PHYSICAL REVIEW A VOLUME 46, NUMBER 5 1 SEPTEMBER 1992

Phase-invariant clock hypothesis for accelerating systems

F. V. Kowalski
Department of Physics, Colorado School of Mines, Golden, Colorado 80401
(Received 30 March 1992)

We study spatially separated clocks that are Einstein synchronized in one inertial frame and then rigidly accelerated into another inertial frame. It is assumed that the clocks remain Einstein synchronized in the new frame in which they are now at rest. The consequences of this assumption are analyzed with respect to the experiments discussed in the literature as supportive of the accepted clock hypothesis.

PACS number(s): 03.30.+p, 04.20.Cv

The predicted frequency shift between two clocks rigidly separated and linearly accelerating in flat space-time has been important in formulating the current theory of gravity. However, this result has not been experimentally verified. The standard opinion is that such a verification would be of pedagogical interest but not of high scientific priority. This reflects the overwhelming agreement on the existence of this shift. There has, however, been some dissent [1–3].

There have been predominantly two arguments used to justify this shift. One employs the longitudinal Doppler shift of light propagating between the two accelerating clocks [4] while the other compares the accelerating clocks via a third clock which is in an inertial frame [5]. Having accepted these arguments, a metric different from that of flat space-time is then necessary to describe this frequency shift in the coordinate system of the accelerating observer [6].

Before proceeding it is useful to consider the assumptions which lead to the above result. These assumptions augment the postulates of special relativity to allow a treatment of accelerating systems in flat space-time. The standard assumptions are that the speed of light is independent of the acceleration of the source and that acceleration in itself affects neither the rate of a standard clock nor the length of a standard rod. The accepted interpretation of the second assumption, hereafter referred to as the accepted clock hypothesis (ACH), is that the rate of an accelerated clock is equal to that of a comoving unaccelerated clock [7].

These assumptions, along with the equivalence principle, form the basis for the accepted theory of gravity. Since this theory has been successful in describing gravity, there has been little challenge to the assumptions dealing with accelerating systems in flat space-time.

While not challenging these assumptions, Desloge [8] claims that measurements made in a uniformly accelerating frame in field-free space will not give the same results as those in a frame at rest in a uniform gravitational field. Mashhoon [9], also accepting these assumptions, argues that the determination of frequency during acceleration is not a local process.

In this work we embrace the postulates of special relativity and the first assumption but revise the second as follows. Consider spatially separated clocks which were Einstein synchronized [10] (hereafter referred to as synchronized) in an inertial frame S and now accelerate rigidly. We assume that when the acceleration vanishes these clocks are still synchronized in the new inertial frame S' in which the clocks are now at rest. That is, the clocks are still in phase. ACH, on the other hand, predicts that these clocks are no longer synchronized in the frame S' in which they are now at rest [11].

It is interesting to note that observers in both frames know which frame is moving relative to the other. That is, there is a preferred frame, the one which does not experience acceleration. Under the assumptions of ACH this distinction does not need to be made during the accelerated part of the motion. An observer in frame S', long after the acceleration terminated, need only note that her clocks are no longer synchronized to determine that her frame accelerated relative to frame S.

In the interest of simplicity we study only rigid motion of the clocks. However, we can generalize the new hypothesis for arbitrary motion as follows: spatially separated clocks, synchronized in one inertial frame, maintain their synchronization in another inertial frame independent of the motion of the clocks as long as their proper separation remains the same when they come to rest with respect to each other in any other inertial frame.

To better understand the argument, consider spatially separated clocks synchronized in an inertial frame S. All the clocks have period T seconds per tick or frequency T^{-1} ticks per second. Two clocks, separated by a distance L, are then accelerated into another inertial frame S', which moves at speed v with respect to frame S after the acceleration has terminated. The motion of the clocks maintains their proper distance [12], L.

In addition, we assume that the clock velocities are much less than the speed of light c and carry all calculations only to second order in v/c. A calculation to higher order in v/c, assuming ACH, is given in Ref. [8]. Also, the motion described is confined to one spatial dimension and we assume that the emission and absorption process is a local phenomenon.

Before applying our hypothesis let us consider a concrete example. Assume the above two clocks to be a source and receiver which resonantly interact in the inertial frame S, an example of which is found in the Mössbauer effect. When in frame S' they again resonantly interact. Do they resonantly interact when they are rigidly accelerating? The answer will depend upon which clock hypotheses is used. Therefore it is to this question that the bulk of our discussion is devoted.

