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Theory of a two-mode phase-sensitive amplifier

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A theory of two-mode phase-sensitive amplification by a three-level atomic system in the cascade configuration is presented, within the framework of the theory of multiwave mixing. Two photons of a strong external pump field induce coherence between the top and bottom levels. It is shown that both quadratures of the field modes acquire unequal gain and added noise. For large values of the dimensionless pump intensity, with a particular choice of its phase, and zero side-mode detuning, the system behaves as a nondegenerate parametric amplifier.

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I. INTRODUCTION

In optical amplifiers the bosonian nature of light adds noise to the signal in order to satisfy the uncertainty principle. The added noise depends on the internal degrees of freedom of the particular system [1]. In phase-insensitive amplifiers, an equal amount of noise is added to each quadrature of the signal field modes and the quadratures acquire equal gain [2–4]. The situation, however, becomes more interesting when we consider phase-sensitive amplification. In such amplifiers, an unequal amount of noise is added to the two quadratures of the field [5–7]. Recently, Scully and Zubairy [8] presented a theory of two-photon phase-sensitive amplification by a three-level atomic system in cascade configuration, where coherence was induced by injecting the atoms in a coherent superposition of the upper and the bottom levels. They showed that both the quadratures of the field are amplified with equal gain and added noise in one of the quadratures goes to zero at the expense of increased noise in the other quadrature, under certain conditions. In another type of two-photon phase-sensitive amplification, Ansari, Gea-Banacloche, and Zubairy [9] considered a three-level atomic system in cascade configuration, where a strong external field induces coherence by coupling the top and bottom levels. They showed that when the Rabi frequency of the classical field is much larger than the atomic level width, the system behaves as a degenerate parametric amplifier. They also predicted certain limits for phase-sensitive and phase-insensitive amplification.

In this paper, we assume the amplifier medium to consist of three-level atoms in cascade configuration. The bottom-to-top-level transition requires two photons of intense pump field. One pump photon detuning is assumed to be large, so that the transition from the bottom level to middle level with pump frequency is negligible. The transition from the top level to the bottom level via the intermediate level results in a buildup of two modes of frequencies ν_1 and ν_3. Our calculations are within the framework of the theory of multiwave mixing given by Sunghyuck and Sargent [10]. They considered this atomic system in cavity configuration and predicted strong squeezing under certain conditions. We show that the amplifier under consideration adds unequal amounts of noise to the two quadratures of the field modes, and they are amplified with unequal gain. For large values of the dimensionless pump intensity, particular choice of its phase and zero side-mode detuning, the added noise to the two quadratures goes to zero, and the system behaves as an ideal nondegenerate parametric amplifier.

II. DENSITY MATRIX EQUATION OF MOTION FOR THE FIELD MODES

We consider a three-level atomic system in cascade configuration as shown in Fig. 1. The upper level a and the bottom level c have the same parity and the intermediate level b has the opposite parity. The dipole-allowed transitions $a\leftrightarrow b$ and $b\leftrightarrow c$ with frequencies ν_1 and ν_3, respectively, are considered weak and treated quantum mechanically up to second order in coupling constant. The $a\leftrightarrow c$ transition requires two pump photons of frequency ν_2. Strong pump field is treated classically up to all orders. The one-photon detuning $\omega_b-\omega_c-\nu_2$ is assumed to be sufficiently large so that the dipole transition $c\leftrightarrow b$ with pump frequency is negligible. The pump frequency ν_2 is exactly one-half the atomic transition frequency $\omega_a-\omega_c$ and the resonance condition $\nu_1+\nu_3=2\nu_2$ is satisfied.

