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Bell's-inequality experiments using independent-particle sources

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In the usual Bell's-inequality experiments two particles carrying spin or polarization are prepared in an entangled state generated from the decay of an unstable quantum-mechanical system. These particles are then delivered to spin or polarization analyzers. The statistics of the measurements reported by the analyzers are incompatible with our notions of local realism. Here we show that Bell's-inequality violations occur even when the initial state is a direct product state. In fact, the two particles can come from two independent widely separated sources.

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In Einstein-Podolsky-Rosen (EPR) experiments [1] of the Bohm type [2] two particles carrying spin are prepared, from the decay of an unstable system, in a spin eigenstate. Due to angular-momentum conservation, the spins of the two particles are correlated. The two particles are in fact in an entangled quantum-mechanical state, and, as Bell [3,4] has shown, when suitable sets of spin components of the two particles are measured, the statistics of the measurements are in conflict with our notions of local realism. Bell formulated this conflict as a mathematical inequality that is violated by quantum mechanics. Here we show that Bell's-inequality violations can be obtained even if the particles are in a direct product state. The two particles can originate from widely separated independent-particle sources. Each particle is directed to a 50-50 beam splitter and entangled with the vacuum. The outputs of the two beam splitters are then directed to two detectors, each consisting of two phase shifters, a beam splitter, and two particle counters. There are thus similarities between our apparatus and the one recently described by Tan, Walls, and Collett in which they [5] point out that a single particle entangled with the vacuum gives rise to Bell's inequalities when homodyne detection with weak local oscillators is employed. Further, the configuration of the experiment described here is equivalent to one described by Reid and Walls [6], although in their case the initial two-particle state was prepared from a parametric down-converter. The emphasis of the present paper is that the two particles can originate from independent widely separated sources. Also, we give a more rigorous and explicit derivation of the violation of local realism taking into account extraneous events that do not occur in the experiment considered by Bell. These spurious events consist of cases in which one detector reports the arrival of two particles while the other detector fails to fire.

The apparatus is shown in Fig. 1. The outputs of two independent-particle sources PS1 and PS2 are fed into the input ports of the beam splitters $S1$ and $S2$, respectively. Vacuum enters the other inputs of $S1$ and $S2$. The outputs of these two beam splitters propagate to two detectors. Detector 1 consists of phase shifters ϕ_{G1} and ϕ_{R1}, the beam splitter $D1$, and the particle counters $R1$ and $G1$. Similarly, detector 2 consists of phase shifters ϕ_{G2} and ϕ_{R2}, the beam splitter $D2$, and the particle counters $R2$ and $G2$. The labels R and G are chosen to be reminiscent of the red and green lights that Mermin employs in his EPR gedanken experiments [7,8]. Other EPR experiments employing detectors consisting of phase shifters, a beam splitter, and a pair of particle counters have been proposed [5,6,9] and even performed [10–12]. An arrangement of beam splitters, phase shifters, and mirrors like that of Fig. 1 was also discussed by Noh, Fougères, and Mandel in their study of homodyne detectors [13].

The annihilation operators of the modes entering the particle counters are labeled $d_{\alpha m}$ where here and

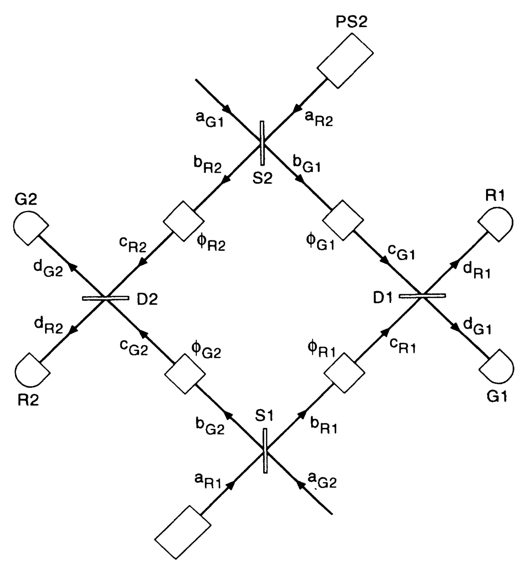

FIG. 1. Schematic of the present Bell's-inequality experiment. See text for detailed explanation.

