Spinor structures of the Bethe-Salpeter irreducible kernels

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In this paper the possible types of the Bethe-Salpeter (BS) irreducible kernels are determined from a phenomenologically reasonable requirement, and the spinor structures of BS wave functions and equations for all the three categories of natural J^{PC} mesons are obtained.

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I. INTRODUCTION

Our motivation for this work is to explore the spin effects of relativistic bound states of fermion antifermion (quark antiquark). This, we hope, will be useful in the incorporation of relativistic effects in quark-model building. Producing the hadron spectrum from quantum chromodynamics has proven to be a formidable task. In phenomenological approaches, as is well known, it is difficult to treat the pion in the potential model and bag model. The pion is composed of the light quarks u and d. Not only the motion of the pion's center of mass, but also the inner motion between the quark q and the antiquark $\bar{q}$, are relativistic. In order to consider the relativistic effects of the inner motion between q and $\bar{q}$, the natural framework is Jacob-Wick helicity formalism [1].

Since hadrons were visualized as relativistic bound states of quarks, the Bethe-Salpeter (BS) equation has been extensively studied [2–7] because it has at least a formal connection with quantum field theory. In this paper we study the general results in the framework of Ref. [6] in view of its good results. As is well known, in the BS framework the bound state must be on its mass shell, since the BS wave function is related to the residue of the Green function at the pole $P^2=-\mu^2$, where P is the total momentum, and μ is the mass of the bound system [8]. Using this fact, we obtain the homogeneous BS equation in the momentum space from the inhomogeneous one [9]; it takes the following form:

$$[i(\hat{p}+\tfrac{1}{2}\hat{P})+m_1]\phi(p,P)[i(\hat{p}-\tfrac{1}{2}\hat{P})+m_2] = -\sum_{I=S,P,V,A,T} W^{(I)}\Gamma^{(I)}\phi(p,P)\Gamma^{(I)} , \quad (1.1)$$

where P is the meson's four-momentum, p is the relative four-momentum between the quark and the antiquark, m_1 and m_2 are, respectively, the mass of the quark and the antiquark, $\Gamma^{(I)}=I,\gamma_5,\gamma_\mu,i\gamma_5\gamma_\mu,\sigma_{\mu\nu}$, $W^{(I)}(p,q;P)$ is the BS irreducible kernel, and $W^{(I)}\phi(p,P)$ is defined by the integral

$$W^{(I)}\phi(p,P)=\int d^4q\; W^{(I)}(p,q;P)\phi(q,P) .$$

In the potential approximation, $W(p,q;P)$ only depends on $p-q$.

When applying Eq. (1.1) to solve the meson spectrum, there are two problems which should be properly solved. First, we must determine the general spinor structures of the BS wave functions for mesons with natural J^{PC} [i.e., the relations $P=(-1)^{l+1}$, $C=(-1)^{l+s}$, $J=|l-s|,|l-s|+1,\ldots,l+s$ exist among the total angular momentum J of the meson, the parity P, the charge conjugate parity C, the orbital angular momentum l, and the total spin S of the quark and the antiquark, $S=0,1$]. The general spinor structures of the meson's BS wave functions are determined from its properties under the Lorentz transformation, the space reflection, the charge conjugation, and the weak space-time inversion. This approach is discussed by Ref. [6] and by Feldman, Fulton, and Townsend in Ref. [10]. In Appendix A we summarize the general covariant spinor structure of the BS wave functions for mesons with natural J^{PC} following Refs. [6] and [11] to fix our notations. They are classified into three categories. In each category the BS wave function includes eight scalar functions. In the literature some authors only considered two or three terms and ignored the other terms based on some argument. But the other terms may also be important, so in the following we will consider all eight terms.

Second, we must determine the possible types of the irreducible BS kernels from phenomenology. If not only the single-gluon-exchange mechanism, but also the contributions of all the irreducible diagrams are considered, then all five types of couplings, S, P, V, A, and T, exist. In the BS equation, the coupling types of the kernels determine the spinor structure of the wave functions. By different choice of the types of the couplings, we obtain the different spinor structures of the BS wave functions. In order to determine the correct types of the couplings from phenomenology, in Appendix B, we review the BS amplitudes of the natural J^{PC} states of the free $q\bar{q}$ system. We require that not only the motion of the center of mass, but also the inner motion between q and $\bar{q}$, be relativistic. Thus we cannot adopt the nonrelativistic spin formalism to construct the natural J^{PC} states; we must adopt the Jacob-Wick helicity formalism [1]. The BS amplitudes of the natural J^{PC} states obtained from Jacob-Wick helicity formalism are independent of dynamics. It is only determined by kinematics, so the results are quite general. We emphasize that in the BS approach, in the vertex with a bound system, from four-momentum conservation it follows that it is impossible to have all legs on their mass shell. This basic feature is represented by the fact that the BS normalization condition [9] cannot be satisfied by the BS amplitudes of the free $q\bar{q}$ system. The results in Appendix B only serve as necessary conditions to determine the possible types of the couplings in the BS equation from the phenomenology discussed in Sec. II. Then, we discuss the BS equation of the mesons with natural J^{PC} in Sec. II. It is a natural requirement that when the interaction between q and $\bar{q}$ approaches zero the solutions of the BS equation should approach the BS amplitudes of the free system obtained in Appendix B [12]. By trying various possibilities, we determine the possible types of the BS irreducible kernels. They are a mixture of S, V, and T types. It is interesting that by the above choice of the couplings the systems of the eight scalar equations of mesons are completely decoupled for all three categories. Thus we obtain three direct results. (1) The spinor structures of the BS wave functions are completely determined by the types of the BS irreducible kernels. When the interaction between the quark and the antiquark approaches zero these BS wave functions approach the BS amplitudes of the free system. (2) Each BS wave function thus obtained only includes one arbitrary scalar function. These scalar functions satisfy the same scalar dynamical equation for all three categories. (3) The BS normalization conditions of the BS wave functions for all three categories are the same also. Summarizing the above results we obtain a unified relativistic model of mesons with natural J^{PC} for all three categories.

In Sec. II we treat the 2^{++} tensor meson to illustrate the above ideas. The reason for treating the tensor meson as the example is that the treatment of the BS wave function and the BS equation for the tensor meson is more difficult than that for the scalar meson and the vector meson. When it is clear how to treat the tensor meson, then it is easy to treat the scalar meson and the vector meson. In Appendix C we tabulate the corresponding results of the other mesons with natural J^{PC}. In Sec. III we show the results in instantaneous approximation [13,14] in which unphysical states in the BS framework are, at least formally, excluded. In Sec. IV we explore the physical meaning of our *Ansatz* of the BS kernel. In Sec. V we show a simple example.

II. THE BS EQUATIONS OF MESONS WITH NATURAL J^{PC} AND THE SOLUTIONS (TO TREAT THE 2^{++} TENSOR MESON AS THE EXAMPLE)

In this section we determine the possible types of the irreducible BS kernel from phenomenology. It is a natural requirement that when the interaction between q and $\bar{q}$ approaches zero the solutions of the BS equation should approach the BS amplitudes of the free system. From the transformation properties of the BS wave functions under the Lorentz transformation, space reflection, charge conjugation, and weak space-time inversion we can determine their general form [10,11]. They are classified into three categories. Generally, the BS wave function $\phi(p,P)$ has eight scalar functions for each category, so (1.1) is a system of eight coupled scalar equations. In general it is difficult to solve. In order to treat (1.1), we introduce the projection operator $P^{(I)}$ of the Γ matrices and the projection potential $V^{(I)}$ [15]. Thus (1.1) can be rewritten as

$$[i(\hat{p}+\tfrac{1}{2}\hat{P})+m_1]\phi(p,P)[i(\hat{p}-\tfrac{1}{2}\hat{P})+m_2] = -\sum_{I=S,P,V,A,T} V^{(I)}[P^{(I)}\phi(p,P)] , \quad (2.1)$$

where projection operators $P^{(I)}$ satisfy

$$P^{(I)}P^{(J)}=\delta^{IJ}P^{(J)}, \quad P^{(I)}\Gamma^{(J)}=\delta^{IJ}\Gamma^{(J)} . \quad (2.2)$$

$V^{(I)}$ is the linear composition of $W^{(I)}$ in (1.1).

