Inversionless amplification of a monochromatic field by a three-level medium

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We consider a three-level medium with low-frequency separation between the two upper levels. The medium interacts with an external field that induces a coherence between the two upper levels. We analyze the amplification condition for the probe monochromatic optical field in a general nonresonant case. We show that amplification without population inversion arises either at line center or at the sidebands, depending on the sign of the population difference between the upper sublevels. This amplification occurs in a limited range of the coherent pump intensity.

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I. INTRODUCTION

A three-level medium is the simplest and most suitable model for understanding the physical mechanisms of amplification without inversion. The possibility of inversionless amplification in a three-level medium with a Λ configuration was demonstrated for the first time in the pulsed regime [1]. It was shown that an ultrashort pulse can be amplified without population inversion at both optical transitions if the low-frequency (LF) coherence is excited just before the pulse interacts with the medium. Different ways of preparing initial medium states were discussed, including resonant pumping by a microwave $\pi/2$ pulse [1,2]. Next, it was shown by Scully, Zhu, and Gavridiles [3] that, in a Λ scheme, a monochromatic field tuned to the middle of the low-frequency splitting and interacting simultaneously with both optical transitions can be amplified without population inversion in a steady regime when the atomic coherence is excited by a strong resonant microwave pump field such that the Rabi frequency equals the low-frequency splitting.

Recently, analyses were presented for all possible resonant three-level configurations where a LF coherent input transfers energy into a monochromatic [5–10] or a bichromatic optical field [4,6,8,10,11], with each component of the amplified field interacting on resonance with only one optical transition. It was shown that there are two different mechanisms of inversionless amplification. One mechanism is a parametric coupling of two optical components mediated by the LF coherence [11]. The role of the microwave field in this mechanism is reduced to the creation of the LF coherence σ_{12}; amplification occurs when this coherence exceeds a critical threshold: $|\sigma_{12}|^2 > |n_{13}n_{23}|$, where n_{13} and n_{23} are the population differences at the optical transitions in the presence of the microwave field. This condition is equivalent to the condition of population inversion in the basis of the dressed states where the LF coherence vanishes [12]. Thus the excitation of the LF coherence leads to the redistribution of populations among the sublevels in the new basis. In the Λ scheme, this mechanism is directly connected with the phenomenon of coherent population trapping [13]. Part of the atoms of the lower levels are trapped in a state that does not interact with the resonant field. As a result, the amplification is possible when the upper-level population exceeds only the population of the untrapped state but not the populations of the original atomic levels. In the V scheme, this mechanism amounts to requiring a population inversion between the sum of the two upper states and the lower state.

Another mechanism appears even in the simplest resonant P scheme, where the microwave field is scattered into the anti-Stokes optical component by means of the atomic coherence of the adjacent optical transition, which in turn is excited as a result of the Stokes scattering of the optical field by the LF coherence created by the microwave field [6,10,14]. This mechanism occurs without effective inversion in the basis where the LF coherence vanishes; hence amplification is possible even when the sum of the upper sublevels population is less than the lower-level population [10].

In this paper we consider the P scheme for the general nonresonant case (Fig. 1): the monochromatic optical field interacts with either one or with both optical transitions, when the two upper sublevels are coupled by either microwave or an external dc field. We analyze first the amplification condition for a simplified model, the F scheme (Fig. 2), where an external field influences the optical polarization via the atomic coherence between the

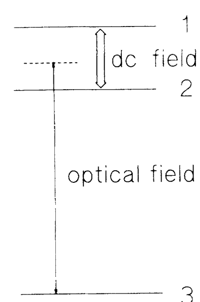

FIG. 1. The P configuration.

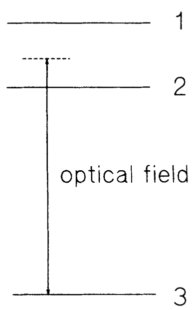

FIG. 2. The F configuration.

sublevels only. Then we establish the equivalence between the P scheme and more complex configurations involving two autoionizing states decaying to an identical continuum and a monochromatic field interacting with both high-frequency transitions [16–19]. Finally we prove that the P scheme and hence the scheme involving a continuum of states are reduced to the F scheme in the dressed-state representation. We find the gain profile and show that the sign of the population difference between the upper sublevels determines if the inversionless amplification occurs at line center or in the sidebands.

