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Possible interference effect in the Stern-Gerlach phenomenon

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We show that if it is possible to manufacture a beam of spin-$\frac{1}{2}$ heavy atoms corresponding to a quantum-mechanical pure state, then the dark lines of the Stern-Gerlach effect should display a fine structure. The two lips of the open-mouth pattern will each be a doublet; the lighter area between the two lines of the doublet arises from an interference between silver atoms on opposite sides of the incoming beam. We obtain a simple classical model giving exactly the same angular distribution around the open mouth. However, this classical model does not give the fine structure.

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I. INTRODUCTION

The Stern-Gerlach phenomenon is what occurs when a beam of spin-$\frac{1}{2}$ atoms (preferably heavy ones; the original experiment was with silver) pass through a transverse inhomogeneous magnetic field. A plate, placed some distance beyond the exit from the magnet, shows a distribution of atoms resembling the pair of lips of a half-open mouth.

The phenomenon is important from the point of view of the quantum theory of measurement for two reasons. The first, which we intend to discuss in a subsequent article, arises from the proposition that the two lips of the distribution contain atoms whose spins are polarized either "up" or "down" with respect to the principal transverse field component. According to such a proposition, it may be possible to design the beam and the magnet so that the two lips become completely separated. We would then have a perfect "spin meter," a device which would enable us to demonstrate [1], possibly in the most convincing manner so far, the property known as quantum nonlocality. The second, which we discuss in this article, is that if the properties of the deflected beam are correctly described by the Pauli-Schrödinger wave equation, then the phases of atoms in different parts of the beam should be correlated, thus giving us a particularly striking illustration of wave interference for such a large composite object as the silver atom.

Bohm [2] argues that, even with the Pauli-Schrödinger description, the interaction with the magnetic field is essentially a "measurement" which destroys the phase coherence. This is because the magnetic field is not deterministic; it is an interaction between the silver atom and the very large array of atoms constituting the magnet, and the average field of the latter is necessarily accompanied by uncontrollable fluctuations. To test his hypothesis Bohm suggested that the beam be passed through a second magnet which is a mirror image of the first. Then on the assumption of a "deterministic" Pauli-Schrödinger evolution (that is one which converts an incident pure state into a final pure state through a unitary transformation), the initial state should be perfectly reproduced after the interaction with the two magnets. For example, if all the incident silver atoms have their spins in the direction of the beam then so do the atoms emerging from the second magnet. Wigner [3], and Englert, Schwinger, and Scully [4] have argued that, in principle, such a process does occur; the latter authors have called it "putting Humpty Dumpty (who is a broken egg) together again." On the Bohm hypothesis the emerging atoms would be completely unpolarized; the pure state evolves into a mixture.

The latter authors also find that the degree of exactness with which the second magnet needs to duplicate the first is a technological impossibility, so it seems rather unlikely that anyone will attempt the miraculous reconstruction of Humpty Dumpty by this method. However, we shall show in the present article that, at least for an inhomogeneous field with a certain geometry, an interference effect is already exhibited, according to the Paul-Schrödinger theory, by the fine structure of the distribution after the first magnet.

II. THE MODEL HAMILTONIAN

We base our study on the magnetic field

$$\mathbf{B}(x,y,x)=(-B'x,0,B_0+B'z) \quad (0<y<Y)_, \tag{2.1}$$

where the beam of silver atoms is traveling along the y axis, and the field is zero for values of y outside the interval $(0, Y)$. We remark that this is the simplest inhomogeneous field which satisfies the (static) Maxwell equations

$$\mathrm{div}\mathbf{B}=0, \quad \mathrm{curl}\mathbf{B}=\mathbf{0} , \tag{2.2}$$

(except of course at the transition points $y=0, Y$; we postpone discussion of these to a later article), and that, since we may move the origin to the point $(0,0,-B_0/B')$, there is no loss of generality in putting $B_0=0$. Then the Pauli-Schrödinger equation is

$$i\hbar\dot{\psi}=(\mathcal{H}_0+\mathcal{H}_1)\psi , \tag{2.3}$$

where $\psi(x,z)$ is a two-component spinor and

$$\mathcal{H}_0=\begin{bmatrix} -\mu B'z & \mu B'x \\ \mu B'x & \mu B'z \end{bmatrix} , \tag{2.4}$$

$$\mathcal{H}_1=-\frac{\hbar^2}{2M}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right] , \tag{2.5}$$