throughout the paper $\alpha \in \{R,G\}$ and $m \in \{1,2\}$. These operators satisfy the usual fermion or boson commutation relations:

$$[d_{\alpha m}, d^{\dagger}_{\alpha' m'}]_{\pm} = \delta_{\alpha,\alpha'}\delta_{m,m'} , \quad (1)$$

$$[d_{\alpha m}, d_{\alpha' m'}]_{\pm} = 0 , \quad (2)$$

where "+" denotes anticommutation and "−" denotes commutation. The beam splitters $D1$ and $D2$ perform the mode transformation

$$\begin{bmatrix} d_{Rm} \\ d_{Gm} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} c_{Rm} \\ c_{Gm} \end{bmatrix} . \quad (3)$$

The phase shifters perform mode transformations of the form

$$c_{\alpha m} = e^{i\phi_{\alpha m}} b_{\alpha m} . \quad (4)$$

Finally, the mode transformations performed by the beam splitters $S1$ and $S2$ are, respectively,

$$\begin{bmatrix} b_{R1} \\ b_{G2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} a_{R1} \\ a_{G2} \end{bmatrix} , \quad (5)$$

$$\begin{bmatrix} b_{R2} \\ b_{G1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} a_{R2} \\ a_{G1} \end{bmatrix} . \quad (6)$$

Each of these beam splitters entangles the particle entering one input port with the vacuum entering the other input port [5].

The mode transformations Eqs. (3)–(6) can be solved to express the modes entering the beam splitters $S1$ and $S2$ in terms of the modes entering the particle counters. In particular, one finds

$$a^{\dagger}_{R1} = \tfrac{1}{2}[e^{i\phi_{R1}}(d^{\dagger}_{R1} + i d^{\dagger}_{G1}) + i e^{i\phi_{G2}}(i d^{\dagger}_{R2} + d^{\dagger}_{G2})] , \quad (7)$$

$$a^{\dagger}_{R2} = \tfrac{1}{2}[e^{i\phi_{R2}}(d^{\dagger}_{R2} + i d^{\dagger}_{G2}) + i e^{i\phi_{G1}}(i d^{\dagger}_{R1} + d^{\dagger}_{G1})] . \quad (8)$$

For the moment, it will be convenient to think of the experiment being operated in the following manner: At a particular instant of time each of the two independent sources emits a single particle. One then observes which particle counters fire. By performing an ensemble of such experiments one can accumulate data of the firing statistics of the particle counters. The state vector for the system is the direct product of the state vector for each individual source. In second quantized notation the state vector is given by

$$|\psi\rangle = a^{\dagger}_{R1} a^{\dagger}_{R2} |0\rangle . \quad (9)$$

By substituting Eqs. (7) and (8) into Eq. (9) it is straightforward to read off the probability amplitudes and, consequently, the probabilities for the various particle counter firing patterns. Let $P(\beta_1\beta_2)$ denote the probability that event β_1 occurs at detector 1 and event β_2 occurs at detector 2. The events β_m are elements of the set $\{0,R,G,R^2,G^2,E\}$ where 0 represents the event in which none of the particle counters of the detector fired, R denotes the event in which the particle counter labeled R counts a single particle, G denotes the event in which the particle counter labeled G counts a single particle, R^2 denotes the event in which the particle counter labeled R counts two particles, G^2 denotes the event in which the particle counter labeled G counts two particles, and E denotes the event in which each particle counter of the detector counts a single particle. Let A denote the set of firing patterns where both of the R particle counters or both of the G particle counters fire:

$$A = \{RR, GG\} . \quad (10)$$

Let B denote the set of firing patterns where each detector counts a single particle and only one of the R particle counters fires:

$$B = \{RG, GR\} . \quad (11)$$

Let C denote the set of firing patterns in which both counters of one detector fire and, as a consequence, none of the particle counters of the other detector fires:

$$C = \{0E, E0\} . \quad (12)$$

Let D denote the set of firing patterns in which one of the particle counters counts two particles and, as a consequence, none of the other particle counters fires:

$$D = \{0R^2, 0G^2, R^2 0, G^2 0\} . \quad (13)$$

For fermions, the Pauli exclusion principle prevents a given mode from being doubly occupied and, hence, events in which one particle counter counts more than one particle (events belonging to D) do not occur. For the fermion case, the probabilities for the particle firing patterns are given by

$$P(\beta_1\beta_2) = \begin{cases} \tfrac{1}{4}\sin^2\phi & \text{if } \beta_1\beta_2 \in A \\ \tfrac{1}{4}\cos^2\phi & \text{if } \beta_1\beta_2 \in B \\ \tfrac{1}{4} & \text{if } \beta_1\beta_2 \in C \\ 0 & \text{otherwise ,} \end{cases} \quad (14)$$

where

$$\phi = \tfrac{1}{2}[\phi_{R1} - \phi_{G1} + \phi_{R2} - \phi_{G2}] . \quad (15)$$

For bosons, because of a destructive interference effect, events for which both particle counters of a given detector fire (events belonging to the set C) cannot occur. For the boson case, the probabilities for the particle firing patterns are given by

$$P(\beta_1\beta_2) = \begin{cases} \tfrac{1}{4}\cos^2\phi & \text{if } \beta_1\beta_2 \in A \\ \tfrac{1}{4}\sin^2\phi & \text{if } \beta_1\beta_2 \in B \\ \tfrac{1}{8} & \text{if } \beta_1\beta_2 \in D \\ 0 & \text{otherwise ,} \end{cases} \quad (16)$$

where ϕ is again given by Eq. (15). In order to facilitate the comparison of this Bell's-inequality experiment with previously described experiments, we introduce the detector phases ϕ_1 and ϕ_2 which are defined as

$$\phi_1 = \phi_{R1} - \phi_{G1} , \quad (17)$$

$$\phi_2=\begin{cases}-(\phi_{R2}-\phi_{G2})+\pi/2 & \text{for fermions}\\ -(\phi_{R2}-\phi_{G2}) & \text{for bosons}\ .\end{cases} \tag{18}$$

With this definition of ϕ_1 and ϕ_2 the fermion and boson cases can be treated in a unified way and Eqs. (14) and (16) can be summarized as

$$P(\beta_1,\beta_2)=\begin{cases}\frac{1}{4}\cos^2(\phi_1-\phi_2) & \text{if } \beta_1\beta_2\in A\\ \frac{1}{4}\sin^2(\phi_1-\phi_2) & \text{if } \beta_1\beta_2\in B\\ \frac{1}{4} & \text{for fermions if } \beta_1\beta_2\in C\\ \frac{1}{8} & \text{for bosons if } \beta_1\beta_2\in D\\ 0 & \text{otherwise}\ .\end{cases} \tag{19}$$

To derive Bell's inequalities it is useful to consider a topological distortion of the apparatus depicted in Figs. 1 and 2(a) to the form depicted in Fig. 2(b). That is, by bringing the two sources together the apparatus resembles that used in the usual Bell's-inequality experiments. One can then ignore the details of how messages are generated by the two sources and simply talk about messages received by the detectors from a central source. Demonstrations of the violation of local realism then can be put forth following arguments that have been presented in the past. It is clear that, if the device of Fig. 2(b) violates local realism, the device of Fig. 2(a) also must violate local realism. Otherwise, one would have a local realistic explanation for how the device of Fig. 2(b) operates. That the standard Bell's inequalities [14,15] follow from Eq. (19) is seen by the following observation. Regardless of the settings of the detector phases ϕ_1 or ϕ_2, from an observation of what happens at one detector one can predict whether the firing pattern belongs to the set $A\cup B$ or C (in the case of fermions) or D (in the case of bosons). Since $A-D$ are disjoint sets, one concludes from a local realism point of view that, if one detector reports the event G or R, a definite instruction has been sent to the other detector to also fire G or R, i.e., the other detector cannot exhibit the event 0, E, R^2, or G^2. This is simply a consequence of the fact that the number of particles is conserved by the phase shifters and beam splitters. Since, from the observation of the outcome at a given detector, one can determine whether or not the firing pattern belongs to $A\cup B$, one can restrict one's attention to only those events in which both detectors fire to see if there is something peculiar about this subset of events. One can then follow standard procedures for obtaining the usual Bell's-inequalities. Here, to give specific examples, we will rederive an inequality due to Wigner [16] and Belinfante [17] and the Clauser-Horne inequality [18] taking special note of the extra events 0, E, R^2, and G^2 which do not occur in the usual Bell's-inequality experiments.