The Hamiltonian for the atom-field system is

$$H=H_0+V \ , \tag{1}$$

where the unperturbed part of the Hamiltonian is

$$H_0=\begin{vmatrix} \omega_a & 0 & 0 \\ 0 & \omega_b & 0 \\ 0 & 0 & \omega_c \end{vmatrix}+\sum_{j=1}^{3}\nu_j a_j^\dagger a_j \ , \tag{2}$$

and the perturbed part is

$$V=\sum_{j=1}^{3} g_j a_j U_j \sigma_j^\dagger+\text{H.c.} \ , \tag{3}$$

where a_1 and a_3 are the annihilation operators for the field modes 1 and 3, a_2 is the effective two-photon annihilation operator for the pump mode, $U_j=U_j(r)$ is the spa-

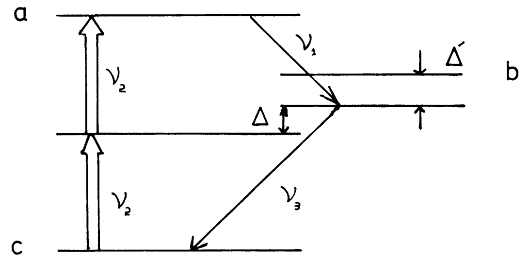

FIG. 1. Three-level atomic system in cascade configuration for phase-sensitive amplification.

tial mode factor for the jth field mode, and g_j is the corresponding atom-field coupling constant. The matrices $\sigma_j^\dagger$ are

$$\sigma_1^\dagger = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \sigma_2^\dagger = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \sigma_3^\dagger = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} .$$

The time dependence of the atom-field density operator ρ_{a-f} can be obtained from the basic density operator equation of motion, as

$$\dot{\rho}_{a-f} = -i[H, \rho_{a-f}] + r , \tag{4}$$

where r denotes the relaxation processes. By considering the slowly varying field modes and taking traces over the atomic states, the density matrix equation of motion for the field modes as obtained in Ref. [10] is

$$\begin{aligned} \dot{\rho} = & -A_1(\rho a_1 a_1^\dagger - a_1^\dagger \rho a_1) - B_1(a_1^\dagger a_1 \rho - a_1 \rho a_1^\dagger) \\ & - A_3(\rho a_3 a_3^\dagger - a_3^\dagger \rho a_3) - B_3(a_3^\dagger a_3 \rho - a_3 \rho a_3^\dagger) \\ & + C_3(a_3^\dagger a_1^\dagger \rho - a_1^\dagger \rho a_3^\dagger) + D_1(\rho a_3^\dagger a_1^\dagger - a_1^\dagger \rho a_3^\dagger) \\ & + \text{adjoint} . \end{aligned} \tag{5}$$

Different coefficients which appear in Eq. (5) are

$$A_1 = \frac{N g_1^2 \mathcal{D}_1}{(1+I_2^2)} \frac{f_a + I_2^2 \mathcal{D}_3^* D_2 / 4T_1T_2}{1 + I_2^2 \mathcal{D}_1 \mathcal{D}_3^* / 4T_1T_2} , \tag{6a}$$

$$B_1 = \frac{N g_1^2 \mathcal{D}_1}{(1+I_2^2)} \frac{f_b}{1 + I_2^2 \mathcal{D}_1 \mathcal{D}_3^* / 4T_1T_2} , \tag{6b}$$

$$A_3 = \frac{N g_3^2 \mathcal{D}_3}{(1+I_2^2)} \frac{f_b}{1 + I_2^2 \mathcal{D}_1^* \mathcal{D}_3 / 4T_1T_2} , \tag{6c}$$

$$B_3 = \frac{N g_3^2 \mathcal{D}_3}{(1+I_2^2)} \frac{f_c - I_2^2 \mathcal{D}_1^* D_2 / 4T_1T_2}{1 + I_2^2 \mathcal{D}_1^* \mathcal{D}_3 / 4T_1T_2} , \tag{6d}$$

$$C_3 = \frac{i N g_3^2 \mathcal{D}_3}{(1+I_2^2)} \frac{I_2}{2(T_1T_2)^{1/2}} \frac{-f_a \mathcal{D}_1^* + D_2}{1 + I_2^2 \mathcal{D}_1^* \mathcal{D}_3 / 4T_1T_2} e^{-i\phi} , \tag{6e}$$

$$D_1 = \frac{i N g_1^2 \mathcal{D}_1}{(1+I_2^2)} \frac{I_2}{2(T_1T_2)^{1/2}} \frac{f_c \mathcal{D}_3^* + D_2}{1 + I_2^2 \mathcal{D}_1 \mathcal{D}_3^* / 4T_1T_2} e^{-i\phi} . \tag{6f}$$