μ being the magnetic moment and M the mass of the silver atom. We have assumed that the initial state is an eigenstate of the momentum p_y, and so the y-dependent part of the wave function has been factored out. Now, defining

$$\Delta^2=\int \psi^\dagger(x,z,t)\psi(x,z,t)(x^2+z^2)dx\,dz , \tag{2.6}$$

we rescale Eq. (2.3) by expressing x and z in units of Δ, t in units of $(\hbar/\mu B'\Delta)$, and M in units of $(\hbar^2/\mu B'\Delta^3)$. This then gives

$$i\dot{\psi}=(H_0+H_1)\psi , \tag{2.7}$$

where

$$H_0=\begin{bmatrix} -z & x \\ x & z \end{bmatrix} \tag{2.8}$$

and

$$H_1=-\frac{1}{2M}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right] . \tag{2.9}$$

We shall confine ourselves, in the present article, to considering the limiting case $M\to\infty$, so that $H\simeq H_0$. This approximation is equivalent to assuming that the silver atoms spend a sufficiently short time in the magnetic field for us to take account of changes in their momentum while neglecting the consequent changes in position. It may be called a *nonconvective approximation* and has been made many times before. For example, Bohm [2] and, more recently, Scully, Lamb, and Barut [5] (who also considered a magnetic field similar to ours) made this approximation in a quantum context, while Singh and Sharma [6] made it in an attempted classical treatment.

III. THE PAULI-SCHRÖDINGER EVOLUTION

We thus consider the Pauli-Schrödinger equation

$$i\dot{\psi}=H_0\psi . \tag{3.1}$$

We shall work in polar coordinates and so write

$$H_0=\begin{bmatrix} -r\cos\theta & r\sin\theta \\ r\sin\theta & r\cos\theta \end{bmatrix} . \tag{3.2}$$

Let us write the state at $t=0$ as

$$\psi(r,\theta,0)=\psi_+(r,\theta)\begin{bmatrix} \cos(\theta/2) \\ -\sin(\theta/2) \end{bmatrix} + \psi_-(r,\theta)\begin{bmatrix} \sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix} , \tag{3.3}$$

so that ψ_+ (ψ_-) may be regarded as the component of ψ with spin parallel (antiparallel) to the local field, whose direction is $(-\sin\theta,0,\cos\theta)$. Then the solution of (3.1) is

$$\psi(r,\theta,t)=\psi_+(r,\theta)e^{irt}\begin{bmatrix} \cos(\theta/2) \\ -\sin(\theta/2) \end{bmatrix} + \psi_-(r,\theta)e^{-irt}\begin{bmatrix} \sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix} . \tag{3.4}$$

If the plate is placed a large distance away from the magnet, it is legitimate to regard the beam issuing from the magnet as a point source, so that the trace on the plate is a picture of the momentum ($\mathbf{p}$) distribution in the outgoing beams, that is

$$W(R,\Theta,t)=\bar{\psi}^\dagger(R,\Theta,t)\bar{\psi}(R,\Theta,t) , \quad (p_x=R\sin\Theta, p_z=R\cos\Theta) , \tag{3.5}$$

where $\bar{\psi}$ is the two-dimensional Fourier transform of ψ, that is

$$\bar{\psi}(R,\Theta,t)=\frac{1}{2\pi}\int_0^\infty r\,dr\int_0^{2\pi}d\theta\,\psi(r,\theta,t)e^{-iRr\cos(\Theta-\theta)} , \tag{3.6}$$

and t, which should be put equal to MY/p_y, is the transit time of the silver atoms through the magnetic field.