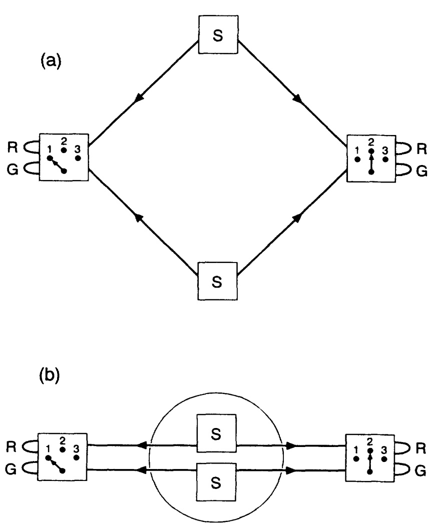

FIG. 2. An abstraction of the apparatus shown in Fig. 1. The particle sources are labeled S. The detectors are represented as boxes with red R and green G lights. The switches which can point to 1, 2, or 3 select the detector phases. The configuration depicted in (a) can be topologically distorted into the configuration depicted in (b). Configuration (b) is like that for conventional EPR experiments where particle pairs are emitted from a central source.

To derive the Wigner-Belinfante inequality [16,17], we restrict the settings of the phase shifters so that the detector phases ϕ_1 and ϕ_2 can only take on one of three values ϕ_a, ϕ_b, and ϕ_c. We will refer to these detector phase settings as setting 1, 2, and 3, respectively. Consider now the case when $\phi_1=\phi_2$, that is, when the detector phase settings are 11, 22, or 33. In this case, if one of the detectors reports R, the other detector also reports R and, similarly, if one detector reports G, the other detector reports G. Since the detector phases can be randomly changed up to the instant before the particle enters the detector and since if one detector reports an R or G the other detector will report an R or G, one concludes from a local realism point of view that it is because definite instructions of the form $\gamma_1\gamma_2\gamma_3;\gamma_1'\gamma_2'\gamma_3'$ have been sent, where the γ_i with $i\in\{1,2,3\}$ are elements of the set $\{R,G\}$. γ_i is the instruction to detector 1 telling it which particle counter to fire when the detector switch position is i. Similarly, γ_i' is the instruction to detector 2 telling it which particle counter to fire when the detector switch position is i. The allowed instruction sets for the case when a detector reports R or G are

$$RRR\ ;RRR,\ \ RRG;RRG\ ,\ \ RGR;RGR,\ \ RGG;RGG\ ,$$
$$GRR;GRR,\ \ GRG;GRG,\ \ GGR;GGR,\ \ GGG;GGG\ . \tag{20}$$

For the fermion case when one detector fails to fire (a 0 event), each particle counter of the other detector fires (an E event) and vice versa. Hence, in addition to the instructions Eq. (20) one has the instruction sets

$$000;EEE,\ \ EEE;000\ . \tag{21}$$

The instructions, Eqs. (20) and (21), form the complete set of legal instructions for the fermion case. The boson case is more complicated. If one detector fails to fire, the other detector will report an R^2 or a G^2. There is more than one way by which this could arise. One possibility is that a message has been sent telling the detector which of the two possible events R^2 or G^2 to report, in which case, in addition to Eq. (20) one has the following legal instructions:

$$\begin{aligned} &000;R^2R^2R^2,\ \ 000;R^2R^2G^2,\ \ 000;R^2G^2R^2,\ \ 000;R^2G^2G^2\ ,\\ &000;G^2R^2R^2,\ \ 000;G^2R^2G^2,\ \ 000;G^2G^2R^2,\ \ 000;G^2G^2G^2\ ,\\ &R^2R^2R^2;000\ \ R^2R^2G^2;000,\ \ R^2G^2R^2;000,\ \ R^2G^2G^2;000\ ,\\ &G^2R^2R^2;000,\ \ G^2R^2G^2;000,\ \ G^2G^2R^2;000,\ \ G^2G^2G^2;000\ . \end{aligned} \tag{22}$$