II. DYNAMICAL EQUATIONS FOR THE P SCHEME

We consider a three-level medium with a P configuration (Fig. 1). The two sublevels of the upper level are coupled by the dc field E_0. The high-frequency field $\bar{E}=(E/2)[\exp(-i\omega t+ikz)+\text{c.c.}]$ excites both optical transitions $1\leftrightarrow 3$ and $2\leftrightarrow 3$. This system is described by the Hamiltonian

$$\hat{H}=\sum_{j=1}^{2}\hbar\omega_{j3}|\Psi_j\rangle\langle\Psi_j| -\bar{E}(\mu_{31}|\Psi_3\rangle\langle\Psi_1|+\mu_{13}|\Psi_1\rangle\langle\Psi_3| +\mu_{32}|\Psi_3\rangle\langle\Psi_2|+\mu_{23}|\Psi_2\rangle\langle\Psi_3|) -E_0(\mu_{21}|\Psi_2\rangle\langle\Psi_1|+\mu_{12}|\Psi_1\rangle\langle\Psi_2|)\ , \tag{2.1}$$

where $\Psi_j=\Psi_j^{(0)}\exp(-iE_jt/\hbar)$ are the atomic states with energy E_j, $\omega_{ij}=(E_i-E_j)/\hbar$ with $i,j=1$, 2, or 3, and $E_3=0$. In the rotating-wave approximation and in the basis $\varphi_{1,2}=\Psi_{1,2}e^{i\omega t}$, $\varphi_3=\Psi_3$, the Hamiltonian (2.1) takes the form

$$H=\hbar\begin{bmatrix}\delta_1 & -\gamma & -\alpha\\ -\gamma^* & \delta_2 & -\eta\alpha\\ -\alpha^* & -\eta^*\alpha^* & 0\end{bmatrix}\ , \tag{2.2}$$

where $\alpha=\mu_{13}E/2\hbar$, $\gamma=\mu_{12}E_0/\hbar$, $\delta_1=(E_1-E_3)/\hbar-\omega$, and $\delta_2=(E_2-E_3)/\hbar-\omega$.

With the usual phenomenological addition of the incoherent pumping and damping processes, we obtain the set of density matrix equations [15]:

$$\frac{\partial\sigma_{13}}{\partial t}=-\left[\frac{1}{T_2^{13}}+i\delta_1\right]\sigma_{13}+i\alpha(n_{31}-\eta\sigma_{12})+i\gamma\sigma_{23}\ , \tag{2.3a}$$

$$\frac{\partial\sigma_{23}}{\partial t}=-\left[\frac{1}{T_2^{23}}+i\delta_2\right]\sigma_{23}+i\alpha(\eta n_{32}-\sigma_{12}^*)+i\gamma^*\sigma_{13}\ , \tag{2.3b}$$

$$\frac{\partial\sigma_{12}}{\partial t}=-\left[\frac{1}{T_2^{12}}+i\omega_{12}\right]\sigma_{12} +i(\alpha\sigma_{23}^*-\eta^*\alpha^*\sigma_{13})+i\gamma n_{21}\ , \tag{2.3c}$$

$$\frac{\partial\rho_{11}}{\partial t}=R_1-2\,\mathrm{Im}(\gamma\sigma_{21}+\alpha\sigma_{31})\ , \tag{2.3d}$$

$$\frac{\partial\rho_{22}}{\partial t}=R_2+2\,\mathrm{Im}(\gamma\sigma_{21}+\alpha^*\eta^*\sigma_{23})\ , \tag{2.3e}$$

with the definitions

$$\sigma_{ij}=\langle\varphi_i|\rho|\varphi_j\rangle\ ,\quad n_{ij}=\rho_{ii}-\rho_{jj}\ ,$$
$$R_i=-\sum_{j=1}^{3}(N_j\rho_{ii}-N_i\rho_{jj})/T_1^{ij}\ ,$$

where N_j is the population of level j in the absence of coherent field ($\alpha=\gamma=0$), T_1^{ij} and T_2^{ij} are the longitudinal and transverse relaxation times for the transition $i\leftrightarrow j$, and $\eta=\mu_{23}/\mu_{13}$.

The wave equation for the complex amplitude α is

$$\frac{\partial\alpha}{\partial z}+\frac{1}{c}\frac{\partial\alpha}{\partial t}=\frac{2\pi i\omega N|\mu_{31}|^2}{\hbar c}(\sigma_{13}+\eta^*\sigma_{23}) \tag{2.4}$$

and pump depletion will be neglected, implying that we work in the intense pump limit.

We need a dc field in order to excite the cw coherence because only in this case is there a steady-state solution of the Maxwell-Bloch equations if the monochromatic field interacts with both optical transitions simultaneously. For this case, the transient processes when the atomic coherence is excited by a microwave field interacts with only one optical transition (in particular, if one of the two transitions is forbidden), there is a steady-state solution of the Maxwell-Bloch equations in the rotating-wave approximation when the low-frequency coherence is excited by the microwave field. This last situation is described by the same set of Bloch equations but with the modified definitions $\varphi_1=\psi_1e^{i\omega t}$, $\varphi_2=\psi_2e^{i(\omega-\omega_p)t}$, $\gamma=\mu_{12}E_0/(2\hbar)$ where E_0 is the complex amplitude of the microwave field, and $\delta_2=\omega_{23}-\omega-\omega_p$ where ω is the frequency of the coherent pumping field and $\eta=0$.