Now we anticipate, and will shortly confirm, that, for large values of t, the distribution W will be zero except for large R. We therefore use the stationary-phase asymptotic approximation [7] for the phase factor in this latter equation, that is

$$e^{-iRr\cos(\Theta-\theta)}\sim\left[\frac{2M}{Rr}\right]^{1/2}[e^{i(\pi/4)-iRr}\delta(\theta-\Theta) + e^{-(i\pi/4)+iRr}\delta(\theta-\Theta-\pi)] . \tag{3.7}$$

Put $\xi=R-t$ and $\eta=R+t$. Then for large R and t we may retain just the terms in $e^{\pm i\xi r}$ giving

$$\bar{\psi}(R,\Theta,t)\sim(2\pi R)^{-1/2}\begin{bmatrix}\cos(\Theta/2)\\-\sin(\Theta/2)\end{bmatrix}\times\int_0^\infty r^{1/2}[e^{-ir\xi+i(\pi/4)}\psi_+(r,\Theta)+e^{ir\xi-i(\pi/4)}\psi_-(r,\Theta+\pi)]dr \tag{3.8}$$

$$=R^{-1/2}\begin{bmatrix}\cos(\Theta/2)\\-\sin(\Theta/2)\end{bmatrix}[\bar{\phi}_+(\xi,\Theta)+\bar{\phi}_-(\xi,\Theta)]\ , \tag{3.9}$$

where

$$\bar{\phi}_\pm(\xi,\Theta)=(2\pi)^{-1/2}\int_{-\infty}^{\infty}\phi_\pm(r,\Theta)e^{-i\xi r}dr\ , \tag{3.10}$$

and

$$\phi_+(r,\Theta)=\begin{cases}r^{1/2}\psi_+(r,\Theta)e^{i(\pi/4)} & (r>0)\\ 0 & (r\le 0)\ ,\end{cases}$$
$$\phi_-(r,\Theta)=\begin{cases}0 & (r\ge 0)\\ (-r)^{1/2}\psi_-(-r,\Theta+\pi)e^{-i(\pi/4)} & (r<0)\ .\end{cases} \tag{3.11}$$

The form of (3.8) confirms what we anticipated; given that ψ is zero for $r\gg 1$ (or $r\gg\Delta$ in the original units) then $\bar{\psi}$ is small for $|R-t|\gg 1$ (or $|R-\mu B't|\gg\hbar/\Delta$ in the original units). So, crudely speaking, the distribution on the plate is confined to a region close to the circle $R=t$ (the half-open mouth is now fully open). This is, of course, all in the asymptotic regime $t\gg 1$. In this same regime there is a perfect correlation between spin and momentum; the spin pattern around the circle $R=t$ is shown in Fig. 1.

The fine structure in (3.8) is exhibited by the interference between $\bar{\phi}_+$ and $\bar{\phi}_-$, and we shall discuss it in Sec. V. But first we propose to examine the coarse structure, that is the distribution around the circle $R=t$. To this end let us form the integral

$$h(\Theta)=\int_0^\infty W(R,\Theta,t)R\ dR\ . \tag{3.12}$$

Formally we expect h to be a function of t, but it is asymptotically independent of t since, from (3.8), it may be written

$$h(\Theta)=\int_{-t}^{\infty}d\xi|\bar{\phi}_+(\xi,\Theta)+\bar{\phi}_-(\xi,\Theta)|^2 \sim\int_{-\infty}^{\infty}d\xi|\bar{\phi}_+(\xi,\Theta)+\bar{\phi}_-(\xi,\Theta)|^2\ . \tag{3.13}$$

By Parseval's theorem, therefore,

$$h(\Theta)\sim\int_{-\infty}^{\infty}dr|\phi_+(r,\Theta)+\phi_-(r,\Theta)|^2 =\int_0^\infty r\,dr[|\psi_+(r,\Theta)|^2+|\psi_-(r,\Theta+\pi)|^2]\ , \tag{3.14}$$

since the cross term is zero.

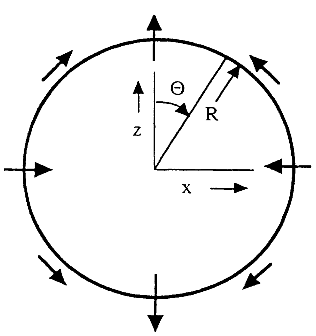

FIG. 1. The pattern of spin orientations in a beam of silver atoms deflected by the magnetic field (2.1).

IV. A CLASSICAL MODEL

In spite of the title which Stern gave to his original paper [8] on the Stern-Gerlach phenomenon ("Experimental demonstration of directional quantization in a magnetic field") his discussion was really classical, as has been pointed out in the more recent literature [9]. However, this more recent literature has established that Stern's classical model, at least for the values of B_0 and B' used in the Stern-Gerlach experiment [10] and its modern refinements [11], does not give a satisfactory explanation of the phenomenon.