Introducing (A2) into (2.1), we obtain the system of equations satisfied by the scalar functions $g_1,\ldots,g_8$ as follows:

$$(m_1m_2+V^{(S)}-p^2-\tfrac{1}{4}\mu^2)\mathbf{p}^2g_1+\{(m_1+m_2)g_2+\mu^2[(m_1+m_2)p_0+\tfrac{1}{2}\mu(m_2-m_1)]p_0g_3 +[(m_1+m_2)p^2-\tfrac{1}{2}\mu(m_2-m_1)p_0]g_4+\mu^2(g_5-p_0^2g_6-\mathbf{p}^2g_7)\}\mathbf{p}^2=0 , \quad (2.3)$$

$$(m_1m_2+V^{(A)}-p^2-\tfrac{1}{4}\mu^2)\mathbf{p}^2g_8+[g_2+(m_1+m_2)g_5+\tfrac{1}{2}\mu(m_2-m_1)p_0g_6]\mathbf{p}^2=0 , \quad (2.4)$$

$$\mu^2(m_1m_2+V^{(V)}+\mathbf{p}^2+p_0^2-\tfrac{1}{4}\mu^2)p_0\mathbf{p}^2g_3-(m_1m_2+V^{(V)}-p^2-\tfrac{1}{4}\mu^2)p_0\mathbf{p}^2g_4 +\{[(m_1+m_2)p_0+\tfrac{1}{2}\mu(m_2-m_1)]g_1+2p_0g_2+\mu(m_2-m_1)(-g_5+p_0^2g_6+\mathbf{p}^2g_7)\}\mathbf{p}^2=0 , \quad (2.5)$$

$$(m_1m_2+V^{(V)}-p^2+\tfrac{1}{4}\mu^2)\mathbf{p}^2p_kg_4+\{-(m_1+m_2)g_1+\mu^2g_8-2g_2-2\mu^2p_0^2g_3 -\mu(m_2-m_1)p_0g_6-\mu[(\mu/2)(m_1+m_2)+(m_2-m_1)p_0]g_7\}\mathbf{p}^2p_k=0 , \quad (2.6)$$

$$(m_1m_2+V^{(V)}+p^2+\tfrac{1}{4}\mu^2)p_kg_2+\{-\mu^2\mathbf{p}^2g_8+\mu[\tfrac{1}{2}\mu(m_1+m_2)+(m_2-m_1)p_0]g_5 +\mu[(m_2-m_1)p_0p^2-\tfrac{1}{2}\mu(m_1+m_2)p_0^2]g_6\}p_k=0 , \quad (2.7)$$

$$-\mu(m_1m_2+V^{(T)}+p^2+\tfrac{1}{4}\mu^2)p_kp_0^2g_6+\mu(m_1m_2+V^{(T)}-\mathbf{p}^2-p_0^2+\tfrac{1}{4}\mu^2)p_kg_5 +\{-\mu(m_1+m_2)\mathbf{p}^2g_8+[\tfrac{1}{2}\mu(m_1+m_2)+(m_2-m_1)p_0]g_2\}p_k=0 , \quad (2.8)$$

$$-(m_1m_2+V^{(T)}+p^2+\tfrac{1}{4}\mu^2)p_k\mathbf{p}^2g_7+[-g_1+(m_1+m_2)g_8+\tfrac{1}{2}(m_1+m_2)g_4+2g_5+\mu(m_2-m_1)p_0g_3]p_k\mathbf{p}^2=0 , \quad (2.9)$$

$$\mu(m_1m_2+V^{(T)}+p^2-\tfrac{1}{4}\mu^2)p_0\mathbf{p}^2g_6+\{\mu[(m_1+m_2)p_0+\tfrac{1}{2}\mu(m_2-m_1)]g_8+2\mu p_0g_5-(m_2-m_1)g_2\}\mathbf{p}^2=0 . \quad (2.10)$$

In the above equations, μ is the meson mass, and $V^{(I)}g_jp_kp_l$ is defined by the integral

$$V^{(I)}g_jp_kp_l \equiv \int d^4q\, V^{(I)}(p,q;P)g_j(q;P)q_kq_l , \ldots ,$$

so in (2.3)–(2.10) we cannot omit the overall factor p_k, $\mathbf{p}^2$, $p_k\mathbf{p}^2$, etc. Only for the free system, $V^{(I)}=0$, can these overall factors be omitted.

In order to determine the types of the BS kernels from phenomenology, we demanded that the solutions of the BS equation should reduce to the BS amplitudes of the free $q\bar{q}$ system when the interaction between q and $\bar{q}$ approaches zero. To ensure this, we examine the system of equations (2.3)–(2.10) and try different possibilities. Notice that if we take $g_1=0$, $\mu^2g_3-g_4=0$ and choose the projected potential such that

$$V^{(A)}=V^{(T)}=0 , \tag{2.11}$$

then we obtain

$$\begin{aligned} &g_2=(m_1m_2+p^2+\tfrac{1}{4}\mu^2)g_8 ,\\ &\mathbf{p}^2g_4=-(m_1m_2+p^2+\tfrac{1}{4}\mu^2)g_8 ,\\ &g_5=-\tfrac{1}{2}(m_1+m_2)g_8 ,\\ &-g_5+p_0^2g_6=\left[\frac{1}{2}(m_1+m_2)+\frac{1}{\mu}(m_2-m_1)p_0\right]g_8 ,\\ &\mathbf{p}^2g_7=-\left[\frac{1}{\mu}(m_2-m_1)p_0+\tfrac{1}{2}(m_1+m_2)\right]g_8 , \end{aligned} \tag{2.12}$$

and the scalar function g_8 satisfies the following equation:

$$[(m_1m_2+V^{(V)}+p^2+\tfrac{1}{4}\mu^2)(m_1m_2+p^2+\tfrac{1}{4}\mu^2)+(m_2-m_1)^2p^2-\mu^2\mathbf{p}^2 -\mu p_0(m_2^2-m_1^2)-\tfrac{1}{4}\mu^2(m_1+m_2)^2]p_kg_8=0 . \tag{2.13}$$

Finally, the BS wave function for the 2^{++} meson is

$$\phi(p,P)=\{i\mu\epsilon_{ij4k}(E_{jl}p_l)p_k\gamma_5\gamma_i+(m_1m_2+p^2+\tfrac{1}{4}\mu^2)[E_{ij}p_j-p_i(E_{kl}n_kn_l)]\gamma_i +i[\tfrac{1}{2}\mu(m_1+m_2)+p_0(m_2-m_1)][E_{lk}p_k-p_l(E_{ij}n_in_j)\sigma_{4l}-(m_2-m_1)(E_{il}p_l)p_j\sigma_{ij}]\}g_8 . \tag{2.14}$$

The BS wave function should satisfy the BS normalization condition [9]. When the BS irreducible kernel $W^{(I)}$ in (1.1) does not explicitly contain P_0, it is easy to show that the BS normalization condition takes the following form:

$$i(2\pi)^4\int d^4p\,\mathrm{Tr}(\bar{\phi}(p,P)\{-\tfrac{1}{2}\gamma_4\phi(p,P)[i(\hat{p}-\tfrac{1}{2}\hat{P})+m_2]+[i(\hat{p}+\tfrac{1}{2}\hat{P})+m_1]\phi(p,P)\tfrac{1}{2}\gamma_4\})=1 . \tag{2.15}$$

By using (2.14), (2.15) is reduced to

$$N^2\int d^4p\{\mu(p^2+m_1m_2+\tfrac{1}{4}\mu^2)^2-\mu(p^2+m_1m_2)[4\mathbf{p}^2+(m_1+m_2)^2]-\mu(m_2^2-m_1^2)p^2 -2p_0(m_2^2-m_1^2)(p^2+m_1m_2-\tfrac{1}{4}\mu^2)\}g^2(p,P)=1 , \tag{2.16}$$

where N is the normalization constant.

It is easy to show that when $V^{(V)}\to 0$, up to a common scalar factor of μ, m_1, m_2, and $\mathbf{p}^2$, (2.14) is reduced to the F_2 term in (B23) and corresponds to the p waves.

Taking $V^{(A)}=V^{(T)}=0$ in the BS equation (2.1) we solve the BS wave functions of the other mesons which are tabulated in Appendix C.

III. THE INSTANTANEOUS APPROXIMATION

When applying the results obtained in the previous sections, we must rule out the unphysical states which are associated with the excitation of the relative time degree of freedom. In the following, we simply adopt the instantaneous approximation to examine qualitatively the physical results of the model. The instantaneous approximation is extensively treated in the literature for example, the work of Cung and co-workers [13,14].

In the instantaneous approximation $V^{(I)}$ is independent of p_0 and p_0'; then (2.1) can be changed into [16]

$$\mu\phi(\mathbf{p})-H_1(\mathbf{p})\phi(\mathbf{p})+\phi(\mathbf{p})H_2(-\mathbf{p}) =\sum_{I=S,P,V,A,T}\int d^3q\, V^{(I)}(\mathbf{p},\mathbf{q};P)\gamma_4\Gamma^{(I)}\left[\frac{1}{\varepsilon_1(\mathbf{q})}H_1(\mathbf{q})\phi(\mathbf{q})-\frac{1}{\varepsilon_2(\mathbf{q})}\phi(\mathbf{q})H_2(-\mathbf{q})\right]\Gamma^{(I)}\gamma_4 , \tag{3.1}$$

where μ is the meson mass, $H_1(\mathbf{p})=\boldsymbol{\alpha}\cdot\mathbf{p}+\gamma_4m_1$, $H_2(-\mathbf{p})=-\boldsymbol{\alpha}\cdot\mathbf{p}+\gamma_4m_2$, $\varepsilon_i(\mathbf{p})=(\mathbf{p}^2+m_i^2)^{1/2}$ $(i=1,2)$. The center-of-mass wave function $\phi(\mathbf{p})$ is independent of p_0, and it is called the three-wave-function in the following. The general forms of the three-wave-functions are tabulated in Appendix A.