To get the *linear* gain from which the amplification condition derives, we need only Eqs. (2.3a), (2.3b), and (2.4) where the functions n_{31}, n_{32}, and σ_{12} are driven by the external field γ and are defined from Eqs. (2.3c)–(2.3e) with $\alpha=0$. In steady state, we have

$$\sigma_{12}=\frac{i\gamma n_{21}}{i\Omega_{12}+1/T_2^{12}}\ ,$$

$$n_{3l}=\frac{n_{21}}{N_{21}}\left[N_{3l}+\frac{4T_e|\gamma|^2}{T_2^{12}[\Omega_{12}^2+(1/T_2^{12})^2]}\right]\ ,\quad l=1 \text{ or } 2\ , \tag{2.5}$$

$$n_{21}=\frac{N_{21}}{1+4T|\gamma|^2/T_2^{12}[\Omega_{12}^2+(1/T_2^{12})^2]}\ ,$$

where the following definitions have been used:

$$T_e=\frac{g}{4}T_2^{23}\frac{N_{31}g_1+N_{32}g_2}{N_1g_1+N_2g_2+N_3g_1g_2}\ ,$$

$$T=\frac{g}{2}T_2^{23}\frac{N_1g_1+N_2g_2+N_3(g_1+g_2)/2}{N_1g_1+N_2g_2+N_3g_1g_2}\ ,$$

$$g_l=T_1^{12}/T_1^{l3}\ ,\quad N_{jl}=N_j-N_l\ ,\quad l=1 \text{ or } 2\ ,$$

$$g=2T_1^{12}/T_2^{23}\ ,\quad \Omega_{12}=\omega_{12}-\omega_p\ .$$

According to Eqs. (2.3a) and (2.3b), there are two ways in which the dc field can induce the atomic polarizations σ_{13} and σ_{23}, and therefore control the high-frequency field absorption. One channel is via the terms $\eta\alpha\sigma_{12}$ and $\alpha\sigma_{12}^*$ (where σ_{12} is proportional to γ), the second channel is via the terms $\gamma\sigma_{23}$ and $\gamma^*\sigma_{13}$. If one of the two optical transitions is forbidden, only the second channel remains. In Refs. [6] and [10], we dealt with a bichromatic field with components α and β, and we found a similar pair of channels (via the terms $\beta\sigma_{12}$, $\alpha\sigma_{12}^*$ and via the terms $\gamma\sigma_{23}$, $\gamma^*\sigma_{13}$, respectively) which were shown to be connected to the two different amplification mechanisms discussed in the Introduction. The question is therefore to determine whether these two mechanisms are also connected to the two channels determined here. To answer this question, we first analyze a simpler model where the direct connection between the sublevels by the dc field is absent. In other words, we set $\gamma=0$ in Eqs. (2.3a) and (2.3b), but keep the terms which are proportional to σ_{12}. To differentiate this situation from the P scheme (where $\gamma\neq 0$), we shall call it the F scheme (Fig. 2). In principle, such a scheme can be realized when the atomic coherence is excited by another monochromatic field resonant to the adjacent transition (compare with the double-Λ scheme of Ref. [6]). In addition, we will show that the P scheme can reduce to the F scheme under certain conditions.

III. AMPLIFICATION CONDITION FOR THE F SCHEME

Let us consider the propagation of a monochromatic field

$$\bar{E}=\frac{E}{2}[\exp(-i\omega t+ikz)+\text{c.c.}]\ ,$$

interacting with a three-level medium in an F configuration (Fig. 2). The cw atomic coherence σ_{12} is supposed to be excited between the two upper sublevels.

The equations for the complex amplitudes of the off-diagonal elements σ_{13} and σ_{23} of the density matrix in the slowly varying amplitude approximation are

$$\begin{aligned}\frac{\partial\sigma_{13}}{\partial t}&=-\left[\frac{1}{T_2^{13}}+i\delta_1\right]\sigma_{13}+i\alpha(n_{31}-\eta\sigma_{12})\ ,\\ \frac{\partial\sigma_{23}}{\partial t}&=-\left[\frac{1}{T_2^{23}}+i\delta_2\right]\sigma_{23}+i\alpha(\eta n_{32}-\sigma_{12}^*)\ .\end{aligned}\tag{3.1}$$

Because we seek the linear gain, we can assume n_{13}, n_{23}, and σ_{12} to be constants defined by some external sources. Let us seek solutions of (2.4) and (3.1) in the form σ_{13}, σ_{23}, $\alpha\sim\exp(-i\tilde{\omega}t+i\tilde{k}z)$. This leads to the dispersion relation

$$\tilde{k}-\frac{\tilde{\omega}}{c}=ig_{13}\left[\frac{n_{31}-\eta\sigma_{12}}{1+iT_2^{13}\Delta_1}+\frac{|\eta|^2n_{32}-(\eta\sigma_{12})^*}{\zeta+iT_2^{13}\Delta_2}\right]\ ,\tag{3.2}$$

where $g_{13}=2\pi N\omega|\mu_{13}|^2T_2^{13}/c\hbar$, $\zeta=T_2^{13}/T_2^{23}$, $\Delta_1=\omega_{13}-\omega-\tilde{\omega}$, $\Delta_2=\Delta_1-\omega_{12}$. The amplification condition $\text{Im}(\tilde{k})<0$ is