We now propose a model, which is as classical in spirit as Stern's, but which reproduces the coarse-structure distribution $h(\Theta)$ of (3.13), and also the associated spin distribution of Fig. 1. We suppose that, more or less immediately on entering the magnetic field, a silver atom has position $\mathbf{x}$, momentum $\mathbf{p}$ and spin parallel [antiparallel] to the local magnetic field with probability density $W_+(\mathbf{x},\mathbf{p})$ [$W_-(\mathbf{x},\mathbf{p})$]. To establish the correspondence with the model in Sec. III we shall suppose that

$$\int W_\pm(\mathbf{x},\mathbf{p})|d^2\mathbf{p}=|\psi_\pm(\mathbf{x})|^2\ . \tag{4.1}$$

This means that a kind of "collapse" occurs to the silver atoms as they enter the magnetic field, their spins aligning either parallel or antiparallel to the local field. As a consequence of this alignment, no subsequent precession of the spins occurs inside the magnetic field, and the momentum change is determined by the standard classical magnetic force $(\boldsymbol{\mu}\cdot\nabla)\mathbf{B}$.

The magnetic force acting on a parallel (antiparallel) aligned atom is of a constant unit magnitude (in the units we are using) and in the direction $\hat{\mathbf{x}}(-\hat{\mathbf{x}})$, that is the unit vector in the outward (inward) radial direction. Hence the momentum distribution at time t is

$$W_{\rm cl}(\mathbf{p},t)=\int[W_+(\mathbf{x},\mathbf{p}-\hat{\mathbf{x}}t)+W_-(\mathbf{x},\mathbf{p}+\hat{\mathbf{x}}t)]d^2x \tag{4.2}$$

$$\sim\int[W_+(r\hat{\mathbf{p}},\mathbf{p}-\hat{\mathbf{x}}t)+W_-(-r\hat{\mathbf{p}},\mathbf{p}-\hat{\mathbf{x}}t)]d^2\mathbf{x} \quad \text{as } t\to\infty\ . \tag{4.3}$$

This last step follows because, for large t, the argument $\mathbf{p}-\hat{\mathbf{x}}t(\mathbf{p}+\hat{\mathbf{x}}t)$ has large modulus, and therefore negligible probability, unless $\hat{\mathbf{x}}=\hat{\mathbf{p}}$ (or $-\hat{\mathbf{p}}$). Hence

$$\int_0^\infty W_{\rm cl}(\mathbf{p},t)R\,dR \sim \int_0^\infty r\,dr\int_0^\infty R\,dR\int_0^{2\pi}d\theta[W_+(r\hat{\mathbf{p}},\mathbf{p}-\hat{\mathbf{x}}t) + W_-(-r\hat{\mathbf{p}},\mathbf{p}-\hat{\mathbf{x}}t)] \; . \tag{4.4}$$

Now

$$\int_0^\infty R\,dR\int_0^{2\pi}d\theta\, f(\mathbf{p}-\hat{\mathbf{x}}t)=\int_0^\infty RJ\,dR'\int_0^{2\pi}d\Theta' f(\mathbf{p}') \; , \tag{4.5}$$

where J is the Jacobian for the change of coordinates $\mathbf{p}'=\mathbf{p}-\hat{\mathbf{x}}t$, for which we find

$$J=R't^{-1}|\sec(\theta-\Theta)| \; . \tag{4.6}$$

But, again for large t, the integrand is small except for R close to t and θ close to Θ. Hence $J\sim R'/R$ and

$$\int_0^\infty W_{\rm cl}(\mathbf{p},t)R\,dR \sim \int_0^\infty r\,dr\int [W_+(r\hat{\mathbf{p}},\mathbf{p}')+W_-(-r\hat{\mathbf{p}},\mathbf{p}')]d^2\mathbf{p}' \; . \tag{4.7}$$

With the identification made in (4.1), this is just Eq. (3.13).