Introducing (A4)–(A6) into (3.1), we obtain the systems of equations satisfied by the scalar functions f_i, g_i, and h_i for three categories as follows: (1) $S=0$, J^{-+} $(J=2n)$, and J^{+-} $(J=2n+1)$:

$$(\mathbf{E}\cdot\mathbf{p})f_1-2(\mathbf{E}\cdot\mathbf{p})(\mathbf{p}^2f_3-f_4)-V^{(P)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right](\mathbf{E}\cdot\mathbf{p})(\mathbf{p}^2f_3-f_4)-(m_1+m_2)(\mathbf{E}\cdot\mathbf{p})f_2 - V^{(P)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right](\mathbf{E}\cdot\mathbf{p})f_2=0\ , \tag{3.2}$$

$$(m_1+m_2)(\mathbf{E}\cdot\mathbf{p})f_1-V^{(A)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right](\mathbf{E}\cdot\mathbf{p})f_1-\mu^2(\mathbf{E}\cdot\mathbf{p})f_2=0\ , \tag{3.3}$$

$$2p_m(\mathbf{E}\cdot\mathbf{p})f_1+V^{(T)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right]p_m(\mathbf{E}\cdot\mathbf{p})f_1-\mu^2[p_m(\mathbf{E}\cdot\mathbf{p})f_3-(\mathbf{E}_m\cdot\mathbf{p})f_4]=0\ , \tag{3.4}$$

(2) $S=1$, J^{--} $(J=2n+1)$, and J^{++} $(J=2n)$:

$$(\mathbf{E}\cdot\mathbf{p})g_1+2(\mathbf{E}\cdot\mathbf{p})(\mathbf{p}^2g_5+g_6)+V^{(S)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right](\mathbf{E}\cdot\mathbf{p})(\mathbf{p}^2g_5+g_6)=0\ , \tag{3.5}$$

$$[p_m(\mathbf{E}\cdot\mathbf{p})g_2+(\mathbf{E}_m\cdot\mathbf{p})g_3]-2[p_m(\mathbf{E}\cdot\mathbf{p})-\mathbf{p}^2(\mathbf{E}_m\cdot\mathbf{p})]g_4-V^{(V)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right][p_m(\mathbf{E}\cdot\mathbf{p})-\mathbf{p}^2(\mathbf{E}_m\cdot\mathbf{p})]g_4 -(m_1+m_2)[p_m(\mathbf{E}\cdot\mathbf{p})g_5+(\mathbf{E}_m\cdot\mathbf{p})g_6]-V^{(V)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right][p_m(\mathbf{E}\cdot\mathbf{p})g_5+(\mathbf{E}_m\cdot\mathbf{p})g_6]=0\ , \tag{3.6}$$

$$V^{(A)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right]p_j[p_k(\mathbf{E}\cdot\mathbf{p})g_2+(\mathbf{E}_k\cdot\mathbf{p})g_3]+2p_j(\mathbf{E}_k\cdot\mathbf{p})g_3+\mu^2p_j(\mathbf{E}_k\cdot\mathbf{p})g_4=0\ , \tag{3.7}$$

$$2p_l(\mathbf{E}\cdot\mathbf{p})g_1-V^{(T)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right]p_l(\mathbf{E}\cdot\mathbf{p})g_1-(m_1+m_2)[p_l(\mathbf{E}\cdot\mathbf{p})g_2+(\mathbf{E}_l\cdot\mathbf{p})g_3] +\mu^2[p_l(\mathbf{E}\cdot\mathbf{p})g_5+(\mathbf{E}_l\cdot\mathbf{p})g_6]+V^{(T)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right][p_l(\mathbf{E}\cdot\mathbf{p})g_2+(\mathbf{E}_l\cdot\mathbf{p})g_3]=0\ , \tag{3.8}$$

(3) $S=1$, J^{--} $[J=2(n+1)]$, and J^{++} $(J=2n+1)$:

$$\mu^2p_j(\mathbf{E}_k\cdot\mathbf{p})h_1+2p_j(\mathbf{E}_k\cdot\mathbf{p})h_3+V^{(V)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right](\mathbf{E}_k\cdot\mathbf{p})p_jh_3-\tfrac{1}{2}(m_1+m_2)p_j(\mathbf{E}_k\cdot\mathbf{p})h_4 -\tfrac{1}{2}V^{(V)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right]p_j(\mathbf{E}_k\cdot\mathbf{p})h_4=0\ , \tag{3.9}$$

$$2[\mathbf{p}^2(\mathbf{E}_m\cdot\mathbf{p})-p_m(\mathbf{E}\cdot\mathbf{p})]h_1+V^{(A)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right][\mathbf{p}^2(\mathbf{E}_m\cdot\mathbf{p})-p_m(\mathbf{E}\cdot\mathbf{p})]h_1+[p_m(\mathbf{E}\cdot\mathbf{p})h_2+(\mathbf{E}_m\cdot\mathbf{p})h_3]=0\ , \tag{3.10}$$

$$2(m_1+m_2)p_j(\mathbf{E}_k\cdot\mathbf{p})h_1-2V^{(T)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right]p_j(\mathbf{E}_k\cdot\mathbf{p})h_1-p_j(\mathbf{E}_k\cdot\mathbf{p})h_4=0\ . \tag{3.11}$$

By the same method illustrated in Sec. II, examining (3.2)–(3.11), we find that if the choice of the potential for the three classes is

$$V^{(A)}=V^{(T)}=0 \tag{3.12}$$

then we can solve (3.2)–(3.11) for all the three classes listed in Appendix A. The space wave functions f, g, and h for the three classes satisfy the same scalar equation:

$$\left[\mu^2-(m_1+m_2)^2-4\mathbf{p}^2-(m_1+m_2)V^{(i)}\left[\frac{m_1}{\varepsilon_1}+\frac{m_2}{\varepsilon_2}\right]-2V^{(i)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right]\mathbf{p}^2\right]F_{JM}^{(i)}(\mathbf{p})=0\quad (i=1,2,3)\ , \tag{3.13}$$

where

$$F_{JM}^{(1)}(\mathbf{p})=\bar{p}^{\,J}Y_{JM}(\theta,\phi)f(\mathbf{p}^2)\ ,$$
$$F_{JM}^{(2)}(\mathbf{p})=\bar{p}^{\,J-1}Y_{J-1,M}(\theta,\phi)g(\mathbf{p}^2)\ , \quad (3.14)$$
$$F_{JM}^{(3)}(\mathbf{p})=\bar{p}^{\,J}Y_{JM}(\theta,\phi)h(\mathbf{p}^2)\ ,$$

and

$$V^{(1)}=V^{(P)},\quad V^{(2)}=V^{(3)}=V^{(V)}\ . \quad (3.15)$$

In (3.13) $V^{(i)}(1/\varepsilon_1+1/\varepsilon_2)\mathbf{p}^2F_{JM}^{(i)}(\mathbf{p})$ is defined by the integral

$$V^{(i)}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right]\mathbf{p}^2F_{JM}^{(i)}(\mathbf{p})\equiv\int d^3q\ V^{(i)}(\mathbf{p},\mathbf{q};P)\times\left[\frac{1}{\varepsilon_1(\mathbf{q})}+\frac{1}{\varepsilon_2(\mathbf{q})}\right]\times\mathbf{q}^2F_{JM}^{(i)}(\mathbf{q})\ ,$$

the angles (θ,ϕ) refer to the direction of n. $\bar{p}\equiv|\mathbf{p}|$.

In (3.14) for classes (1) and (3) $l=J$, for class (2) $l=J-1$. The factor $\bar{p}^{\,J}Y_{lm}$, representing the contribution of the orbital angular momentum, comes from the factor $(\mathbf{E}\cdot\mathbf{p})\equiv E_{i_1\cdots i_J}p_{i_1}\cdots p_{i_J}$ in the three-wave-function $\phi(\mathbf{p})$ (c.f. Appendixes A and C).

Equations (3.1) and (3.13) contain the quark's energy denominator term $\varepsilon_i^{-1}(\mathbf{q})$ $(i=1,2)$. We adopt the heavy-quark approximation to solve it.