Another possibility is that a message has been sent to the second detector telling it to report either R^2 or G^2, but it is left up to the detector to decide which of the two to report. That is, one attributes the firing of R^2 or G^2 to detector noise. In this case the legal instruction set, in addition to Eq. (20), would contain the instructions

$$\begin{aligned} &0,0,0;R^2 \text{ xor } G^2,R^2 \text{ xor } G^2,R^2 \text{ xor } G^2\ ,\\ &R^2 \text{ xor } G^2,R^2 \text{ xor } G^2,R^2 \text{ xor } G^2;0,0,0\ , \end{aligned} \tag{23}$$

where xor denotes exclusive or and commas have been inserted between entries for clarity. We can, of course, imagine intermediate situations in which whether the second detector reports R^2 or G^2 is determined by a combination of a message sent to the detector and detector noise. As we will see, instruction sets Eqs. (21)–(23) do not enter into the derivation of the Bell's inequality.

Let $P(\beta_1,\beta_2,\phi_1,\phi_2)$ denote the probability that detector 1 reports the event β_1 and detector 2 reports the event β_2, given that the detector phase of detector 1 is set to ϕ_1 and the detector phase of detector 2 is set to ϕ_2. Further, let $P(\gamma_1\gamma_2\gamma_3;\gamma'_1\gamma'_2\gamma'_3)$ denote the probability that the instruction set $\gamma_1\gamma_2\gamma_3;\gamma'_1\gamma'_2\gamma'_3$ is sent. Then, one has

$$P(R,G,\phi_a,\phi_b)=P(RGR;RGR)+P(RGG;RGG)\ , \tag{24}$$

$$P(R,G,\phi_a,\phi_c)=P(RRG;RRG)+P(RGG;RGG)\ , \tag{25}$$

$$P(G,R,\phi_b,\phi_c)=P(RGR;RGR)+P(GGR;GGR)\ . \tag{26}$$

From Eqs. (24) and (26) one thus has

$$P(R,G,\phi_a,\phi_b)\geq P(RGG;RGG)\ , \tag{27}$$

$$P(G,R,\phi_b,\phi_c)\geq P(RGR;RGR)\ . \tag{28}$$

From these last two equations and Eq. (25) one then obtains the Bell's inequality:

$$P(R,G,\phi_a,\phi_b)\leq P(R,G,\phi_a,\phi_c)+P(G,R,\phi_b,\phi_c)\ . \tag{29}$$

Substituting Eq. (19) into this, one obtains

$$\sin^2(\phi_a-\phi_b)\leq \sin^2(\phi_a-\phi_c)+\sin^2(\phi_b-\phi_c)\ . \tag{30}$$

Taking

$$\phi_a-\phi_c=\phi_c-\phi_b=\theta\ , \tag{31}$$

this inequality becomes

$$\sin^2(2\theta)\leq 2\sin^2(\theta) \tag{32}$$

which is violated, for example, when $0<|\theta|<\pi/4$.

We now derive the Clauser-Horne [18] inequality for this system, using the methods of Wigner [16] and Belinfante [17], following Clauser and Shimony [14]. In this case, the number of detector phase settings to be considered is increased to four. The detector phases ϕ_1 and ϕ_2 will now be allowed to take on the values ϕ_a, ϕ_b, ϕ_c, and ϕ_d. We will refer to these phase settings as 1, 2, 3, and 4 respectively. For notational simplicity, we will consider only the fermion case. By considering the phase settings 11, 22, 33, and 44 and using local realism arguments similar to those used above, one again concludes that instruction sets are transmitted to the detectors telling them how to fire. These instruction sets can be represented in tabular form $\gamma_1\gamma_2\gamma_3\gamma_4;\gamma'_1\gamma'_2\gamma'_3\gamma'_4$ where again γ_i represents the instruction for what detector 1 is to do if its detector phase setting is i where $i\in\{1,3,3,4\}$ and γ'_i is the instruction for what detector 2 is to do when its detector phase setting is i. By considering switch settings of the form 12, 13, 14, 23, 24, and 34, one concludes that the only instruction sets allowed in which one detector fails to fire or both counters in one detector fire are