The complex Lorentzian for the field modes 1 and 3 is

$$\mathcal{D}_{1,3} = \frac{1}{\gamma_{1,3} + i\Delta_{1,3}} , \tag{7a}$$

where $\Delta_1 = \omega_a - \omega_b - \nu_1 = -\Delta'$ and $\Delta_3 = \omega_b - \omega_c - \nu_3 = \Delta'$. The side-mode detuning $\Delta' = \omega_b - \omega_c - \nu_2 - \Delta$ and $\Delta = \nu_2 - \nu_1$ is the beat frequency. The dipole decay constants for the $a \leftrightarrow b$ and $b \leftrightarrow c$ transitions are γ_1 and γ_3, respectively, and

$$D_2 = \frac{1}{\gamma_2} , \tag{7b}$$

where $\gamma_2 \equiv 1/T_2$ is the two-photon coherent decay rate between the levels a and c. The dimensionless pump intensity I_2 is

$$I_2 = 2|V_2|(T_1T_2)^{1/2} , \tag{7c}$$

where $V_2 = g_2 U_2 (n_2)^{1/2}$ is the effective two-photon interaction energy. The population difference decay time T_1 is

$$T_1 = \frac{1}{\Gamma_a} \left[1 + \frac{\Gamma_1}{2\Gamma_3} \right] , \tag{7d}$$

where $\Gamma_a (= \Gamma_1 + \Gamma_2)$ is the upper level decay rate to the lower levels b and c. Γ_1 and Γ_3 are the decay constants for the $a \to b$ and $b \to c$ transitions and Γ_2 allows for the nonradiative decay of level a to c. The probability factors f_k are

$$f_a = \frac{\Gamma_3}{\Gamma_1 + 2\Gamma_3} I_2^2 , \tag{7e}$$

$$f_b = \frac{\Gamma_1}{\Gamma_1 + 2\Gamma_3} I_2^2 , \tag{7f}$$

$$f_c = 1 + f_a . \tag{7g}$$

Also ϕ is the phase of the classical pump field which can be obtained from the relation

$$V_2 = |V_2| e^{-i\phi} , \tag{7h}$$

and N is the total number of interacting atoms.

The physical interpretation of these coefficients is as follows. The terms A_j and B_j with their complex conjugates are the gain and absorption coefficients for the jth mode, respectively, and the phase-dependent coefficients C_3 and D_1 lead to phase-sensitive amplification.

III. TWO-MODE LINEAR AMPLIFIER

In this section we consider the case of a two-mode linear amplifier. We first calculate the time-dependent solutions of the various operator expectation values and discover certain conditions under which we can factor out real gain and added noise in the two quadratures of the field modes.

We define the linear superposition of the coupled-mode annihilation operators for the field modes 1 and 3 as

$$X = \frac{1}{\sqrt{2}}(a_1 + a_3) . \tag{8}$$

The canonical-conjugate Hermitian operators are

$$X_1=\frac{1}{2\sqrt{2}}(X+X^\dagger)\ , \qquad (9a)$$

$$X_2=\frac{1}{2i\sqrt{2}}(X-X^\dagger)\ . \qquad (9b)$$

The equations of motions for the expectation values of the annihilation operators for the field modes 1 and 3 are

$$\frac{d}{dt}\langle a_1\rangle=-\alpha_1\langle a_1\rangle-D_1\langle a_3^\dagger\rangle\ , \qquad (10a)$$

$$\frac{d}{dt}\langle a_3\rangle=-\alpha_3\langle a_3\rangle+C_3\langle a_1^\dagger\rangle\ , \qquad (10b)$$

where $\alpha_1=B_1-A_1$ and $\alpha_3=B_3-A_3$.