$$\text{Im}(\mu_{31}\mu_{23}\sigma_{12})\left[\frac{\Delta_1(T_2^{13})^2}{1+(\Delta_1T_2^{13})^2}-\frac{\Delta_2(T_2^{23})^2}{1+(\Delta_2T_2^{23})^2}\right]+\text{Re}(\mu_{31}\mu_{23}\sigma_{12})\left[\frac{T_2^{13}}{1+(\Delta_1T_2^{13})^2}+\frac{T_2^{23}}{1+(\Delta_2T_2^{23})^2}\right]>\frac{n_{31}T_2^{13}|\mu_{13}|^2}{1+(\Delta_1T_2^{13})^2}+\frac{n_{32}T_2^{23}|\mu_{23}|^2}{1+(\Delta_2T_2^{23})^2}\ .\tag{3.3}$$

In the limit $1/T_2^{13},1/T_2^{23}\to 0$, this condition takes the simple form

$$\frac{\omega_{12}}{\Delta_1\Delta_2}\text{Im}(\mu_{31}\mu_{23}\sigma_{12})<0\ .$$

Thus if the optical field is tuned between levels 1 and 2, such that $\Delta_1\Delta_2<0$, it is necessary to have $\text{Im}(\mu_{31}\mu_{23}\sigma_{12})>0$.

In the symmetrical case, defined by the conditions $T_2^{13}=T_2^{23}=T_2$, $\Delta_1=-\Delta_2=\omega_{12}/2$, we have, from (3.3),

$$\omega_{12}T_2\,\text{Im}(\mu_{31}\mu_{23}\sigma_{12})+2\,\text{Re}(\mu_{31}\mu_{23}\sigma_{12})>n_{31}|\mu_{31}|^2+n_{32}|\mu_{32}|^2\ .\tag{3.4}$$

We stress that this inequality is quite different from the inversionless amplification condition of a bichromatic field:

$$|\sigma_{21}|^2>n_{13}n_{23}\ ,\tag{3.5}$$

which is equivalent to the condition of population inversion in the basis where the atomic coherence vanishes [6].

If the first term in the left-hand side (3.4) vanishes, the inequality (3.4) is more restrictive than condition (3.5). However, the contrary is also possible, i.e., condition (3.4) can be easier to verify than condition (3.5). In that case, inversionless amplification of a monochromatic field is possible even when $\rho_{11}+\rho_{22}<\rho_{33}$ and amplification of a bichromatic field is impossible [6]. However, in that case, $\rho_{11}+\rho_{22}$ has to exceed a critical threshold. This requires a finite frequency splitting ($\omega_{12}\neq 0$) and field interactions with both optical transitions simultaneously: $\mu_{13}\neq 0$, $\mu_{23}\neq 0$.

To observe inversionless amplification in the F scheme, it is necessary that both optical transitions be dipole-allowed transitions: $\mu_{13}\neq 0$, $\mu_{23}\neq 0$. However, it is also sufficient to have a large enough atomic polarization at only one of the transitions: the other atomic polarization can be small as a result of fast relaxing process. For instance, in the limit $T_2^{23}\rightarrow 0$ and $\sigma_{23}\rightarrow 0$, Eqs. (2.4) and (3.1) reduce to

$$\frac{\partial\sigma_{13}}{\partial t}=-\left[\frac{1}{T_2^{13}}+i\delta_1\right]\sigma_{13}+i\frac{\alpha}{\mu_{13}}(\mu_{13}n_{31}-\mu_{23}\sigma_{12}) , \tag{3.5a}$$

$$\frac{\partial\alpha}{\partial z}+\frac{1}{c}\frac{\partial\alpha}{\partial t}=\frac{2\pi i\omega N|\mu_{13}|^2}{\hbar c}\sigma_{13} . \tag{3.5b}$$

The amplification condition (3.3) then takes the form

$$\Delta_1\,\mathrm{Im}(\mu_{31}\mu_{23}\sigma_{12})+\mathrm{Re}(\mu_{31}\mu_{23}\sigma_{12})/T_2^{13} > n_{31}|\mu_{31}|^2/T_2^{13} . \tag{3.6}$$

In this limit, the physical mechanism leading to inversionless amplification is especially simple. The optical field α acting on the transition $2\leftrightarrow 3$ creates an atomic polarization at the optical transition $1\leftrightarrow 3$ via the interaction with the LF coherence σ_{12} as described by the source term $i\alpha\mu_{23}\sigma_{12}/\mu_{13}$ in (3.5a). Under appropriate conditions on σ_{12}, the atomic polarization σ_{13} can drive the optical field α and result in a net gain of emission over absorption.

Hence this inversionless amplification is due to the fact that the external source exciting the atomic coherence σ_{12} couples sublevels in such a way that the interaction of the field with the transition $2\leftrightarrow 3$ leads to the depletion of the upper states, even if the sum of their populations is less than the lower-level population.