The answer to this question is also independent of the frame in which the calculation is done. It is simply a matter of counting both wavecrests and clock cycles at the same space-time point. That is, the absorption process in frame independent. In what follows, we choose the inertial frame S in which to calculate the interaction of the Mössbauer source and receiver as they accelerate.

Let us now return to the example where the two clocks move and assume that they remain synchronized in the inertial frame in which they now are at rest. The observer in S, however, concludes that the trailing clock leads by a time $\tau = Lv/c^2$. That is, two clocks synchronized in the inertial frame in which they are at rest appear unsynchronized in any other inertial frame. The trailing clock, as seen in frame S, has therefore registered τ/T more ticks than the leading clock.

These two clocks are now made to move at a new velocity $v+\Delta v$ in a time Δt, as seen in S. In order to maintain the synchronization of the clocks in the inertial frame in which they are at rest, the observer in S claims that the trailing clock has advanced a number of ticks ΔN, given by

$$\Delta N \simeq \frac{\Delta\tau}{T} \simeq \frac{L\Delta v}{Tc^2} \ .$$

During the change in velocity, the frequency of the trailing clock must therefore have increased by an amount $\Delta\nu$, given by

$$\Delta\nu \simeq \frac{\Delta N}{\Delta t} \simeq \frac{L\Delta v}{Tc^2\Delta t} \simeq \frac{La}{Tc^2} \ , \tag{1}$$

where a is the acceleration. During this acceleration, the frequency of the trailing clock must increase relative to that of the leading clock, as seen by the observer in frame S, if the two clocks are to maintain synchrony in the new inertial frame.

We expect this frequency shift to remain constant for an infinite sequence of identical accelerations in the limit of Δt going to zero. Therefore, as seen by an observer in S, the frequency of the trailing clock, relative to that of the leading clock, is given by

$$\nu_{\text{trailing}} - \nu_{\text{leading}} \simeq \frac{1}{T}\frac{La}{c^2} \ . \tag{2}$$

Before proceeding with a discussion of the consequences of the new clock hypothesis it is important to agree upon an operational definition of the clock comparison. In what follows, the comparison is carried out by one clock transmitting information about its frequency to the other spatially separated clock. The second clock then determines its rate relative to the first using this information. This definition is closely related to the way in which resonance absorption occurs in the Mössbauer example discussed above.

An alternate choice is to bring the clocks slowly together after the acceleration has terminated [13]. However, such a procedure does not address the issue of the clock comparison during the acceleration process.

We now consider the effect of the new clock hypothesis on the two examples used to justify the frequency shift, starting with the longitudinal Doppler-shift argument. For simplicity let the leading clock emit a light wave train when it is momentarily at rest with respect to the inertial frame S. The inertial observer measures the frequency of the wave train to be the frequency of the leading clock. The trailing clock, however, receives this wave train when it is moving with respect to the inertial frame. An observer in S, using the Doppler equations, can then determine the frequency received by the trailing clock. The frequency received by an observer moving with the trailing clock is the same as that of a momentarily comoving unaccelerated observer, which is given by

$$\nu_{\text{received}} \simeq \nu_{\text{emitted}} \left[1 + \frac{aL}{c^2} \right] ,$$

where $\nu_{\text{emitted}} = T^{-1}$. The unaccelerated comoving clock ticks at a rate T^{-1} and therefore a blueshift is observed for this clock. However, the frequency of the accelerated trailing clock, as determined by an observer in S, is given by Eq. (2) and it coincides with that of the received frequency. The observer in frame S concludes that if the clocks are a Mössbauer source and receiver accelerating as described above then they will resonantly interact during the acceleration.

In the time dilation example [5] there are three clocks: clocks A and B are the leading and trailing clocks, respectively, that rigidly accelerate as described above. In this case the clock comparison is made using the information carried by an inertial clock. To make the comparison, the observer at the leading clock "releases," or puts into the comoving inertial frame S, an observer with clock C which is identical to hers. Clock C ticks with period $T_C = T_A = T$. This inertial observer then compares his clock rate with that of the trailing clock, clock B, as it passes by him and measures the period

$$T_B \simeq T^* \left[1 + \frac{v_B^2}{2c^2} \right] , \tag{3}$$

where v_B is the velocity of clock B and $T^* = 1/\nu^*$ is the period of clock B as determined by the observer in S. Eliminating the period using Eq. (2), we find

$$T_B \simeq T \left[1 - \frac{aL}{c^2} + \frac{v_B^2}{2c^2} \right] = T_A \ ,$$

since $v_B^2 \simeq 2aL$. The inertial observer at clock C then relays the information, to the observer moving with clock B, that the periods of clocks A and B are the same.