Now as to the organization of the spin distribution, leading to Fig. 1, this is clearly obtained from the latter equation, because the antiparallel spin direction at $-r\hat{\mathbf{p}}$ is the same as the parallel direction at $r\hat{\mathbf{p}}$. The corresponding process in the quantum evolution is more complex; it is explained as a phase cancellation of those components of ψ having the "wrong" spin direction. We conclude that there is nothing really wavelike in the coarse structure of (3.13) nor in the spin pattern of Fig. 1; any wavelike properties which silver atoms may have must be sought in the fine structure of (3.8).

V. THE FINE STRUCTURE

To see an example of this structure, we consider the special case of a minimal Gaussian state with spin aligned in the direction ($\sin\theta_0\cos\phi_0$, $\sin\theta_0$, $\sin\phi_0$, $\cos\theta_0$). The wave function for this state is

$$\psi(r,\theta,0)=\pi^{-1/2}e^{-(1/2)r^2}\begin{bmatrix}\cos(\theta_0/2)\\ \sin(\theta_0/2)e^{i\phi_0}\end{bmatrix} , \tag{5.1}$$

for which

$$\psi_+(r,\theta)=\psi_-(r,\theta+\pi) =\pi^{-1/2}e^{-(1/2)r^2} \times\left[\cos\left[\frac{\theta}{2}\right]\cos\left[\frac{\theta_0}{2}\right] - \sin\left[\frac{\theta}{2}\right]\sin\left[\frac{\theta_0}{2}\right]e^{i\phi_0}\right] \tag{5.2}$$

Then, using the integral

$$\int_0^\infty r^{1/2}e^{-(1/2)r^2}\cos\left[r\xi-\frac{\pi}{4}\right]dr =\left[\frac{\pi}{2}\right]^{1/2}D_{1/2}(\xi)e^{-(1/4)\xi^2} , \tag{5.3}$$

where $D_{1/2}$ is the parabolic cylinder function, (3.8) gives

$$W(R,\Theta,t)\sim(2\pi R)^{-1}[1+\cos\Phi(\Theta)] \times[D_{1/2}(\xi)]^2e^{-(1/2)\xi^2} , \tag{5.4}$$

where, as before, $\xi=R-t$, and

$$\cos\Phi(\Theta)=\cos\Theta\cos\theta_0-\sin\Theta\sin\theta_0\cos\phi_0 , \tag{5.5}$$

so that $\Phi(\Theta)$ is the angle between the incident spin direction and the local field direction ($-\sin\theta$,0, $\cos\theta$) for $\theta=\Theta$.

By way of comparison, we consider also a classical model. It is by no means unique; the only constraint on such models is that they should satisfy Eq. (4.1). However, one model which does this very naturally is

$$W_\pm(\mathbf{x},\mathbf{p})=\frac{1}{2\pi^2}[1\pm\cos\Phi(\theta)]e^{-r^2-R^2} . \tag{5.6}$$

This is just the Wigner distribution [12] for a spinless, Gaussian packet multiplied by the standard "collapse factor" of $\frac{1}{2}(1\pm\cos\Phi)$, for which there is a well-known classical model [13]. Substituted into (4.2), this gives

$$W_{\rm cl}(\mathbf{p},t)=\frac{1}{\pi}\exp(-R^2-t^2)[I_0(2Rt) +I_1(2Rt)\cos\Phi(\Theta)] , \tag{5.7}$$

where I_n is the Bessel function of imaginary argument. This has the asymptotic behavior, as $t\to\infty$, of

$$W_{\rm cl}(\mathbf{p},t)\sim\frac{1}{2\pi R\sqrt{\pi}}e^{-\xi^2}[1+\cos\Phi(\Theta)] . \tag{5.8}$$

A comparison of (5.4) and (5.8) shows that both of them, in agreement with the result of Sec. IV, have the form

$$W(\mathbf{p},t)=R^{-1}h(\Theta)f(\xi) , \tag{5.9}$$

where $h(\Theta)$ gives the same coarse structure for both models, that is

$$h(\Theta)=(2\pi)^{-1}[1+\cos\Phi(\Theta)] . \tag{5.10}$$

However, the fine structure, given by $f(\xi)$, is different for the quantum (Q) and classical (cl) cases, that is

$$f_Q(\xi)=[D_{1/2}(\xi)]^2e^{-(1/2)\xi^2} , \tag{5.11}$$

$$f_{\rm cl}(\xi)=\pi^{-1/2}e^{-\xi^2} . \tag{5.12}$$

In Fig. 2 we plot these two "line shapes." The striking difference is that, because $D_{1/2}(\xi)$ has a zero at $\xi\simeq-0.76$, there are, according to the quantum theory, two concentric dark circles with a lighter area between them.

