given as

$$a_b LT = \pi[1+(\Delta\omega\tau)^2] , \tag{29a}$$

$$r = \frac{1}{m}[m_0 \pm (m_0^2-1)^{1/2}] , \tag{29b}$$

$$m_0=\left[1+\frac{R_2}{m}\right]\cosh(\pi\Delta\omega\tau)+\frac{R_2}{m}\ . \tag{29c}$$

(ii) $\varphi=\pm\pi/2$. The self-oscillation conditions are given as

$$a_b LT=\frac{\pi[1+(\Delta\omega\tau)^2]}{|\Delta\omega\tau|}\ , \tag{30a}$$

$$r=\frac{1}{m}\exp\left[2\sinh^{-1}\left\{\left[\frac{R_2}{m}\right]^{1/2}\cosh\left[\frac{\pi}{2\Delta\omega\tau}\right]\right\}\pm\frac{\pi}{|\Delta\omega\tau|}\right]\ . \tag{30b}$$

For $mr=1$, we have $r=R_2^{-1}\tanh^2(\pi/2\Delta\omega\tau)$.

C. Phase-conjugate oscillator (PCO)

The self-oscillation condition for the PCO is, in general, complicated and can only be determined using numerical methods. The situation is highly simplified in the special case $mr=1$. The self-oscillation condition then can be compared with that for the PCO of Paper I. In fact, if we replace γ_+, γ_-, a, R_1, and R_2 of Paper I by S_+, S_-, a_bT, mR_1, and R_2/m, respectively, the oscillation condition for the PCO here coincides with that for PCO of Paper I. Let us consider the degenerate and nondegenerate cases separately.

(a) *Degenerate case.* We have $\Delta\omega\tau=0$. For $\varphi=0$ and $2kd=p\pi$, where p is an integer, the self-oscillation condition can be written as

$$\cos(a_bLT)=-\frac{(1-mR_1)\left[1-\frac{R_2}{m}\right]}{(1+mR_1)\left[1+\frac{R_2}{m}\right]\pm4\sqrt{R_1R_2}}\ . \tag{31}$$

The $+$ and $-$ signs apply for p even or odd. For $\varphi=\pm\pi/2$, the self-oscillation condition is given by

$$\cosh(a_bLT)=-\frac{(1+mR_1)\left[1+\frac{R_2}{m}\right]+4\sqrt{R_1R_2}\cos(2kd)}{(1-mR_1)\left[1-\frac{R_2}{m}\right]}\ . \tag{32}$$

Since the numerator of Eq. (32) is always positive, the denominator must be negative. This implies that r should not be between R_1 and R_2^{-1}, i.e., $r<R_1$ or $r>R_2^{-1}$. The interesting point to note here is that unlike the parallel-polarized case, self-oscillation here is possible for $\varphi=\pm\pi/2$.

(b) *Nondegenerate case.* Here we have $\Delta\omega\tau\neq0$. For $\varphi=0$ and $2kd=p\pi$, where p is an integer, the self-oscillation condition becomes

$$\cosh\left[a_bLT\frac{\Delta\omega\tau}{1+(\Delta\omega\tau)^2}\right]=\frac{(1-mR_1)\left[1-\frac{R_2}{m}\right]}{(1+mR_1)\left[1+\frac{R_2}{m}\right]\pm4\sqrt{R_1R_2}}\ . \tag{33}$$

The $+$ and $-$ signs apply for p even or odd. Equation (33) cannot be satisfied since the right-hand side is less than 1. Therefore, self-oscillation is not possible for these cases. For $\varphi=\pm\pi/2$, the self-oscillation condition for $2kd=p\pi$, where p is an integer, is

$$\left|(\Delta\omega\tau)\cosh^{-1}\left[\frac{(1+mR_1)\left[1+\frac{R_2}{m}\right]\pm4\sqrt{R_1R_2}}{(1-mR_1)\left[1-\frac{R_2}{m}\right]}\right]\right|=\pi. \tag{34}$$

The $+$ and $-$ signs apply for p even or odd. For $2kd=(2p+1)\pi/2$, the self-oscillation condition is

$$\left|(\Delta\omega\tau)\cosh^{-1}\left[\frac{(1+mR_1)\left[1+\frac{R_2}{m}\right]}{(1-mR_1)\left[1-\frac{R_2}{m}\right]}\right]\right|=\pi\ . \tag{35}$$

In Eqs. (34) and (35) r must satisfy the inequality $R_1<r<R_2^{-1}$.

V. CONCLUSIONS

In this paper, we have applied the theory of photorefractive phase-conjugate Fabry-Pérot oscillators developed earlier [1] in Paper I to the case when the off-axis counterpropagating pump beams are orthogonally polarized instead of parallel polarized. Two cases were considered in this paper. For the first case, which applies to the sillenite family of crystals such as $Bi_{12}SiO_{20}$, the amplitudes of the coupling coefficients for forward and backward gratings are the same but the phases differ by π. For the second case, which applies to the family of crystals such as $BaTiO_3$ and SBN, the amplitudes of the coupling coefficients are different but they have the same phases. The self-oscillation conditions for the phase-conjugate mirror, phase-conjugate resonator, and phase-conjugate oscillator were obtained for both these cases.