IV. SPINOR STRUCTURES OF THE BS KERNELS

By the *Ansätze* (2.11) and (3.12) only the "projection potentials" $V^{(P)}$ and $V^{(V)}$ appear in the equations. The projection potentials $V^{(I)}$ are the composition of the BS kernel $W^{(I)}$ $(I=S,P,V,A,T)$. One possible result obtained by the *Ansätze* (2.11) and (3.12) is that only the composition of the scalar $W^{(S)}$, the vector $W^{(V)}$, and the tensor $W^{(T)}$ appears in the equations. $W^{(V)}$ can be taken as the Coulomb potential, $W^{(S)}$ as the linear (confinement) potential, and $W^{(T)}$ as the spin-spin coupling. Just $W^{(T)}$ leads to the mass splitting between the $S=0$ meson and $S=1$ meson. In fact in (1) in the nonrelativistic approximation the term $\gamma_4\Gamma^{(T)}\Gamma^{(T)}\gamma_4\sim\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}$ which is just the spin-spin coupling. Thus the results presented here incorporate the phenomenologically reasonable spinor structure of the BS kernel which is useful in the study of the bound system in quark models. There is another interesting approach to incorporate spin effects in a relativistic bound system [17]. The spin effects are found to be phenomenologically large in the hadron spectrum, therefore they are important for quark-model building. Our results are sensitive to the spinor structures of the BS kernels, and give all the spin effects. In the following we consider a simplified potential to show that our model works well.

V. A SIMPLE EXAMPLE

We now consider the potentials $V^{(V)}$ and $V^{(P)}$.

It has been observed by Buchmüller and Tye [18] that the bound-state quarkonium energy levels are sensitive to the form of the potential mainly in the region $0.1\lesssim r\lesssim 1$ fm. In this region the various potentials proposed in the literature are close to each other, so in order to examine the qualitative character of our model, for simplicity we take the $V^{(V)}$ to be a linear potential

$$V^{(V)}=V_0+r/a^2\ , \quad (5.1)$$

where a and V_0 are constants. This assumption should be a good one if at least one of the quarks in the meson is not too heavy (u, d, or s). This is because the mean distance of u, d, or s quark from the other (anti) quark is large enough so that the wave function is most sensitive to the potential at distances where the linear approximation is a good one. However, if both quarks are heavy, we may underestimate the splitting between the 1S and 2S levels with a linear potential. This is because the wave function in the 1S level is particularly sensitive to the short-distance behavior of the potential, which, according to perturbative QCD, is Coulomb-like, rather than linear as we have assumed.

The splittings between $S=1$ and 0 mesons are determined by

$$V^{(V)}-V^{(P)}=W^{(T)} \quad (5.2)$$

which is just the tensor-type BS irreducible kernel. In order to estimate these splittings we treat $W^{(T)}$ as a constant perturbation, which is in qualitative agreement with the observation of Martin [19] that the difference of the mass-squared values between $S=0$ and 1 mesons are approximately constant in the 1S level.

Thus we obtain the following eigenequation for the $S=1$ meson by using (3.13) and (5.1):

$$[\mu^2-(m_1+m_2)^2-2(m_1+m_2)V_0+4\alpha\nabla^2-2(m_1+m_2)r/a^2]f(\mathbf{r})=0\ . \quad (5.3)$$

The mass-squared eigenvalues are

$$\mu_{nl}^2=(m_1+m_2)^2+2(m_1+m_2)V_0+[16\alpha(m_1+m_2)^2/a^4]^{1/3}\zeta_{nl}\ . \quad (5.4)$$

The mass-squared splittings between $S=1$ and 0 mesons are

$$\Delta\mu^2=-\alpha^{-1/3}(m_1+m_2)\delta\ . \quad (5.5)$$

In (5.3)–(5.5) m_1 and m_2 are, respectively, the quark and antiquark mass,

$$\alpha=1+[(m_1+m_2)(4m_1m_2-m_1^2-m_2^2)/8m_1^2m_2^2]V_0\ . \quad (5.6)$$

n and l are the radial and orbital quantum numbers. The scaleless quantities ζ_{nl} are eigenvalues of the equation

$$[d^2/d\rho^2-l(l+1)/\rho^2-\rho+\zeta_{nl}]U_{nl}(\rho)=0\ .$$

The parameters take the values

TABLE I. The mass spectrum of the $S=1$ mesons (in GeV).

Meson	Method	1s	1p[a]	2s	1d	2p	3s	2d	4s	5s	6s
ρ	Theor.	0.78	1.24	1.48	1.53	1.70	1.86	1.89	2.15		
	Expt. [20]	0.768	a_1, 1.260±0.030	1.450±0.008			1.700±0.020				
K^*	Theor.	0.91	1.35	1.59	1.64	1.82	1.98				
	Expt.	0.892	K_0^* 1.429±0.006		1.678 ±0.064						
			K_2^*, 1.425±0.001								
ϕ	Theor.	1.02	1.45	1.69	1.74	1.92	2.09				
	Expt.	1.019	f_1, 1.425±0.001	1.680 +0.050							
ψ	Theor.	3.08	3.45	3.69	3.73	3.92	4.10	4.13	4.44	4.73	4.98
	Expt.	3.097	3.511	3.686	3.770		4.040 ±0.010	4.159 ±0.020	4.415 ±0.006		
D^*	Theor.	2.13	2.53	2.78							
	Expt.	D^{*+}, 2.010 D^{*0}, 2.007									
D_S^*	Theor.	2.18	2.58	2.82							
	Expt.	2.110	2.536								
Υ	Theor.	9.46	9.72	9.90	9.94	10.10	10.26	10.28	10.56	10.82	11.07
	Expt.	9.460	9.892	10.023		10.255	10.355		10.580 ±0.003	10.865 ±0.008	11.019 ±0.008
B^*	Theor.	5.43									
	Expt.										

[a]The experimental value refers to the 1^{++} meson if it is not specified.

$$V_0=-1.274\ \text{GeV},\quad a=1.866\ (\text{GeV})^{-1},$$
$$\delta=0.299\ (\text{GeV})^{2/3},$$
$$m_u=m_d=0.809\ \text{GeV},\quad m_s=0.879\ \text{GeV},$$
$$m_c=1.850\ \text{GeV},\quad m_b=5.097\ \text{GeV}. \tag{5.7}$$

Our values of m_u, m_d, and m_s are larger than those in most other models. However, constituent masses are model dependent, and we choose to make these masses larger than usual (compensated by a large and negative V_0) so as to be consistent with our heavy-quark approximation.

The interesting fact is that the parameters of the potential V_0, a, and δ are flavor independent, but the factors α and (m_1+m_2) in (5.4) and (5.5) are flavor dependent, so we can get the unified explanation of the mass spectrum for all the mesons. Tables I and II show that our model

TABLE II. The mass spectrum of the $S=0$ mesons (in GeV).

Meson	Method	1s	1p	2s	2p	3s
π	Theor.	0.14	0.97	1.26	1.52	1.70
	Expt.	$\pi^{\pm}$, 0.140 π^0, 0.135		1.300 ±0.100		
K	Theor.	0.51	1.12	1.40	1.66	1.84
	Expt.	K^+, 0.494 K^0, 0.498	K_1, 1.270±0.010			
η_c	Theor.	2.96	3.33	3.57	3.82	4.00
	Expt.	2.980				
D	Theor.	2.01	2.43	2.69		
	Expt.	D^+, 1.869 D^0, 1.865				
D_s	Theor.	2.07	2.48	2.73		
	Expt.	1.969				
η_b	Theor.	9.38	9.64	9.83	10.03	10.18
	Expt.					
B	Theor.	5.39				
	Expt.	B^+, 5.278 B^0, 5.279				

works well. The success of the model cannot be purely accidental. There must be some underlying truth behind it.

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APPENDIX A: THE GENERAL SPINOR STRUCTURE OF THE BS WAVE FUNCTIONS FOR MESONS WITH THE NATURAL J^{PC} QUANTUM NUMBER

From the transformation properties of the BS wave function under the Lorentz transformation, the space reflection, the charge conjugation, and the weak space-time reflection [10,11], we obtain the meson BS wave functions with the natural J^{PC} quantum number which are classified into three categories.