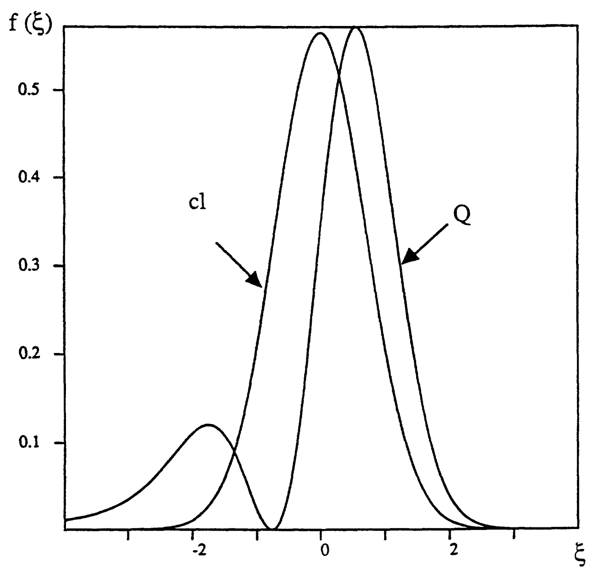

FIG. 2. The fine structure of the circle depicted in Fig. 1, according to the quantum model (Q) of Sec. III and the classical model (cl) of Sec. IV.

It would not, of course, really be possible to observe this asymptotic line shape, even if we were able to manufacture the necessary form of the magnetic field and the minimal Gaussian packet of silver atoms. The neglect of the convection part H_1 of the Hamiltonian, for sufficiently large B', can be justified for a certain range of t, but certainly not in the limit $t \rightarrow \infty$. However, we have obtained a complete asymptotic expansion for $\bar{\psi}$, valid for the region $R - t \ll t$, and also a matched expansion valid for the region $R < t(1-\delta)$, the details of which we shall publish elsewhere. With these it is possible to obtain accurate values of $W(R,\Theta,t)$ for quite moderate values of t. In Fig. 3 we have made a computer simulation of the intensity distribution of silver atoms at the plate for the case $t=10$, $\theta_0=0$, with the corresponding classical distribution, given by (5.7), displayed for comparison. The fringe pattern is already quite evident for this value of t.

So is this just another *gedanken* experiment, like the double Stern-Gerlach referred to in Sec. I? In a sequel [14] to Ref. [4], the same authors warn us that "overidealized models are not good guides to suggesting experiments." We accept this warning and therefore hope that the "possible interference effect" of the present article will not be described as a "phenomenon." However, we do think that a more accurate single Stern-Gerlach experiment has a better chance of being realized than the double Stern-Gerlach one, to which a considerable literature has been devoted. It seems to us that the biggest challenge lies in the manufacture of a minimal, or near-minimal, packet of silver atoms.

The case $B_0 \neq 0$ may be treated by simply shifting the origin to the point $(0,0-a)$, where $a = B_0/B'\Delta$. This means that, in the initial wave function, the factor $\exp(-\frac{1}{2}r^2)$ is replaced by $\exp(-\frac{1}{2}r^2 - \frac{1}{2}a^2 + ar\cos\theta)$, with the result that Eq. (5.4) is replaced by

$$W(R,\Theta,t) \sim (2\pi R)^{-1}[1+\cos\Phi(\Theta)]|D_{1/2}(\xi + ia\cos\Theta)|^2 \times \exp(-\tfrac{1}{2}\xi^2 + \tfrac{1}{2}a^2\cos^2\Theta - a^2) \; . \tag{5.13}$$

We propose to discuss this distribution in a subsequent article, which will be devoted to the question of whether it is possible for a Stern-Gerlach apparatus to completely separate the "spin-up" and "spin-down" components. As far as the interference is concerned, we note that the replacement of a real by a complex argument in the parabolic cylinder actually decreases the contrast between the light and dark fringes. The interference effect we predict is thus best seen for the case $B_0 = 0$. Nevertheless, for $\Theta = \pi/2$, the fringe visibility, even with $B_0 \neq 0$, is still 100%. We remark that it seems to be the case with experimental realizations [10,11] of the Stern-Gerlach