The time-dependent solutions of Eqs. (10a) and (10b) are

$$\langle a_1\rangle_t=\left[\frac{1}{s}\left[s-\frac{\alpha_1-\alpha_3^*}{2}\right]\langle a_1\rangle_0 -\frac{D_1}{s}\langle a_3^\dagger\rangle_0\right]\frac{1}{2}e^{(-r+s)t} +\left[\frac{1}{s}\left[s+\frac{\alpha_1-\alpha_3^*}{2}\right]\langle a_1\rangle_0 +\frac{D_1}{s}\langle a_3^\dagger\rangle_0\right]\frac{1}{2}e^{-(r+s)t}\ , \qquad (11a)$$

$$\langle a_3\rangle_t=\left[\frac{1}{s^*}\left[s^*+\frac{\alpha_1^*-\alpha_3}{2}\right]\langle a_3\rangle_0 +\frac{C_3}{s^*}\langle a_1^\dagger\rangle_0\right]\frac{1}{2}e^{(-r^*+s^*)t} +\left[\frac{1}{s^*}\left[s^*-\frac{\alpha_1^*-\alpha_3}{2}\right]\langle a_3\rangle_0 -\frac{C_3}{s^*}\langle a_1^\dagger\rangle_0\right]\frac{1}{2}e^{-(r^*+s^*)t}\ , \qquad (11b)$$

where $r=(\alpha_1+\alpha_3^*)/2$ and $s=[(\alpha_1-\alpha_3^*)^2-4D_1C_3^*]^{1/2}/2$.

Equations (11a) and (11b) along with their complex conjugate give the gain of the two quadratures of the field modes. At first sight of these equations it seems very difficult to factor out gain in the two quadratures. If we impose certain conditions like $\alpha_1=\alpha_3=\alpha_1^*=\alpha_3^*=\alpha$ and $C_3=C_3^*=-D_1=-D_1^*=C_3$, then we can get real gain from Eqs. (11a) and (11b). For $I_2^2>1$ and $\Gamma_a=1$, $\Gamma_1=\Gamma_3=1$, $\gamma_1=\gamma_2=\gamma_3=\gamma=1$ we get $f_a=f_b\approx f_c$. Also for zero side-mode detuning, i.e., $\Delta'=0$, $g_1=g_3=g$, and $\phi=\pi/2$, the above conditions are satisfied and we get the simplified expressions for the coefficients of Eqs. (6a)–(6f), as

$$A_1\approx\frac{3C}{6+I_2^2}\ , \qquad (12a)$$

$$A_3=B_1\approx\frac{2C}{6+I_2^2}\ , \qquad (12b)$$

$$B_3\approx\frac{C}{6+I_2^2}\ , \qquad (12c)$$

$$C_3=-D_1\approx\frac{-2\sqrt{1.5}I_2C}{3(6+I_2^2)}\ , \qquad (12d)$$

where $C=Ng^2/\gamma$. Using Eqs. (9), (11a) and (11b) along with their complex conjugate and (12a)–(12d), we get

$$(X_1)_t=(G_1)^{1/2}(X_1)_0\ , \qquad (13a)$$

$$(X_2)_t=(G_2)^{1/2}(X_2)_0\ , \qquad (13b)$$

where

$$(G_1^{1/2})\approx e^{(-\alpha+c_3)t}\approx\exp\left[\frac{3-2\sqrt{1.5}I_2}{3(6+I_2^2)}\right]Ct\ , \qquad (14a)$$

$$(G_2^{1/2})\approx e^{-(\alpha+c_3)t}\approx\exp\left[\frac{3+2\sqrt{1.5}I_2}{3(6+I_2^2)}\right]Ct\ . \qquad (14b)$$

From Eqs. (14a) and (14b) it is clear that both the quadratures acquire unequal gain. For $I_2>1$, the first quadrature is deamplified, i.e., $G_1<1$, and the second quadrature is amplified with some gain, $G_2>1$. In the following calculations we will replace the $\approx$ sign with $=$, in order to avoid any complications.