(1) $S=0$, J^{-+} ($J=2n$), and J^{+-} ($J=2n+1$):

$$\phi(p,P)=\gamma_5(Ep)f_1+i\gamma_\mu\epsilon_{\mu\nu\rho\sigma}(E_\nu p)P_\rho p_\sigma(Pp)f_2+i\gamma_5\gamma_\mu[P_\mu(Ep)f_3+p_\mu(Ep)(Pp)f_4+(E_\mu p)(Pp)f_5] + i\sigma_{\mu\nu}\gamma_5[P_\mu p_\nu(Ep)f_6+P_\mu(E_\nu p)f_7+p_\mu(E_\nu p)(Pp)f_8] . \quad \text{(A1)}$$

(2) $S=1$, J^{--} ($J=2n+1$), and J^{++} ($J=2n$):

$$\phi(p,P)=i(Ep)g_1+\gamma_\mu[P_\mu(Ep)(Pp)g_2+p_\mu(Ep)g_3+(E_\mu p)g_4]+\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}(E_\nu p)P_\rho p_\sigma g_5 + \sigma_{\mu\nu}[P_\mu p_\nu(Ep)g_6+P_\mu(E_\nu p)g_7+p_\mu(E_\nu p)(Pp)g_8] . \quad \text{(A2)}$$

(3) $S=1$, J^{--} [$J=2(n+1)$], and J^{++} ($J=2n+1$):

$$\phi(p,P)=\gamma_5(Ep)(Pp)h_1+i\gamma_\mu\epsilon_{\mu\nu\rho\sigma}(E_\nu p)P_\rho p_\sigma h_2+i\gamma_5\gamma_\mu[P_\mu(Ep)(Pp)h_3+p_\mu(Ep)h_4+(E_\mu p)h_5] + i\sigma_{\mu\nu}\gamma_5[P_\mu p_\nu(Ep)(Pp)h_6+P_\mu(E_\nu p)(Pp)h_7+p_\mu(E_\nu p)h_8] . \quad \text{(A3)}$$

In (A1)–(A3), $n=0,1,2,\ldots$. The space wave functions f_i, g_i, and h_i ($i=1,2,\ldots,8$) are real scalar functions of p^2, $(\mathbf{p}\cdot\mathbf{P})^2$. The polarization tensors $E_{\mu_1\mu_2\cdots\mu_J}$ satisfy the following conditions:

$$E_{\cdots\mu_l\cdots\mu_k\cdots}=E_{\cdots\mu_k\cdots\mu_l\cdots} \quad \text{(symmetry)} ,$$

$$E_{\cdots\mu\cdots}P_\mu=0 \quad \text{(Lorentz condition)} ,$$

$$E_{\cdots\mu\cdots\mu\cdots}=0 \quad \text{(traceless)} .$$

The Greek index runs over 1,2,3,4. We have used the symbols $(Pp)\equiv P_\mu p_\mu$,

$$(Ep)\equiv E_{\mu_1\mu_2\cdots\mu_J}p_{\mu_1}p_{\mu_2}\cdots p_{\mu_J} ,$$

$$(E_\mu p)\equiv E_{\mu\mu_2\cdots\mu_J}p_{\mu_2}\cdots p_{\mu_J} .$$

The angular momentums are included in (Ep) and $(E_\mu p)$. For example, for vector meson $J=1$, (Ep) is reduced to $\mathbf{e}\cdot\mathbf{p}\sim Y_{1M}\bar{p}$ ($M=+1,0,-1$) in the c.m. system. If we make $(Ep)\rightarrow 1$, $(E_\mu p)\rightarrow 0$, then (A1)–(A3) are reduced to the $J=0$ BS wave functions.

In the approximation of the instantaneous interaction, the general forms of the three wave functions $\phi(\mathbf{p})$ in the c.m. system are the following.

(1) $S=0$, J^{-+} ($J=2n$), and J^{+-} ($J=2n+1$):

$$\phi(\mathbf{p})=\gamma_5(\mathbf{E}\cdot\mathbf{p})f_1-\gamma_5\gamma_4\mu(\mathbf{E}\cdot\mathbf{p})f_2 - \sigma_{4l}\gamma_5\mu[p_l(\mathbf{E}\cdot\mathbf{p})f_3-(\mathbf{E}_l\cdot\mathbf{p})f_4] . \quad \text{(A4)}$$

(2) $S=1$, J^{--} ($J=2n+1$), and J^{++} ($J=2n$):

$$\phi(\mathbf{p})=i(\mathbf{E}\cdot\mathbf{p})g_1+\gamma_l[p_l(\mathbf{E}\cdot\mathbf{p})g_2+(\mathbf{E}_l\cdot\mathbf{p})g_3] + i\gamma_5\gamma_l\mu\epsilon_{lk4j}(\mathbf{E}_k\cdot\mathbf{p})p_j g_4 + i\sigma_{4l}\mu[p_l(\mathbf{E}\cdot\mathbf{p})g_5+(\mathbf{E}_l\cdot\mathbf{p})g_6] . \quad \text{(A5)}$$

(3) $S=1$, J^{--} [$J=2(n+1)$], and J^{++} ($J=2n+1$):

$$\phi(\mathbf{p})=-\gamma_l\mu\epsilon_{lk4j}(\mathbf{E}_k\cdot\mathbf{p})p_j h_1 + i\gamma_5\gamma_l[p_l(\mathbf{E}\cdot\mathbf{p})h_2+(\mathbf{E}_l\cdot\mathbf{p})h_3] + i\sigma_{ij}\gamma_5(\mathbf{E}_i\cdot\mathbf{p})p_j h_4 . \quad \text{(A6)}$$

where

$$(\mathbf{E}\cdot\mathbf{p})=E_{i_1i_2\cdots i_J}p_{i_1}p_{i_2}\cdots p_{i_J} ,$$

$$(\mathbf{E}_l\cdot\mathbf{p})\equiv E_{li_2\cdots i_J}p_{i_2}\cdots p_{i_J} .$$

APPENDIX B: TWO-PARTICLE HELICITY STATES OF THE FREE $q\bar{q}$ SYSTEM WITH DEFINITE J^{PC} AND THE CORRESPONDING BS AMPLITUDES (TO TREAT THE 2^{++} TENSOR MESONS AS THE EXAMPLE)

1. Two-particle helicity states with definite J^{PC}

Suppose a spin-$\frac{1}{2}$ single-particle helicity state with mass m and momentum $\mathbf{p}$ is described by $|\mathbf{p},\mu\rangle$, where $\mu=\pm\frac{1}{2}$. The normalization of the state $|\mathbf{p},\mu\rangle$ is

$$\langle \mathbf{p}'\mu'|\mathbf{p}\mu\rangle=(2\pi)^3 2\varepsilon\delta(\mathbf{p}-\mathbf{p}')\delta_{\mu\mu'} , \tag{B1}$$

where $\varepsilon=(\mathbf{p}^2+m^2)^{1/2}$ is the energy of the particle.

A noninteracting two-particle helicity state can be described by the direct product of two single-particle helicity states, i.e.,

$$|\mathbf{p}_1\mu_1\rangle|\mathbf{p}_2\mu_2\rangle=|\mathbf{p}_1\mathbf{p}_2\mu_1\mu_2\rangle . \tag{B2}$$

The c.m. two-particle helicity state is

$$|\mathbf{p},-\mathbf{p}\mu_1\mu_2\rangle\equiv|\bar{p}\theta\phi\mu_1\mu_2\rangle =(2\pi)^3(4\sqrt{s}/\bar{p})^{1/2}|\theta\phi\mu_1\mu_2\rangle|P\rangle , \tag{B3}$$

where $\mathbf{p}=\frac{1}{2}(\mathbf{p}_1-\mathbf{p}_2)$ is the relative momentum, $(\theta\phi)$ are the polar angles of $\mathbf{p}$, $\bar{p}=|\mathbf{p}|$, P is the c.m. four-momentum, $s=(m_1^2+\mathbf{p}^2)^{1/2}+(m_2^2+\mathbf{p}^2)^{1/2}$. Normalization of the state $|\theta\phi\mu_1\mu_2\rangle$ is

$$\langle\theta'\phi'\mu_1'\mu_2'|\theta\phi\mu_1\mu_2\rangle=\delta(\cos\theta'-\cos\theta)\delta(\phi'-\phi)\times\delta_{\mu_1'\mu_1}\delta_{\mu_2'\mu_2} . \tag{B4}$$

We write the eigenstates of $\mathbf{J}^2$ and J_z with eigenvalue $J(J+1)$ and M as $|\bar{p}JM\mu_1\mu_2\rangle$ which is represented by

$$|\bar{p}JM\mu_1\mu_2\rangle=(4\sqrt{s}/\bar{p})^{1/2}|P\rangle|JM\mu_1\mu_2\rangle . \tag{B5}$$

Normalization of the state $|JM\mu_1\mu_2\rangle$ is

$$\langle J'M'\mu_1'\mu_2'|JM\mu_1\mu_2\rangle=\delta_{JJ'}\delta_{MM'}\delta_{\mu_1\mu_1'}\delta_{\mu_2\mu_2'} . \tag{B6}$$

Normalization of the state (B5) is

$$\langle\bar{p}J'M'\mu_1'\mu_2'|\bar{p}JM\mu_1\mu_2\rangle=\frac{4\sqrt{s}}{\bar{p}}\delta^4(P-P')\delta_{JJ'}\delta_{MM'}\times\delta_{\mu_1\mu_1'}\delta_{\mu_2\mu_2'} . \tag{B7}$$