IV. AMPLIFICATION CONDITION FOR THE P SCHEME

Let us return to the analysis of amplification condition in P scheme. Seeking solutions of Eqs. (2.3) and (2.4) in the form σ_{13}, σ_{23}, and $\alpha\sim\exp(-i\tilde{\omega}t+i\tilde{k}z)$ leads to the dispersion relation

$$\tilde{k}-\frac{\tilde{\omega}}{c}=g_{13}\left[\frac{\sigma_{13}+\eta^*\sigma_{23}}{\alpha T_2^{13}}\right] ,$$

where

$$\sigma_{13}=i\alpha T_2^{13}[(n_{31}-\eta\sigma_{12})(i\tilde{\Delta}_2+\zeta)+i\gamma T_2^{13}(\eta n_{32}-\sigma_{12}^*)]/\mathcal{P} ,$$

$$\sigma_{23}=i\alpha T_2^{13}[(\eta n_{32}-\sigma_{12}^*)(i\tilde{\Delta}_1+1)+i\gamma^* T_2^{13}(n_{31}-\eta\sigma_{12})]/\mathcal{P} ,$$

$$\mathcal{P}=(1+i\tilde{\Delta}_1)(\zeta+i\tilde{\Delta}_2)+P , \quad P=|\gamma|^2(T_2^{13})^2 ,$$

$$\tilde{\Delta}_1=(\omega_{13}-\omega-\tilde{\omega})T_2^{13}, \quad \tilde{\Delta}_2=\tilde{\Delta}_1-(\omega_{12}-\omega_p)T_2^{13} .$$

The gain $G=-\mathrm{Im}(k)$ is given by

$$G=g_{13}\,\mathrm{Re}\{[(-n_{31}+\eta\sigma_{12})(\zeta+i\tilde{\Delta}_2)+(\eta^*\sigma_{12}^*-|\eta|^2 n_{32})(1+i\tilde{\Delta}_1)-i\gamma T_2^{13}(\eta n_{32}-\sigma_{12}^*)-i\gamma^* T_2^{13}(\eta^* n_{31}-|\eta|^2\sigma_{12})]/\mathcal{P}\} . \tag{4.1}$$

In these notations, the linear gain/absorption of a two-level system would be $G=g_{13}N_{13}$. In the symmetrical case defined by $\tilde{\Delta}_1=-\tilde{\Delta}_2$, $T_2^{13}=T_2^{23}$, $|\mu_{13}|^2=|\mu_{23}|^2$, and $\omega_p=0$, this formula coincides with (3.4). Hence in this case, the action of the field γ is reduced to the excitation of the atomic coherence and the P scheme becomes equivalent to the F scheme. If the transition $2\leftrightarrow 3$ is forbidden ($\mu_{23}=0$), the gain (4.1) in the resonant case $\bar{\omega}_{12}=0$ takes the form

$$G=g_{13}\frac{[qP(N_{21}-4T_e/T_2^{23})-N_{31}\zeta](\zeta+P)-\tilde{\Delta}_1^2\{N_{31}+qP[N_{21}+4T_e/(T_2^{23}\zeta)]}{1+4qPT/(\zeta T_2^{23})]|\mathcal{P}|^2} , \tag{4.2}$$

where $q=T_2^{12}/T_2^{13}$ and $\zeta=T_2^{13}/T_2^{23}$.

When dealing with amplification without population inversion between atomic levels, it is sensible to differentiate two situations. One is the absence of initial population inversion, i.e., before switching on the coherent pumping. Another is the absence of the population inversion in the steady state, i.e., under the action of the coherent pumping. When population inversion at the optical transitions is not realized initially, it cannot occur in steady state under the action of the microwave pumping. This follows directly from the steady-state solution (2.5). However, the opposite situation, when there was initial population inversion but it disappeared in steady state, is possible. In particular, it is realized when the transition $2\leftrightarrow 3$ is forbidden and all the pumping and relaxation processes are radiative. This situation was analyzed in [7,9]. Indeed using the detailed balance condition (i.e., introducing the transition probabilities in the form $w_{ik}=N_k/T_1^{ik}$), we can verify easily that the amplification condition found in [7,8] implies initial population inversion $N_{23}<0$. However, in general when all transitions are allowed or when there are nonradiative decay processes, the Maxwell-Bloch equations (2.3) and (2.4) allow for the possibility of amplification in the ab-

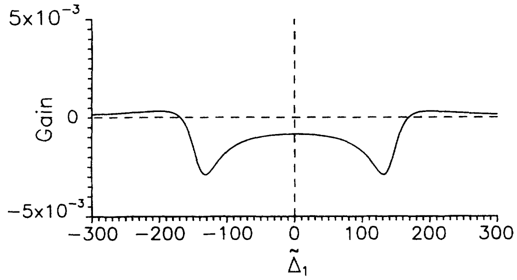

FIG. 3. The gain $G=-\text{Im}(k)$ defined by Eq. (4.2) in units of g_{13} vs detuning in the case $N_{21}<0$ corresponding to an amplification in the sidebands. The parameters are $N_1=0.45$, $N_2=0$, $N_3=0.55$, $g_1=0.01$, $g=0.45$, $g_2=0.001$, $\zeta=45.45$, $q=0.022$, and $P=20\,600$.

sence of the initial population inversion $N_{13}<0$ and $N_{23}<0$.