The results of these two examples are therefore consistent. That is, one consequence of the new clock hypothesis is that no "redshift" exists between spatially separated clocks which are rigidly accelerating. This holds for all spatially separated clocks located in a grid which accelerates rigidly. That is, any one of these clocks can be the source, described above, while any other clock in the grid can be the receiver.

We now consider perhaps the most unusual consequence of this hypothesis. The relative frequency between the accelerating clocks as seen in frame S is given by Eq. (2). An observer in frame S can ask, which clock, in the rigidly accelerating grid of clocks, ticks at the rate of the unaccelerated clock $1/T$?

We already discussed the fact that there is a preferred frame in the problem. This question addresses a preferred point in that frame. That is, some point from which L is measured and not just a relative separation as is given in Eq. (2).

The answer to this question may involve the proper length of a rigidly accelerating object, which has a maximum value given by $L_m = c^2/a_F$, where a_F is the proper acceleration of the front of the object [11]. The acceleration of the rear end is infinite as seen in S and this end travels at the speed of light. The front end of the rod can be assumed to have arbitrarily small proper acceleration. Therefore, for the observer in S, both ends of the rigidly accelerating rod are unique points, from which a specific clock frequency can be defined.

Another possibility for the accelerating clock which ticks at the rate $1/T$, as seen in frame S, might be the one which coincided with the point from which the grid of clocks was synchronized, prior to the acceleration. This synchronization procedure is necessary if the grid is to accelerate rigidly. It is also interesting to note that to synchronize all the clocks requires an infinite time. Therefore the acceleration occurs before complete synchronization is achieved. We limit our discussion of this issue for two reasons: first, none of the experiments, described in the literature as pertinent to ACH, address this aspect of the hypothesis presented here, and second, there appears to be no experimental results, in flat space-time, relevant to this discussion.

We now consider how predictions of our clock hypothesis agree with the experimental results which are discussed in the literature [14] as forming the foundation for our understanding of accelerated systems in flat space-time. Unfortunately, most of the experimental work has been done on systems undergoing centripetal acceleration. Our clock hypothesis, however, has to be consistent with the results of three types of experiments that involve linear motion.

The first is the twin paradox; a related experimental example is the transport of atomic clocks around the earth in commercial aircraft [15]. Consider three clocks synchronized in an inertial frame, two of which are spatially separated and linearly accelerated into another inertial frame, as discussed above, while the third remains in the initial inertial frame. Our hypothesis is that clocks, initially in an inertial frame, which accelerate into another inertial frame remain synchronized in the new frame in which they are now at rest. This is true for every frame change they undergo. Therefore it is only necessary to calculate the effect of the motion on one of the traveling clocks to determine the reading on the other clock in the frame in which they are now both at rest. The result is that both clocks which were accelerated will read a shorter time than the one which remained in the first inertial frame when the accelerated clocks return to their starting positions. This conclusion is independent of the acceleration that the clocks experience, by construction.

ACH predicts that the two clocks which accelerate into a new inertial frame have different readings in the frame in which they are now at rest [13]. They then decelerate back to the inertial frame in which they started and have identical readings. The prediction of our new hypothesis is therefore the same as that of ACH when the clocks return to their initial positions. However, no such spatially separated clock experiment, analogous to the one described above, has been performed.

The second type of experiment investigates the frequency received in an inertial frame from accelerating sources. One example uses sources which executed harmonic motion due to the thermal vibration of the lattice [16] (thermal effect). Since the lifetime of the excited state is long compared to the period of oscillation, effects linear in velocity and acceleration cancel [9]. Therefore the term linear in acceleration, given in Eq. (2), will not be observed. Sherwin [16] interprets this experiment as an example of the twin paradox for which both clock hypotheses have been shown above to yield the same result when the clocks return to their initial positions. Therefore this experiment cannot distinguish between the two hypotheses.

An experiment of this type, done in a gravitational field, is discussed in Ref. [17]. The results of such an experiment, repeated in flat space-time, would determine the point from which L is measured in our clock hypothesis.

The third type of experiment is that in which both S and R accelerate. This directly measures the prediction of Eq. (2), since L is the relative separation of S and R in that equation. Such an experiment, in which a spatially separated Mössbauer source and receiver were accelerated (distance effect), is described in Ref. [18]. The source and receiver executed harmonic motion with the period of oscillation comparable to the lifetime of the excited state. The intensity of the radiation transmitted through the receiver depended on the source-receiver separation. This was interpreted as evidence for ACH. However, the experimental results were never published.