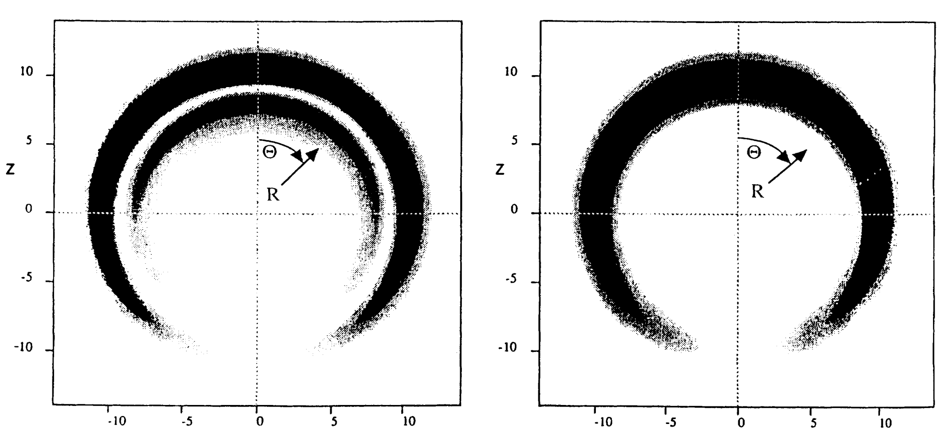

FIG. 3. Computer simulated "photoplate" of the distribution of deflected silver atoms in an ideal Stern-Gerlach experiment, according to the quantum (left-hand plate) and classical (right-hand plate) models. It has been supposed that the incident beam has been prepared in a pure "spin-up" state, so this figure corresponds to one of the two separate Stern-Gerlach lines which arise in the (fictitious) case of a one-dimensional magnetic field.

phenomenon that the parameter a is of order 1; the total magnetic field must, of necessity, vary significantly over the cross section of the packet. This is another point we propose to treat in our subsequent article. For the effect we predict here, the implication is that any magnetic field which is capable of producing a Stern-Gerlach phenomenon also produces the interference effect.

The achievement of a magnetic field with the above property, combined with a near-minimal packet of silver atoms, is likely to be a severe technological challenge. But, by way of compensation, Fig. 3 indicates that the field intensity itself does not need to be very big; if $B'\Delta$ is of the order of B_0, then $t=10$ corresponds, in ordinary units, to $\omega t \simeq 10$, where $\omega = eB_0/2mc$, m being the electron mass. In standard Stern-Gerlach experiments, using "thermal" packets of atoms, ωt is [9] of the order of 10^8.

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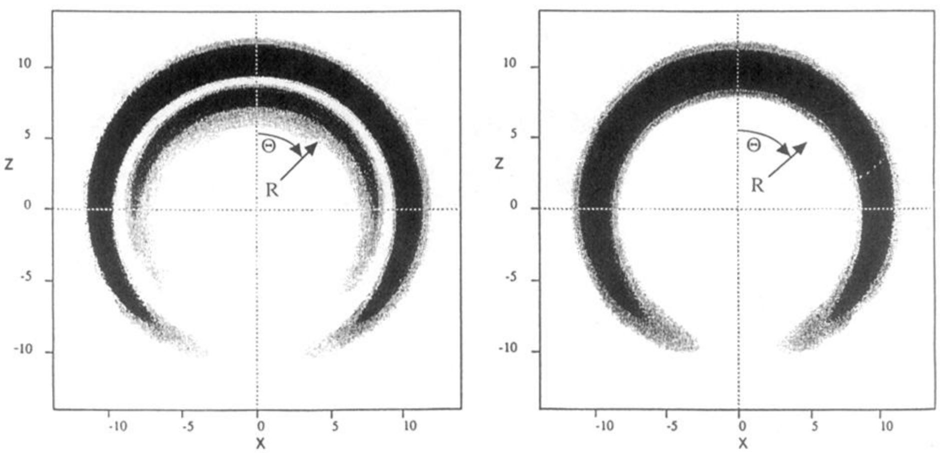

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