Using Eqs. (5) and (12a)–(12d), the equations of motion for various expectation values of second-order moments are

$$\frac{d}{dt}\langle a_1^2\rangle=-2\alpha\langle a_1^2\rangle+2C_3\langle a_1a_3^\dagger\rangle\ , \qquad (15a)$$

$$\frac{d}{dt}\langle a_3^{\dagger 2}\rangle=-2\alpha\langle a_3^{\dagger 2}\rangle+2C_3\langle a_1a_3^\dagger\rangle\ , \qquad (15b)$$

$$\frac{d}{dt}\langle a_1a_3^\dagger\rangle=-2\alpha\langle a_1a_3^\dagger\rangle+C_3(\langle a_1^2\rangle+\langle a_3^{\dagger 2}\rangle)\ , \qquad (15c)$$

$$\frac{d}{dt}(\langle a_1^\dagger a_1\rangle+\langle a_3^\dagger a_3\rangle)=-2\alpha(\langle a_1^\dagger a_1\rangle+\langle a_3^\dagger a_3\rangle)+2C_3(\langle a_1a_3\rangle+\langle a_1^\dagger a_3^\dagger\rangle)+2(A_1+A_3)\ , \qquad (15d)$$

$$\frac{d}{dt}(\langle a_1a_3\rangle+\langle a_1^\dagger a_3^\dagger\rangle)=-2\alpha(\langle a_1a_3\rangle+\langle a_1^\dagger a_3^+\rangle)+2C_3(\langle a_1^\dagger a_1\rangle+\langle a_3^\dagger a_3\rangle)+2C_3\ . \qquad (15e)$$

The time-dependent solutions of Eqs. (15a)–(15e) are

$$\langle a_1^2\rangle_t=\left[\frac{\langle a_1^2\rangle_0}{2}(\tfrac{1}{2}+\cosh 2C_3t)+\frac{\langle a_3^{\dagger 2}\rangle_0}{2}(-\tfrac{1}{2}+\cosh 2C_3t)+2\langle a_1a_3^\dagger\rangle_0\sinh 2C_3t\right]e^{-2\alpha t}\ . \qquad (16a)$$

$$\langle a_3^{\dagger 2}\rangle_t=\left[\frac{\langle a_3^{\dagger 2}\rangle_0}{2}(\tfrac{1}{2}+\cosh 2C_3t)+\frac{\langle a_1^2\rangle_0}{2}(-\tfrac{1}{2}+\cosh 2C_3t)+2\langle a_1a_3^\dagger\rangle_0\sinh 2C_3t\right]e^{-2\alpha t}\ . \tag{16b}$$

$$\langle a_1a_3^\dagger\rangle_t=[\langle a_1a_3^\dagger\rangle_0\cosh 2C_3t+(\langle a_1^2\rangle+\langle a_3^{\dagger 2}\rangle)_0\sinh 2C_3t]e^{-2\alpha t} \tag{16c}$$

$$(\langle a_1^\dagger a_1\rangle+\langle a_3^\dagger a_3\rangle)_t=[(\langle a_1^\dagger a_1\rangle+\langle a_3^\dagger a_3\rangle)_0\cosh 2C_3+(\langle a_1a_3\rangle+\langle a_1^\dagger a_3^\dagger\rangle)_0\sinh 2C_3t]e^{-2\alpha t}$$
$$+\frac{(A_1+A_3)\alpha+C_3^2}{C_3^2-\alpha^2}(\cosh 2C_3te^{-2\alpha t}-1)+\frac{(A_1+A_3)C_3+C_3\alpha}{C_3^2-\alpha^2}\sinh 2C_3te^{-2\alpha t}\ , \tag{16d}$$

$$(\langle a_1a_3\rangle+\langle a_1^\dagger a_3^\dagger\rangle)_t=[(\langle a_1a_3\rangle+\langle a_1^\dagger a_3^\dagger\rangle)_0\cosh 2C_3t+(\langle a_1^\dagger a_1\rangle+\langle a_3^\dagger a_3\rangle)_0\sinh 2C_3t]e^{-2\alpha t}$$
$$+\frac{(A_1+A_3)C_3+C_3\alpha}{C_3^2-\alpha^2}(\cosh 2C_3te^{-2\alpha t}-1)+\frac{(A_1+A_3)\alpha+C_3^2}{C_3^2-\alpha^2}\sinh 2C_3te^{-2\alpha t}\ . \tag{16e}$$

To find out the added noise in the two quadratures defined by the Eqs. (13a) and (13b), we must consider the variance in both the quadratures, i.e.,

$$(\Delta X_i)_t^2=\langle X_i^2\rangle_t-\langle X_i\rangle_t^2\quad(i=1,2)\ . \tag{17}$$