According to the expression (4.2) for the gain, it is possible to get this amplification either at line center if $N_{21}>0$ or at the sidebands if $N_{21}<0$. This is illustrated in Figs. 3 and 4 where we take $N_{21}=-0.45$ and 0.45, respectively; the other available parameters were chosen (i) to fulfill the inequality (4.5) and (ii) to maximize the gain in each case. In particular, for $\zeta=1$ (i.e., $T_2^{13}=T_2^{23}$), the condition for inversionless amplification in both cases can be expressed in the same form. Namely, it implies large enough population difference at the transition $1\leftrightarrow 2$:

$$q|N_{21}|>4T_e/T_2^{23}\ , \tag{4.3}$$

and strong enough pump

$$qP>N_{31}/(|N_{21}|-4T_e/T_2^{23})\ . \tag{4.4}$$

In the case $N_{21}>0$, the maximum gain occurs at line center while absorption takes place in the sidebands. On the contrary, if $N_{21}<0$, there is absorption at line center and gain in the sidebands. The maximum gain in the limit of a strong pump $\gamma\gg 1/T_2^{ik}$ is achieved when the detuning equals the Rabi frequency: $|\tilde{\Delta}_1|\simeq\gamma T_2^{13}$.

It is worth stressing that Maxwell-Bloch equations with the phenomenological incoherent pump and relaxation rates are valid as long as the Rabi frequency is small

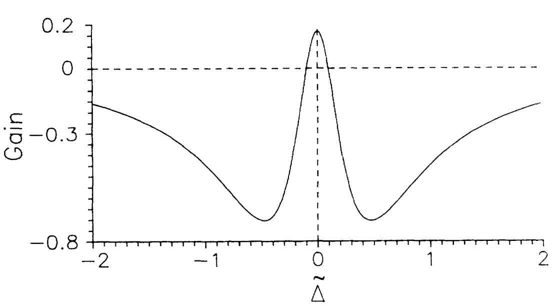

FIG. 4. The gain $G=-\text{Im}(k)$ defined by Eq. (4.2) in units of g_{13} vs detuning in the case $N_{21}>0$ corresponding to an amplification at line center. The parameters are $N_1=0$, $N_2=0.45$, $N_3=0.55$, $g_1=0.001$, $g_2=g=0.001$, $\zeta=0.002$, $q=1$, and $P=0.2$.

enough as compared to the frequency transition [15]: $\gamma\ll\omega_{12}$. Hence one has to be careful when considering the resonant case for a dc field because it implies level degeneracy ($\omega_{12}=0$). However, for the microwave field, the analysis of the resonant case is correct even in the limit $\gamma T_2^{ik}\gg 1$ provided $\omega_{12}T_2^{ik}\gg 1$. Thus inversionless amplification both at line center and in the sidebands can appear although the magnitude of the gain tends to zero when $\gamma T_2^{ik}\gg 1$ as seen from Eq. (4.2).

The amplification conditions at line center ($\tilde{\Delta}=0$) were analyzed previously [6]. It was shown that the necessary condition of amplification (4.3) is compatible, in a wide domain of the material parameters, with the inequalities for the relaxation rates in a three-level medium:

$$\frac{2}{T_2^{mk}}\geq\phi(m,j,k)=\frac{1}{T_1^{mk}}+N_j\left[\frac{1}{T_1^{jm}}+\frac{1}{T_1^{jk}}-\frac{1}{T_1^{mk}}\right] \tag{4.5}$$

and with the condition of no population inversion at both optical transitions: $N_{31}>0$, $N_{32}>0$. In particular, the most favorable limit is $2/T_2^{32}=\phi(m,j,k)$, $N_1=0$ and $T_1^{12}\ll T_1^{13}\ll T_1^{23}$. The amplification occurs here when the population of level 2 exceeds a very small critical value $N_2^*=(T_1^{12}/T_1^{13})^{1/2}\ll 1$.

In case it is the transition $1\leftrightarrow 3$ which is forbidden ($\mu_{13}=0$), a similar analysis applies. In this situation, inversionless amplification occurs at line center if $N_{12}>0$ and at the sidebands if $N_{12}<0$. Let us mention that the same mechanism of inversionless amplification appears also in the three-level configurations with lower-level splitting.

If we keep the two-photon resonance condition ($\tilde{\Delta}=\delta T_2^{13}$) but not the one-photon resonance condition (i.e., nonresonant Raman scattering), it follows from the analysis of Eq. (4.1) that the amplification condition takes the usual form of population inversion between the levels 2 and 3: $N_{23}>0$.