The relations between the states $|JM\mu_1\mu_2\rangle$ and $|\theta\phi\mu_1\mu_2\rangle$ are [1]

$$|\theta\phi\mu_1\mu_2\rangle=\sum_{J,M}\left[\frac{2J+1}{4\pi}\right]^{1/2}D^J_{M,\mu}(\phi,\theta,-\phi)\times|JM\mu_1\mu_2\rangle , \tag{B8}$$

$$|JM\mu_1\mu_2\rangle=\left[\frac{2J+1}{4\pi}\right]^{1/2}\times\int d\Omega\, D^{J*}_{M,\mu}(\phi,\theta,-\phi)|\theta\phi\mu_1\mu_2\rangle , \tag{B9}$$

where $\mu=\mu_1-\mu_2$, $D^{J*}_{M,\mu}(\phi,\theta,-\phi)$ is the complex conjugate of the rotation matrix, $d\Omega=\sin\theta d\theta d\phi$. From (B3), (B5), and (B9), we get

$$|\bar{p}JM\mu_1\mu_2\rangle=\frac{1}{(2\pi)^3}\left[\frac{2J+1}{4\pi}\right]^{1/2}\times\int d\Omega\, D^{J*}_{M,\mu}(\phi,\theta,-\phi)|\mathbf{p},-\mathbf{p}\mu_1\mu_2\rangle . \tag{B10}$$

From the transformation properties under space reflection P and charge conjugate C of the two-particle helicity state with definite total angular momentum, we can determine the two-particle helicity state with J^{PC}. There is a phase difference between the helicity states $|\mathbf{p}\mu\rangle$ and $|-\mathbf{p}\mu\rangle$. We emphasize that the correct determination of the phase factor of the two-particle helicity state under space reflection is very important.

The $J^{PC}=2^{++}$ helicity state for the free $q\bar{q}$ system is [taking the relative inner parity between q and $\bar{q}$ as -1; $(+)(+)$ denotes $(P=+1)(C=+1)$]

$$|\bar{p}2M\mu_1\mu_2;(+)(+)\rangle=\tfrac{1}{2}\{|\bar{p}2M\mu_1\mu_2\rangle+|\bar{p}2M,-\mu_1,-\mu_2\rangle+(\mu_1\leftrightarrow\mu_2)\text{terms}\}\quad(2^{++}\text{ state}) . \tag{B11}$$

(B11) can be represented by the plane wave

$$|\bar{p}2M\mu_1\mu_2;(+)(+)\rangle\sim\int d\Omega\, D^{2*}_{M,\mu_1-\mu_2}(\phi,\theta,-\phi)\{|\mathbf{p},-\mathbf{p}\mu_1\mu_2\rangle+|\mathbf{p},-\mathbf{p},-\mu_1,-\mu_2\rangle+(\mu_1\leftrightarrow\mu_2)\text{terms}\} . \tag{B12}$$

Sum the μ_1 and μ_2, then (B12) is reduced to

$$\begin{aligned}|\mathbf{p}2M;(+)(+)\rangle&=\sum_{\mu_1,\mu_2=\pm\frac{1}{2}}c(\mu_1,\mu_2)|\mathbf{p}2M\mu_1\mu_2;(+)(+)\rangle\\&=c_1\int d\Omega\, D^{2*}_{M,0}(\phi,\theta,-\phi)\{|\mathbf{p},-\mathbf{p}\tfrac{1}{2}\tfrac{1}{2}\rangle+|\mathbf{p},-\mathbf{p},-\tfrac{1}{2},-\tfrac{1}{2}\rangle\}\\&\quad+c_2\int d\Omega\{D^{2*}_{M,1}(\phi,\theta,-\phi)|\mathbf{p},-\mathbf{p}\tfrac{1}{2},-\tfrac{1}{2}\rangle+D^{2*}_{M,-1}(\phi,\theta,-\phi)|\mathbf{p},-\mathbf{p},-\tfrac{1}{2}\tfrac{1}{2}\rangle\} ,\end{aligned} \tag{B13}$$

where c_1 and c_2 are two independent constants.

The two-particle helicity states with other J^{PC} values are

$$|\bar{p}00\mu_1\mu_2;(-)(+)\rangle=\tfrac{1}{2}\{|\bar{p}00\mu_1\mu_2\rangle-|\bar{p}00,-\mu_1,-\mu_2\rangle+(\mu_1\leftrightarrow\mu_2)\text{ terms}\}\,,$$
$$0^{-+}\text{ state }(S=0,\ s\text{ wave})\,, \tag{B14}$$

$$|\bar{p}1M\mu_1\mu_2;(+)(-)\rangle=\tfrac{1}{2}\{|\bar{p}1M\mu_1\mu_2\rangle-|\bar{p}1M,-\mu_1,-\mu_2\rangle+(\mu_1\leftrightarrow\mu_2)\text{ terms}\}\,,$$
$$1^{+-}\text{ state }(S=0,\ p\text{ wave})\,, \tag{B15}$$

$$|\bar{p}1M\mu_1\mu_2;(-)(-)\rangle=\tfrac{1}{2}\{|\bar{p}1M\mu_1\mu_2\rangle+|\bar{p}1M,-\mu_1,-\mu_2\rangle+(\mu_1\leftrightarrow\mu_2)\text{ terms}\}\,,$$
$$1^{--}\text{ state }(S=1,\ s\text{ wave})\,, \tag{B16}$$

$$|\bar{p}00\mu_1\mu_2;(+)(+)\rangle=\tfrac{1}{2}\{|\bar{p}00\mu_1\mu_2\rangle+|\bar{p}00,-\mu_1,-\mu_2\rangle+(\mu_1\leftrightarrow\mu_2)\text{ terms}\rangle\}\,,$$
$$0^{++}\text{ state }(S=1,\ p\text{ wave})\,, \tag{B17}$$

$$|\bar{p}1M\mu_1\mu_2;(+)(+)\rangle=\tfrac{1}{2}\{|\bar{p}1M\mu_1\mu_2\rangle-|\bar{p}1M,-\mu_1,-\mu_2\rangle-(\mu_1\leftrightarrow\mu_2)\text{ terms}\}\,.$$
$$1^{++}\text{ state }(S=1,\ p\text{ wave})\,. \tag{B18}$$

2. BS amplitudes for the free $q\bar{q}$ system

The BS amplitude for the "meson" is defined as

$$\phi^{ab}_{\alpha\beta}(x_1,x_2)=\langle 0|T\{\Psi^a_\alpha(x_1)\bar{\Psi}^b_\beta(x_2)\}|\zeta\rangle\,, \tag{B19}$$

where $|\zeta\rangle$ is the "meson" state, $\Psi(x_i)$ $(i=1,2)$ are the fermions' local field operators, x_i $(i=1,2)$ are space-time coordinates, a and b are the flavor and color indices (in the following we shall omit such indices), and α and β are the spinor indices. Introducing the c.m. coordinate $X=(x_1+x_2)/2$ and the relative coordinate $x=x_1-x_2$, (B19) is reduced to

$$\phi_{\alpha\beta}(x,X)=e^{iPX}\left\langle 0\left|T\left\{\Psi_\alpha\left[\frac{X}{2}\right]\bar{\Psi}_\beta\left[-\frac{x}{2}\right]\right\}\right|\zeta\right\rangle$$
$$\equiv e^{iPX}\phi_{\alpha\beta}(x)\,, \tag{B20}$$

where P is the total four-momentum of the system. In the momentum representation,

$$\phi_{\alpha\beta}(p)=\frac{1}{(2\pi)^4}\int d^4x\, e^{-ipx}\phi_{\alpha\beta}(x)\,. \tag{B21}$$