In the first case, the PCM was earlier [3] shown to exhibit degenerate self-oscillation for $\varphi_b=\pi/2$. We have shown in this paper that for $\varphi_b\neq0$ and $\pi/2$, PCM can exhibit nondegenerate self-oscillation. It is also shown that when the pump beams have equal intensities the presence of a mirror in PCR makes it possible to obtain degenerate self-oscillation at φ_b other than $\pi/2$. The coupling coefficient a_bL required for degenerate self-oscillation is one-half of that for the PCM when the mirror is perfectly reflecting. However, for a perfectly reflecting mirror, nondegenerate self-oscillation is impossible. For nondegenerate self-oscillation to occur, the mirror reflection coefficient R_2 must be less than $\cos^2\varphi_b$. For the PCO, presence of two mirrors in fact increases the value of the coupling coefficient $a_b L$ required for self-oscillation by a factor $(1+R_1R_2)/(1-R_1R_2)$. The self-oscillation condition for the PCO depends on the reflection coefficients R_1 and R_2 of two mirrors, the cavity length d, the wavelength λ of the pump waves, and linear refractive index n_0 of the photorefractive material at λ.

For the second case, the results are quite similar to those in the isotropic mixing. In fact, the results of Paper I are special cases of the results obtained for the second case of anisotropic mixing. When the ratio of the coupling coefficients $m=\gamma_b/\gamma_f$ is unity, results for the isotropic and anisotropic mixing coincide. The PCM for the second case can exhibit self-oscillation except when $\varphi=\pi/2$. For the PCR, self-oscillation is possible for $\varphi=\pi/2$ in the presence of a mirror. The coupling coefficient $a_b L$ required for degenerate self-oscillation in PCR is one-half that for PCM when $R_2=m$ and $r=1/m$. For the PCO, degenerate self-oscillation is possible when $\varphi=\pm\pi/2$, which was not possible in the isotropic mixing (see Paper I). However, as in the isotropic case, PCO cannot exhibit nondegenerate self-oscillation for $\varphi=0$.

For both crystal types considered in this paper with anisotropic mixing, the self-oscillation conditions for PCO do not depend upon the position of the photorefractive crystal inside the cavity.

APPENDIX A

In this appendix, we give the matrix elements of the matrix **K** for a sillenite family of crystals:

$$M_\pm=\frac{1}{\alpha_\pm^*}\exp(-S_\pm^*L/2)\left[\alpha_\pm^{*2}-\frac{\gamma_{b\pm}^{*2}}{I_0^2}|A_5|^2|A_6|^2\beta_\pm^{*2}\right],$$

$$N_\pm=\frac{1}{\alpha_\pm}\exp(-S_\pm L/2),$$

$$P_\pm=-\frac{1}{\alpha_\pm^*}\exp(-S_\pm^*L/2)\left[\frac{\gamma_{b\pm}^*}{I_0}A_5A_6\beta_\pm^*\right],$$

$$Q_\pm=\frac{1}{\alpha_\pm}\exp(-S_\pm L/2)\left[\frac{\gamma_{b\pm}}{I_0}A_5A_6\beta_\pm\right],$$

where

$$S_\pm=\gamma_{b\pm}\left[\frac{1-r}{1+r}\right]=-\gamma_{f\pm}\left[\frac{1-r}{1+r}\right],$$

$$\alpha_\pm=\left[\frac{1-r}{2\sqrt{r}}\right]\frac{1}{\sinh\left[\frac{S_\pm L}{2}-\frac{\ln r}{2}\right]},$$

$$\beta_\pm=\left[\frac{1+r}{\sqrt{r}}\right]\frac{\sinh(S_\pm L/2)}{\gamma_{b\pm}\sinh\left[\frac{S_\pm L}{2}-\frac{\ln r}{2}\right]},$$

$$I_0=|A_5|^2+A_6|^2,$$

and

$$r=|A_5|^2/|A_6|^2.$$

APPENDIX B

In this appendix, we give the matrix elements of the matrix **K** for a family of crystals such as $BaTiO_3$ and SBN:

$$M_\pm=\frac{1}{\alpha_\pm^*}\exp(S_\pm^*L/2)\left[\alpha_\pm^{*2}+\frac{\gamma_{f\pm}^*\gamma_{b\pm}^*}{I_0^2}|A_5|^2|A_6|^2\beta_\pm^{*2}\right],$$

$$N_\pm=\frac{1}{\alpha_\pm}\exp(S_\pm L/2),$$

$$P_\pm=\frac{1}{\alpha_\pm^*}\exp(S_\pm^*L/2)\left[\frac{\gamma_{f\pm}^*}{I_0}A_5A_6\beta_\pm^*\right],$$

$$Q_\pm=\frac{1}{\alpha_\pm}\exp(S_\pm L/2)\left[\frac{\gamma_{b\pm}}{I_0}A_5A_6\beta_\pm\right],$$

where

$$S_\pm=\gamma_{b\pm}T,$$

$$T=\left[\frac{m^{-1}+r}{1+r}\right],$$

$$\alpha_\pm=\left[\frac{1+mr}{2\sqrt{mr}}\right]\operatorname{sech}\left[\frac{S_\pm L}{2}+\frac{\ln mr}{2}\right],$$

$$\beta_\pm=\sqrt{m}\left[\frac{1+r}{\sqrt{r}}\right]\frac{\sinh(S_\pm L/2)}{\gamma_{b\pm}\cosh\left[\frac{S_\pm L}{2}+\frac{\ln mr}{2}\right]},$$

$$I_0=|A_5|^2+|A_6|^2,$$

$$r=|A_5|^2/|A_6|^2,$$

and

$$m=a_b/a_f.$$

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