V. AMPLIFICATION CONDITION IN THE BASIS OF DRESSED STATES

To further clarify the role of the external field, let us rewrite the dynamical Eqs. (2.3a), (2.3b), and (2.4) in the basis of the states dressed by the external field, i.e., in the basis of states diagonalizing the Hamiltonian of the atomic subsystem (1,2):

$$\begin{bmatrix}\delta_1 & -\gamma\\ -\gamma^* & \delta_2\end{bmatrix}\tilde{\varphi}_j=\tilde{\delta}_j\tilde{\varphi}_j,\quad j=1,2\ . \tag{5.1}$$

The solution of the eigenvalue problem (5.1) is

$$\begin{aligned}\tilde{\varphi}_1&=\frac{1}{(\Theta^2+|\gamma|^2)^{1/2}}\begin{bmatrix}\Theta\\ -\gamma^*\end{bmatrix},\\ \tilde{\varphi}_2&=\frac{1}{(\Theta^2+|\gamma|^2)^{1/2}}\begin{bmatrix}\gamma\\ \Theta\end{bmatrix},\\ \tilde{\delta}_{1,2}&=[\delta_1+\delta_2\pm(\Omega_{12}^2+4|\gamma|^2)^{1/2}]/2\ ,\\ \Theta&=[\Omega_{12}+(\Omega_{12}^2+4|\gamma|^2)^{1/2}]/2\ .\end{aligned} \tag{5.2}$$

In this basis, Eqs. (2.3a), (2.3b), and (2.4) take the form

$$\frac{\partial\tilde{\sigma}_{13}}{\partial t}=-\tilde{\Gamma}_{11}\tilde{\sigma}_{13}+\Gamma_{12}\tilde{\sigma}_{23}+i\tilde{\alpha}(\tilde{n}_{31}-\tilde{\eta}\tilde{\sigma}_{12}) ,$$
$$\frac{\partial\tilde{\sigma}_{23}}{\partial t}=-\tilde{\Gamma}_{22}\tilde{\sigma}_{23}+\Gamma_{21}\tilde{\sigma}_{13}+i\tilde{\alpha}(\tilde{\eta}\tilde{n}_{32}-\tilde{\sigma}^*_{12}) , \quad (5.3)$$
$$\frac{\partial\tilde{\alpha}}{\partial z}+\frac{1}{c}\frac{\partial\tilde{\alpha}}{\partial t}=\frac{2\pi i\omega N|\tilde{\mu}_{13}|^2}{\hbar c}(\tilde{\sigma}_{13}+\tilde{\eta}^*\tilde{\sigma}_{23}) .$$

We have used the notations

$$\tilde{\sigma}_{ij}=\langle\tilde{\varphi}_i|\hat{\rho}|\varphi_j\rangle, \quad \tilde{n}_{ij}=\tilde{\sigma}_{ii}-\tilde{\sigma}_{jj}, \quad \varphi_3=\varphi_3 ,$$
$$\tilde{\mu}_{ij}=\langle\tilde{\varphi}_i|\hat{\mu}|\tilde{\varphi}_j\rangle, \quad \tilde{\alpha}=\tilde{\mu}_{13}E/2\hbar ,$$
$$1/\tau_2^{k3}=1/T_2^{k3}+i\delta_k, \quad k=1 \text{ or } 2 ,$$
$$\tilde{\eta}=\tilde{\mu}_{23}/\tilde{\mu}_{13}, \quad \Gamma_{11}=\frac{\Theta^2/\tau_2^{13}+|\gamma|^2/\tau_2^{23}}{\Theta^2+|\gamma|^2} ,$$
$$\Gamma_{22}=\frac{\Theta^2/\tau_2^{23}+|\gamma|^2/\tau_2^{13}}{\Theta^2+|\gamma|^2} ,$$
$$\Gamma_{12}=\frac{\gamma\Theta}{\Theta^2+|\gamma|^2}\left[\frac{1}{\tau_2^{23}}-\frac{1}{\tau_2^{13}}\right], \quad \Gamma_{21}=(\Gamma_{12})^* .$$