In (B19)–(B21), taking (B13) as the "meson" state $|\zeta\rangle$, by some tedious calculation, we obtain the BS amplitude for the free $q\bar{q}$ 2^{++} state

$$\begin{aligned}\phi(p,P)=&F_1(p,P)D^{2*}_{M,0}[(a_1+a_2)I+i(1-a_1a_2)n_l\gamma_l-(a_1-a_2)\gamma_4+(1+a_1a_2)n_l\sigma_{l4}]\\&+F_2(p,P)\{-i(1+a_1a_2)(N_+^2D^{2*}_{M,1}\xi_l+N_-^2D^{2*}_{M,-1}\eta_l)\gamma_l+(a_1+a_2)(N_+^2D^{2*}_{M,1}\xi_l-N_-^2D^{2*}_{M,-1}\eta_l)(i\gamma_5\gamma_l)\\&+(a_1-a_2)(N_+^2D^{2*}_{M,1}\xi_l-N_-^2D^{2*}_{M,-1}\eta_l)\Sigma_l-(1-a_1a_2)(N_+^2D^{2*}_{M,1}\xi_l+N_-^2D^{2*}_{M,-1}\eta_l)\sigma_{l4}\}\,,\end{aligned} \tag{B22}$$

where $\Sigma_l=\frac{1}{2}\theta_{ljk}\sigma_{jk}$, $F_i(p,P)$ $(i=1,2)$ are two independent scalar functions of p^2 and (pP), $n_l=p_l/\bar{p}$, $a_i=\bar{p}/(\varepsilon_i+m_i)$, $\varepsilon_i=(\mathbf{p}^2+m_i^2)^{1/2}$ $(i=1,2)$. ξ_l and η_l $(l=1,2,3)$ are two mutual orthogonal unit vectors,

$$\begin{aligned}&\xi_1=i\sin\phi-\cos\theta\cos\phi\,,\\&\xi_2=-(i\cos\phi+\cos\theta\sin\phi),\quad \xi_3=\sin\theta\,,\end{aligned} \tag{B23}$$

$$\begin{aligned}&\eta_1=i\sin\phi+\cos\theta\cos\phi\,,\\&\eta_2=-i\cos\phi+\cos\theta\sin\phi,\quad \eta_3=-\sin\theta\,.\end{aligned} \tag{B24}$$

N_+ and N_- are, respectively, the phase of the Dirac spinor $U_{1/2}(\mathbf{p})$ and $u_{-1/2}(\mathbf{p})$; Ref. [18] shows that

$$\begin{aligned}&N_+^2=-i\sqrt{2}N\exp(i\phi),\quad N_-^2=-i\sqrt{2}N\exp(-i\phi)\,,\\&N_+N_-=-i\sqrt{2}N\,,\end{aligned} \tag{B25}$$

where N is some constant. In (B22), $D^{2*}_{M,0}(\phi,\theta,-\phi)$ and $N_+^2D^{2*}_{M,1}(\phi,\theta,-\phi)\xi_l+N_-^2D^{2*}_{M,-1}(\phi,\theta,-\phi)\eta_l$ should be related to polarization tensor $f^{(M)}_{lk}$ and n_l. The results are

$$D^{2*}_{M,0}(\phi,\theta,-\phi)=\sqrt{3/2}f^{(M)}_{ij}n_in_j\,, \tag{B26}$$

$$N_+^2D^{2*}_{M,1}(\phi,\theta,-\phi)\xi_l+N_-^2D^{2*}_{M,-1}(\phi,\theta,-\phi)\eta_l=i\sqrt{2}[f^{(M)}_{lm}n_m-n_l(f^{(M)}_{km}n_kn_m)]\,, \tag{B27}$$

$$N_+^2D^{2*}_{M,1}(\phi,\theta,-\phi)\xi_l-N_-^2D^{2*}_{M,-1}(\phi,\theta,-\phi)\xi_l=i\sqrt{2}[-i(f^{(M)}_{ik}n_k)\epsilon_{lij4}n_j]\quad(M=2,1,0,-1,-2)\,, \tag{B28}$$

where $f^{(M)}_{ij}$ are the polarization tensors which can be represented by the polarization vector as follows:

$$\begin{aligned}&f^{(\pm2)}_{ij}=e_i^\pm e_j^\pm\,,\\&f^{(\pm1)}_{ij}=(e_i^\pm e_j^0+e_i^0e_j^\pm)/\sqrt{2}\,,\\&f^{(0)}_{ij}=\sqrt{2/3}[e_i^0e_j^0+(e_i^+e_j^-+e_i^-e_j^+)/2]\,.\end{aligned} \tag{B29}$$

The polarization vectors e_i^m $(m=+1,0,-1)$ satisfy the orthogonal normalization condition $e_i^me_i^{m'}=\delta_{mm'}$ and the Lorentz condition $P_le_l^m=0$. From these conditions it is easy to show that the polarization tensors $f^{(M)}_{ij}$ satisfy the following relations:

$$f_{ij}^{(M)}=f_{ji}^{(M)},\quad P_i f_{ij}^{(M)}=f_{ij}^{(M)}P_j=0\ ,$$
$$\sum_i f_{ii}^{(M)}=0,\quad f_{ij}^{(M)}f_{ij}^{(M')*}=\delta_{MM'}\ . \tag{B30}$$

Using (B26)–(B28), we rewrite (B22) as

$$\begin{aligned}\phi(p,P)=&F_1(p,P)(f_{ij}^{(M)}n_in_j)[(a_1+a_2)I+i(1-a_1a_2)n_l\gamma_l-(a_1-a_2)\gamma_4+(1+a_1a_2)n_l\sigma_{l4}]\\&+F_2(p,P)\{-i(1+a_1a_2)[f_{li}^{(M)}n_i-n_l(f_{ij}^{(M)}n_in_j)]\gamma_l+(a_1+a_2)(f_{ik}^{(M)}n_k)\epsilon_{lij4}n_j\gamma_5\gamma_l\\&-i(a_1-a_2)(f_{ik}^{(M)}n_k)\epsilon_{lij4}n_j\Sigma_l-(1-a_1a_2)[f_{li}^{(M)}n_i-n_l(f_{ij}^{(M)}n_in_j)]\sigma_{l4}\}\ ,\end{aligned} \tag{B31}$$

where the two independent scalar functions $F_i(p,P)$ $(i=1,2)$ correspond to the component of the p and the f wave in the 2^{++} state. Notice that the BS amplitude (B31) for the 2^{++} state of the free $q\bar{q}$ system in the momentum representation is reduced to the 1^{--} BS wave function if the factor $f_{ij}^{(M)}n_j$ is replaced by the polarization vector e_i^m.

The BS amplitudes for the free $q\bar{q}$ system with other J^{PC} are, for the 0^{-+} state,

$$\begin{aligned}\phi(p,P)=f_1(p,P)\gamma_5\{&(1+a_1a_2)I-(1-a_1a_2)\gamma_4\\&+i(a_1-a_2)n_l\gamma_l\\&-(a_1+a_2)n_l\sigma_{4l}\}\ ,\end{aligned} \tag{B32}$$

and for the 1^{+-} state,

$$\begin{aligned}\phi(p,P)=f_2(p,P)(\mathbf{e}\cdot\mathbf{n})\gamma_5\{&(1+a_1a_2)I+i(a_1-a_2)n_l\gamma_l\\&-(1-a_1a_2)\gamma_4\\&-(a_1+a_2)n_l\sigma_{4l}\}\ .\end{aligned} \tag{B33}$$

The 1^{--} state has two components, the s wave and the d wave; $1^{--}s$ wave:

$$\begin{aligned}\phi(p,P)=f_3(p,P)\{&(1+a_1a_2)[e_l-(\mathbf{e}\cdot\mathbf{n})n_l]\gamma_l-i(a_1+a_2)(\mathbf{e}\times\mathbf{n})_l\gamma_l\gamma_5-(a_1-a_2)(\mathbf{e}\times\mathbf{n})_l\Sigma_l\\&+i(1-a_1a_2)[e_l-(\mathbf{e}\cdot\mathbf{n})n_l]\sigma_{4l}\}\ ,\end{aligned} \tag{B34}$$

$1^{--}d$ wave:

$$\begin{aligned}\phi(p,P)=f_4(p,P)(\mathbf{e}\cdot\mathbf{n})\{&-i(a_1+a_2)I+(1-a_1a_2)n_l\gamma_l\\&+i(a_1-a_2)\gamma_4\\&+i(1+a_1a_2)n_l\sigma_{4l}\}\ ,\end{aligned} \tag{B35}$$

0^{++} state:

$$\begin{aligned}\phi(p,P)=f_5(p,P)\{&-i(a_1+a_2)I+(1-a_1a_2)n_l\gamma_l\\&+i(a_1-a_2)\gamma_4\\&+i(1+a_1a_2)n_l\sigma_{4l}\}\ ,\end{aligned} \tag{B36}$$

1^{++} state:

$$\begin{aligned}\phi(p,P)=f_6(p,P)\{&-(1+a_1a_2)(\mathbf{e}\times\mathbf{n})_l\gamma_l\\&+i(a_1+a_2)[e_l-(\mathbf{e}\cdot\mathbf{n})n_l]\gamma_5\gamma_l\\&-(a_1-a_2)[e_l-(\mathbf{e}\cdot\mathbf{n})n_l]\Sigma_l\\&+i(1-a_1a_2)(\mathbf{e}\times\mathbf{n})_l\sigma_{l4}\}\ .\end{aligned} \tag{B37}$$

As we emphasized in the Introduction, in the BS framework the BS wave function is related to the residue of Green function at the pole $P^2=-\mu^2$, so the bound state must be on its mass shell. From four-momentum conservation it follows that in the vertex with bound state it is impossible to have all legs on their mass shells. This basic feature is represented by the fact that the BS normalization condition [9] cannot be satisfied by the BS amplitude of the free $q\bar{q}$ system. In fact, it is easy to show that (2.15) cannot be satisfied by (B31)–(B37).