More recently an explanation of the distance effect was given in terms of an effective refractive-index dependence on the ultrasonic amplitude and frequency and by the forced Mandelshtam-Brillouin effect [19]. The distance effect is predicted to occur even for nonsynchronous motion of the source and receiver. Until further experiments are performed, the distance effect should not be considered supportive of ACH.

We now give a simplified explanation of why effects linear in acceleration cancel when the period of oscillation is shorter than the lifetime of the excited state. Upon completion of one cycle either the source (thermal effect) or both the source and receiver (distance effect) return to their initial positions. We assume for simplicity that resonant absorption by the receiver is only a function of the number of wavecrests received during a time comparable to the lifetime of the transition. The source starts in an inertial frame emitting a frequency unaffected by acceleration. It then accelerates for one cycle. When the source (source and receiver in the distance effect) returns to the starting position, any linear acceleration-dependent frequency effect has resulted in more wavecrests being counted by the receiver for the acceleration in one direction while fewer are counted in the other direction. Therefore the total number of wavecrests counted during this period is independent of any effect which is linear in acceleration. In this case the distance effect cannot confirm ACH nor can the thermal effect repudiate our clock hypothesis.

Finally we discuss an experiment which might distinguish between these two hypotheses. This would be similar to Bommel's, with the period of oscillation much smaller than the lifetime of the Mössbauer transition. Therefore, during a lifetime, the acceleration is essentially constant. As the acceleration varies, the receiver, which moves synchronously with the source, would go in and out of resonance according to ACH. Our hypothesis predicts that it would remain in resonance. A detailed analysis of such a proposed experiment is not appropriate for this work. However, it is interesting to note that commercially available piezoelectric crystals are capable of providing harmonic motion with an amplitude of 1 μm at 30 kHz [20] oscillation frequency. This corresponds to a maximum acceleration of 3×10^4 m/sec^2, which is large enough to shift the resonant frequency by one-third of the full width at half maximum of the absorption linewidth, for a source-receiver separation of 1 m and the following Mössbauer transition: ^{57}Co in Rh metal foil as the source with ^{57}Fe in $K_4[\mathrm{Fe(CN)}_6]\cdot 3\mathrm{H}_3\mathrm{O}$ as the receiver. Such a shift is large enough to measure. Synchronizing the motion of the source and receiver would not be easy, however.

We have proposed an alternative clock hypothesis for accelerating systems which is consistent with the experimental evidence discussed in the literature as supportive of ACH. It differs from the accepted clock hypothesis in predicting no frequency shift between two spatially separated clocks rigidly accelerating.

I would like to express my gratitude to Professor Geoff E. Stedman for his interest in and helpful discussions of this work.

[1] J. E. Romain, Rev. Mod. Phys. **35**, 376 (1963).
[2] I. Khan, Int. J. Theor. Phys. **6**, 383 (1972).
[3] M. Sachs, Am. J. Phys. **41**, 748 (1973).
[4] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, New York, 1973), pp. 189 and 190.
[5] L. I. Schiff, Am. J. Phys. **28**, 340 (1960).
[6] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Ref. [4]), pp. 172–174 and 187–191.,
[7] W. Rindler, *Introduction to Special Relativity* (Clarendon, Oxford, 1982), p. 31.
[8] E. A. Desloge, Am. J. Phys. **58**, 856 (1990).
[9] B. Mashhoon, Phys. Lett. A **122**, 67 (1987).
[10] R. W. Brehme, Am. J. Phys. **56**, 811 (1987).
[11] C. Giannoni and Ø. Grøn, Am. J. Phys. **47**, 431 (1979).
[12] E. F. Taylor and A. P. French, Am. J. Phys. **51**, 889 (1983).
[13] In the following work the separation of the accelerating clocks is constant in frame *S* rather than the proper length being fixed, as we assume: E. A. Desloge and R. J. Philpott, Am. J. Phys. **59**, 280 (1991).
[14] D. Newman, G. W. Ford, A. Rich, and E. Sweetman, Phys. Rev. Lett. **40**, 1355 (1978).
[15] J. C. Hafele and F. E. Keating, Science **177**, 166 (1972).
[16] C. W. Sherwin, Phys. Rev. **120**, 17 (1960).
[17] R. F. C. Vessot and M. W. Levine, Gen. Relativ. Gravit. **10**, 181 (1979).
[18] H. E. Bömmel, *Proceedings of the Second International Conference on the Mössbauer Effect, Saclay, France* (Wiley, New York, 1962), pp. 229–232.
[19] V. G. Baryshevsky, J. Phys. C **18**, 191 (1985).
[20] Physik Instrumente, Model P-835.10, Waldbronn, Germany.