$$0000;EEEE\quad EEEE;0000\ . \tag{33}$$

In addition, the only instruction sets allowed in which one of the detectors reports an R or a G are those for which $\gamma_i=\gamma'_i$ and $\gamma_i\in\{R,G\}$ for all $i\in\{1,2,3,4\}$. Consequently, one can shorten the notation for the instruction set for this class to $\gamma_1\gamma_2\gamma_3\gamma_4$. There are 16 such instruction sets, examples of which are *RRRR*, *RRRG*, *RRGG*, *RGRR*, and *GGGG*. These 16 instructions together with the two instructions Eq. (33) form the complete list of legal instructions. Let $P(\beta_1,\beta_2,\phi_1,\phi_2)$ denote the probability that detector 1 reports the event β_1 when its detector phase is set to ϕ_1 and detector 2 reports the event β_2 when its detector phase is set to ϕ_2. Similarly, let $P(\gamma_1\gamma_2\gamma_3\gamma_4)$ denote the probability that the instruction set $\gamma_1\gamma_2\gamma_3\gamma_4$ is sent. Then, one has

$$\begin{aligned} P(R,R,\phi_a,\phi_c)=&P(RRRR)+P(RRRG)\\ &+P(RGRR)+P(RGRG)\ , \end{aligned} \tag{34}$$

$$\begin{aligned} P(R,R,\phi_a,\phi_d)=&P(RRRR)+P(RRGR)\\ &+P(RGRR)+P(RGGR)\ , \end{aligned} \tag{35}$$

$$P(R,R,\phi_b,\phi_c)=P(RRRR)+P(RRRG)+P(GRRR)+P(GRRG)\ , \quad (36)$$

$$P(R,R,\phi_b,\phi_d)=P(RRRR)+P(RRGR)+P(GRRR)+P(GRGR)\ . \quad (37)$$

Further, let $P_1(\beta_1,\phi_1)$ denote the probability that detector 1 reports the event β_1 when its detector phase is set to ϕ_1 and let $P_2(\beta_2,\phi_2)$ be the corresponding probability for detector 2. Then, one has

$$P_1(R,\phi_b)=P(RRRR)+P(RRRG)+P(RRGR)+P(RRGG)+P(GRRR)+P(GRRG)+P(GRGR)+P(GRGG)\ , \quad (38)$$

$$P_2(R,\phi_c)=P(RRRR)+P(RRRG)+P(RGRR)+P(RGRG)+P(GRRR)+P(GRRG)+P(GGRR)+P(GGRG)\ . \quad (39)$$

From Eqs. (34)–(39) one obtains

$$P(R,R,\phi_a,\phi_c)-P(R,R,\phi_a,\phi_d)+P(R,R,\phi_b,\phi_c)+P(R,R,\phi_b,\phi_d)-P_1(R,\phi_b)-P_2(R,\phi_c)$$
$$=-P(RRGR)-P(RGRR)-P(RRGG)-P(RGGR)-P(GRRG)-P(GGRR)-P(GRGG)-P(GGRG)\ . \quad (40)$$

Now, the sum over the probabilities of all legal instruction sets is 1:

$$\sum_{\text{over all}} P(\gamma_1\gamma_2\gamma_3\gamma_4;\gamma'_1\gamma'_2\gamma'_3\gamma'_4)=1\ . \quad (41)$$

However, from Eq. (19) one sees that the probability that one detector will report a 0 or an E is $\frac{1}{2}$, that is,

$$P(0,E)+P(E,0)=\tfrac{1}{2}\ . \quad (42)$$

It thus follows that half the time an instruction set will be emitted telling one of the detectors not to fire, that is, $P(0000;EEEE)+P(EEEE;0000)=\frac{1}{2}$. From Eqs. (41) and (42) it thus follows that half the time an instruction set is sent telling the detectors to report R or G:

$$\sum_{\text{over all}} P(\gamma_1\gamma_2\gamma_3\gamma_4)=\tfrac{1}{2}\ . \quad (43)$$