APPENDIX C: THE BS WAVE FUNCTIONS OF MESONS WITH NATURAL J^{PC}

Assuming the types of coupling in (3.12)–(3.14), by the same method as in Sec. II, we obtain the BS wave functions of mesons with natural J^{PC} as follows (in the c.m. system).

(1) $S=0$, J^{-+} $(J=2n)$, and J^{+-} $(J=2n+1)$:

$$\begin{aligned}\phi(p,P)=\gamma_5\{&(p^2+m_1m_2+\tfrac{1}{4}\mu^2)+i(m_2-m_1)p_l\gamma_l\\&-[\tfrac{1}{2}\mu(m_1+m_2)+(m_2-m_1)p_0]\gamma_4-\mu p_l\sigma_{4l}\}(\mathbf{E}\cdot\mathbf{p})f(p,P)\ .\end{aligned} \tag{C1}$$

(2) $S=1$, J^{--} $(J=2n+1)$, and J^{++} $(J=2n)$:

$$\phi(p,P)=\{-i\mu\epsilon_{lj4k}(\mathbf{E}_j\cdot\mathbf{p})p_k\gamma_5\gamma_l+(p^2+m_1m_2+\tfrac{1}{4}\mu^2)[(\mathbf{E}_l\cdot\mathbf{p})-n_ln_j(\mathbf{E}_j\cdot\mathbf{p})]\gamma_l-(m_2-m_1)(\mathbf{E}_l\cdot\mathbf{p})p_k\sigma_{lk}$$
$$+i[\tfrac{1}{2}\mu(m_1+m_2)+(m_2-m_1)p_0][(\mathbf{E}_l\cdot\mathbf{p})-n_ln_j(\mathbf{E}_j\cdot\mathbf{p})]\sigma_{4l}\}g(p,P)\ . \quad \text{(C2)}$$

(3) $S=1$, J^{--} $[J=2(n+1)]$, and J^{++} $(J=2n+1)$:

$$\phi(p,P)=\{i\mu[(\mathbf{E}_l\cdot\mathbf{p})\bar{p}^{\,2}-(\mathbf{E}\cdot\mathbf{p})p_l]\gamma_5\gamma_l+(p^2+m_1m_2+\tfrac{1}{4}\mu^2)\epsilon_{lj4k}(\mathbf{E}_j\cdot\mathbf{p})p_k\gamma_l-(m_2-m_1)[(\mathbf{E}_l\cdot\mathbf{p})\bar{p}^{\,2}-(\mathbf{E}\cdot\mathbf{p})p_l]\Sigma_l$$
$$+i[\tfrac{1}{2}\mu(m_1+m_2)+(m_2-m_1)p_0]\epsilon_{lj4k}(\mathbf{E}_j\cdot\mathbf{p})p_l\sigma_{4l}\}h(p,P)\ . \quad \text{(C3)}$$

It is easy to show that for the above three categories (C1)–(C3) the BS normalization condition (2.15) is reduced to

$$N_i^2\int d^4p\{\mu(p^2+m_1m_2+\tfrac{1}{4}\mu^2)^2-\mu(p^2+m_1m_2)[4\bar{p}^{\,2}+(m_1+m_2)^2]-\mu(m_2^2-m_1^2)p^2$$
$$-2p_0(m_2^2-m_1^2)(p^2+m_1m_2-\tfrac{1}{4}\mu^2)\}[F_J^{(i)}(p,P)]^2=1\ , \quad \text{(C4)}$$

where N_i is the normalization constant. For $i=1$, $(F_J^{(1)})^2=\bar{p}^{\,2J}f^2(p,P)$; for $i=2$, $(F_J^{(2)})^2=\bar{p}^{\,2J-2}g^2(p,P)$; for $i=3$, $(F_J^{(3)})^2=\bar{p}^{\,2J}h^2(p,P)$.

The corresponding three-wave-functions in the instantaneous approximation are the following.

(1) $S=0$, J^{-+} $(J=2n)$, and J^{+-} $(J=2n+1)$:

$$\phi(\mathbf{p})=\gamma_5\left[1-\frac{1}{\mu}(m_1+m_2)\gamma_4-\frac{2}{\mu}p_l\sigma_{4l}\right](\mathbf{E}\cdot\mathbf{p})f(\mathbf{p}^2)\ . \quad \text{(C5)}$$

(2) $S=1$, J^{--} $(J=2n+1)$, and J^{++} $(J=2n)$:

$$\phi(\mathbf{p})=\left[\left[\gamma_l+i\frac{1}{\mu}(m_1+m_2)\sigma_{4l}\right][n_l(\mathbf{E}\cdot\mathbf{p})-(\mathbf{E}_l\cdot\mathbf{p})]+i\frac{2}{\mu}\gamma_5\gamma_l\epsilon_{lk4j}(\mathbf{E}_k\cdot\mathbf{p})p_j\right]g(\mathbf{p}^2)\ . \quad \text{(C6)}$$

(3) $S=1$, J^{--} $[J=2(n+1)]$, and J^{++} $(J=2n+1)$:

$$\phi(\mathbf{p})=\left[\left[\gamma_l+i\frac{1}{\mu}(m_1+m_2)\sigma_{4l}\right]\epsilon_{lk4j}(\mathbf{E}_k\cdot\mathbf{p})p_j-i\frac{2}{\mu}\gamma_5\gamma_l[p_l(\mathbf{E}\cdot\mathbf{p})-\bar{p}^{\,2}(\mathbf{E}\cdot\mathbf{p})]\right]h(\mathbf{p}^2)\ , \quad \text{(C7)}$$

where

$$(\mathbf{E}\cdot\mathbf{p})\equiv E_{i_1i_2\cdots i_J}p_{i_1}p_{i_2}\cdots p_{i_J}\ ,$$
$$(\mathbf{E}_l\cdot\mathbf{p})\equiv E_{li_2\cdots i_J}p_{i_2}\cdots p_{i_J}\ .$$

In (C1)–(C3), (C5)–(C7), taking $(\mathbf{E}\cdot\mathbf{p})\to 1$, $(\mathbf{E}_l\cdot\mathbf{p})\to 0$, we obtain the $J=0$ BS wave functions.

[1] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **1**, 404 (1959).
[2] C. H. Llewellyn Smith, Ann. Phys. (N.Y.) **53**, 251 (1969).
[3] M. Bohm, H. Joos, and M. Krammer, Nucl. Phys. B **51**, 397 (1973).
[4] J. R. Henley, Phys. Rev. D **20**, 532 (1979).
[5] Y. An, Z. Y. Zhu, and J. Z. Zhang, in *Proceedings of the 1980 Guangzhou Conference on Theoretical Particle Physics*, edited by H. Y. Zhu *et al.* (Science, Beijing, China, distributed by Van Nostrand Reinhold, New York, 1980), Vol. 1, p. 475.
[6] J. Z. Zhang, Phys. Lett. **112B**, 165 (1982).
[7] J. Z. Zhang (unpublished).
[8] S. Mandelstam, Proc. R. Soc. London, Ser. A **233**, 248 (1955).
[9] D. Lurie, *Particles and Fields* (Interscience, New York, 1968), Chap. 9.
[10] G. Feldman, T. Fulton, and J. Townsend, Ann. Phys. (N.Y.) **82**, 501 (1974).
[11] J. Z. Zhang, Physica Energine Fortis et Physca Nuclearis **7**, 688 (1983).
[12] Here we mean spinor structures (which are determined by kinematics), but not scalar functions f_i, g_i, or h_i in (A1)–(A3) (which are determined by dynamics). For scalar functions we emphasize that when boundary conditions are incorporated into an integral equation, there then occurs a kind of "discontinuity" between the bound and the scattering cases, so that it is not straightforward to infer behavior from one case to the other.
[13] V. K. Cung, T. Fulton, and W. E. Repko, Ann. Phys. (N.Y.) **96**, 261 (1976).
[14] V. K. Cung, T. Fulton, W. E. Repko, A. Schaum, and A. Devoto, Ann. Phys. (N.Y.) **98**, 516 (1976).
[15] Y. An and Z. Y. Zhu, Sci. Sin. **4**, 387 (1978).
[16] N. Hu, Sci. Sin. **22**, 295 (1979).
[17] W. Dykshoorn, R. Koniuk, and R. Munoz-Tapia, Phys. Rev. A **41**, 60 (1990); W. Dykshoorn and R. Koniuk, *ibid.* **41**, 64 (1990); T. Zhang and R. Koniuk, Phys. Lett. B **261**, 311 (1991).
[18] W. Buchmüller and S.-H. H. Tye, Phys. Rev. D **24**, 132 (1981).
[19] A. Martin, Phys. Lett. **100B**, 511 (1981).
[20] J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).