One can see that this transformation eliminates the terms which were proportional γ in Eqs. (2.3a) and (2.3b). The dynamical (or Hamiltonian) part of Eqs. (5.3) for the P scheme in the new basis is similar to the dynamical part of Eqs. (3.1) and (2.4) for the F scheme in the old basis. The difference between these two sets of equations is due to relaxation terms only. In particular, if $\tau_2^{13}=\tau_2^{23}=T_2$, we have $\Gamma_{11}=\Gamma_{22}=1/T_2$, $\Gamma_{12}=\Gamma_{21}=0$, and Eqs. (5.3) in a new basis coincide with Eqs. (3.1) and (2.4) in the old basis. Hence if we substitute in the amplification condition (3.3) for the F scheme the variables $\tilde{\sigma}_{ij}$, $\tilde{n}_{ij}$, $\tilde{\mu}_{ij}$ for the variables σ_{ij}, n_{ij}, μ_{ij}, and express them in the old basis, we obtain the amplification condition for the P scheme in the form $G>0$, where G is defined by (4.1). This means that there is only one mechanism of inversionless amplification in the P scheme (with microwave or dc pump). This mechanism appears already in the simplest F scheme where a monochromatic field interacts simultaneously and therefore nonresonantly with two nondegenerate optical transitions. As shown above, in the P scheme (in contradistinction with the F scheme), inversionless amplification occurs even in the case $\Omega_{12}=0$ and even if one transition is forbidden. This is due to the fact that the microwave field not only creates the atomic coherence σ_{12} but also splits and mixes the upper atomic states. In fact, if $\Omega_{12}=0$ and $\mu_{23}=0$, we have in the dressed-state basis (5.2): $\tilde{\mu}_{13}=\tilde{\mu}_{23}=\mu_{13}/\sqrt{2}$ and $\tilde{\omega}_{12}=2\gamma$. Hence in the effective F scheme, there is a splitting of the upper level and interaction of the monochromatic field with both optical transitions simultaneously.

Thus the P scheme in the dressed-state representation is reduced to the effective F scheme. It is not difficult to calculate the population differences between the dressed states and the lower level under resonant pumping ($\Omega_{12}=0$, i.e., $\omega_p=\omega_{12}$):

$$\tilde{n}_{31}=\tilde{n}_{32}=(N_{31}+4\gamma^2T_eT_2^{12}-N_{12}/2)/(1+4\gamma^2TT_2^{12}) , \quad (5.4)$$

and to verify that population inversion in this basis never arises ($\tilde{n}_{31}=\tilde{n}_{32}>0$) if it did not exist initially between the atomic states ($N_{31},N_{32}>0$). Moreover it is also obvious that it cannot appear in the dressed states ($\tilde{\rho}_{11}+\tilde{\rho}_{22}<\tilde{\rho}_{33}$) if it is absent in steady state between the atomic levels ($\rho_{11}+\rho_{22}<\rho_{33}$) because the unitary transformation (5.2) does not change the trace of the matrix. Thus from (5.3) it follows that it is low-frequency coherence between the dressed states $\tilde{\sigma}_{12}$ which is responsible for the inversionless amplification. The principal role of the low-frequency coherence between the dressed states in this mechanism has been stressed recently in the cases of amplification without inversion both on the initial- and on the steady-state population [8] and also in the steady-state population only [9]. At the resonant pumping ($\omega_p=\omega_{12}$) the low-frequency coherence in the steady state is expressed by

$$\tilde{\sigma}_{12}=N_{21}(2i\gamma T_2^{12}-1)/(2+8\gamma^2TT_2^{12}) . \quad (5.5)$$

It tends to zero when $\gamma T_2^{ik}\gg 1$ and hence inversionless amplification vanishes in this limit as already mentioned. This result is in full agreement with the fact that it is in the dressed-state basis that the diagonal distribution has to be established in steady state, at least in the framework of the traditional rotating-wave, Born-Markoff, and secular approximations. The last assumption implies strong enough coherent pumping [20]: $\gamma T_2^{ik}\gg 1$. Thus this mechanism can appear only beyond these approximations.

This is quite different from the mechanism based on the coherent population trapping, where the necessary and sufficient condition for amplification when $\gamma T_2^{ik}\gg 1$ takes the form of population inversion between dressed states [21].

Recently, the possibility of inversionless amplification was demonstrated in a more complex scheme involving two autoionizing states decaying to an identical continuum and a monochromatic field interacting with both optical transitions [16–18]. The comparison of density matrix and wave equations [17,19] for this situation with the corresponding Eqs. (2.3) and (2.4) for the P scheme shows that the decaying into the identical continuum is described by similar terms and hence plays a similar role as the pumping in the P scheme. Thus the physical mechanism of inversionless amplification in these more complex schemes is essentially the same as in the simpler P and F schemes.

VI. CONCLUSIONS

The main results of this paper are the following.

(1) A monochromatic field can be amplified without population inversion in a three-level medium of the F configuration (Fig. 2) if it interacts simultaneously with both optical transitions and the atomic coherence σ_{12} between the two upper sublevels is excited by the external

source. The condition of amplification can be less stringent than for a bichromatic field, where each component interacts with its own transition. Furthermore, amplification does not imply population inversion in the basis where the atomic coherence vanishes.

(2) The P scheme where two nearby sublevels are coupled by a dc field (or by a microwave field) is equivalent to the F scheme in the dressed-state basis. The coupling field plays three major roles in the inversionless amplification process: (a) It induces atomic coherence between dressed states. (b) It splits the upper atomic states and hence reduces adsorption for the resonant optical field. (c) It mixes the upper atomic sublevels and therefore leads to the effective interaction of the monochromatic field with both dressed states. Because of this effect, inversionless amplification becomes possible even when one optical transition is forbidden.

(3) Amplification without inversion for the P scheme occurs at line center or at the sidebands, depending on the sign of the sublevel population difference.

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