From this and from the fact that the probabilities are positive, one obtains from Eq. (40) the inequalities

$$-\tfrac{1}{2}\le P(R,R,\phi_a,\phi_c)-P(R,R,\phi_a,\phi_d)+P(R,R,\phi_b,\phi_c)+P(R,R,\phi_b,\phi_d)-P_1(R,\phi_b)-P_2(R,\phi_c)\le 0\ . \quad (44)$$

From the right-hand inequality of Eq. (40) one obtains

$$\frac{P(P,R,\phi_a,\phi_c)-P(R,R,\phi_a\phi_d)+P(R,R,\phi_b,\phi_c)+P(R,R,\phi_b,\phi_d)}{P_1(R,\phi_b)+P_2(R,\phi_c)}\le 1\ . \quad (45)$$

One can evaluate the various probabilities appearing in this expression using Eq. (19). In particular, one has

$$P_1(R,\phi_b)+P_2(R,\phi_c)=\tfrac{1}{2}\ , \quad (46)$$

$$P(R,R,\phi_1,\phi_2)=\tfrac{1}{4}\cos^2(\phi_1-\phi_2)\ . \quad (47)$$

Substituting Eqs. (46) and (47) into Eq. (45), and making the following choice for the angles:

$$\phi_a-\phi_c=\theta\ , \quad (48)$$

$$\phi_a-\phi_d=3\theta\ , \quad (49)$$

$$\phi_b-\phi_c=-\theta\ , \quad (50)$$

$$\phi_b-\phi_d=\theta\ , \quad (51)$$

one obtains

$$3\cos^2(\theta)-\cos^2(3\theta)\le 2\ . \quad (52)$$

This inequality is violated over a range of θ. The maximum violation occurs when $\theta=\pi/8$, in which case Eq. (52) becomes $1+\sqrt{2}\le 2$.

We have shown that the apparatus appearing in Figs. 1 and 2 gives rise to Bell's-inequality violations when the input state consists of the direct product state Eq. (9). Such states are generated from a parametric down-converter having separate signal and idler modes, provided that the pump intensity is low enough that events, in which more than one signal-idler photon pair is emitted during a coherence time, are sufficiently rare that they can be neglected. The state Eq. (9) need not be generated from a single source, however. Two independent sources each of which emits a particle at the same time will do. In fact, two free-running sources will work. A detailed treatment of this situation requires a wideband analysis [9] in which multitime correlation functions for the four counters employed are evaluated. We have carried out such an analysis for the case when the particle sources emit a beam of thermal fermions that has been momentum selected [19]. Here, however, we will confine ourselves to a qualitative discussion.

Consider first the fermion case. For each source, let the mean time between the emission of a particle be T and the coherence time be τ. The probability of the emission of a particle in a coherence time τ is thus τ/T. Because of the Pauli exclusion principle no more than two particles will arrive at the detectors in coincidence. In fact, if two particles arrive in coincidence, one infers that each particle came from a separate source (this can, in principle, be verified by monitoring the change in the number of particles in each source) and, consequently, one infers that the input state was of the form Eq. (9). In order for Eq. (9) to apply, the two particles must arrive within a time that is of the order of the coherence time τ. The rate at which such particle pairs arrive is $(\tau/T)^2$. Hence, provided one has detectors that can resolve the arrival of individual fermions on a time scale comparable to the coherence time, one can accumulate statistics exhibiting violations of local realism even if two independent free-running fermion sources are employed.

For the boson case, one no longer has a Pauli exclusion principle to take advantage of. However, the experiment will still work provided the light sources are sufficiently antibunched that one can certify that only one particle is emitted from the source during a coherence time τ. Alternatively, one could use two parametric down-converters in which the idler modes are monitored to determine when a particle is present in each signal mode [20,21].

In summary, we have shown that violations of local realism can occur even when the particles originate from independent-particle sources. An entangled state generated from the decay of an unstable quantum-mechanical system is not a prerequisite for a Bell's-inequality violation experiment. EPR effects are thus more ubiquitous than previously realized. In addition, we have recently shown that EPR effects of the Greenberger-Horne-Zeilinger type can also be performed with independent well-separated particle sources [19].

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