Using Eqs. (12a)–(12d), (11a), (11b), and (16a)–(16e) along with their complex conjugate and (17), we finally get

$$(\Delta X_1)_t^2=G_1(\Delta X_1)_0^2+(G_1-1)N_1\ , \tag{18a}$$

$$(\Delta X_2)_t^2=G_2(\Delta X_2)_0^2+(G_2-1)N_2\ , \tag{18b}$$

where G_1 and G_2 are given by Eqs. (14a) and (14b), and

$$N_1=\frac{A_1+A_3+\alpha}{4(C_3-\alpha)}\ ,\qquad =\frac{3}{3-2.45I_2}\ , \tag{19a}$$

$$N_2=-\frac{A_1+A_3+\alpha}{4(C_3+\alpha)}\ ,\qquad =\frac{3}{3+2.45I_2}\ . \tag{19b}$$

Equations (14a), (14b), (19a), and (19b) give the approximate expressions for the gain and added noise in both quadratures of the coupled field modes. These expressions show that the two quadratures are amplified with unequal gain and an unequal amount of noise is added to them. Using Eqs. (14) and (19) Caves's theorem for phase-sensitive amplifiers becomes

$$N'\geq G'\ , \tag{20}$$

where

$$N'=|N_1N_2|\ , \tag{21a}$$

and

$$G'=\left|\frac{[(G_1G_2)^{1/2}-1]^2}{16(G_1-1)(G_2-1)}\right|\ . \tag{21b}$$

Equation (20) is also known as the amplifier uncertainty principle.

From Eqs. (14) and (19) it is clear that when $I_2\rightarrow\infty$, N_1 and $N_2\rightarrow 0$, i.e., both the noises approach zero. Also for $Ct\rightarrow\infty$ and $I_2\rightarrow\infty$, $G_1=1/G_2$ where $G_2>1$. Thus for very large values of dimensionless pump intensity the system behaves as an ideal nondegenerate parametric amplifier. As discussed in Ref. [9], we can also check it from Eqs. (5) and (12a)–(12d). For large values of I_2, the coefficients A_1, A_3, B_1, and B_3 approach zero much faster than C_3 and D_1 and (C_3+D_1) becomes zero. Under such conditions Eq. (5) reduces to the master equation for the nondegenerate parametric amplifier, in the absence of pump depletion. The amplifier uncertainty principle is also satisfied where both sides of Eq. (20) are equal to zero. In Fig. 2 we have plotted gain term versus dimensionless pump intensity I_2, for $Ct=1$. We start from $I_2=5$ so that $I_2^2=25>1$ to satisfy the above conditions which are essential to factor out real gain and added noise terms. The figure illustrates that G_1 is always less than one while the second quadrature amplifies with some gain. For large values of I_2 both the gains approach unity. In Fig. 3 we have plotted N_1 and N_2 versus I_2. This graph shows that N_1 noise is negative in order to keep the second term of Eq. (18a) positive for $G_1<1$. Here first we have phase-sensitive amplification

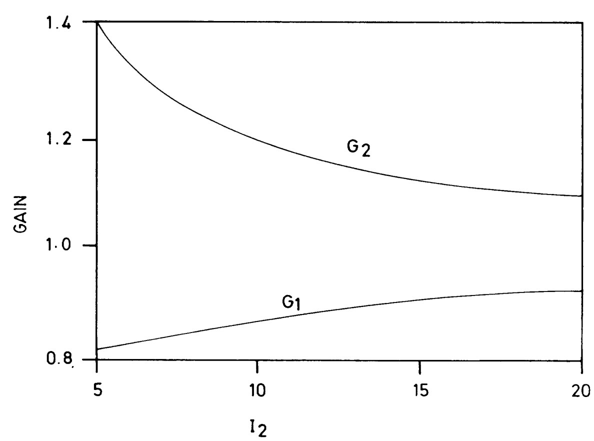

FIG. 2. Plot of gains G_1, G_2 vs the dimensionless pump intensity I_2. Pump phase angle $\phi=\pi/2$ and $Ct=1$, $\gamma_1=\gamma_2=\gamma_3=1$, $\Gamma_a=1$, and $\Gamma_1=\Gamma_3=1$.

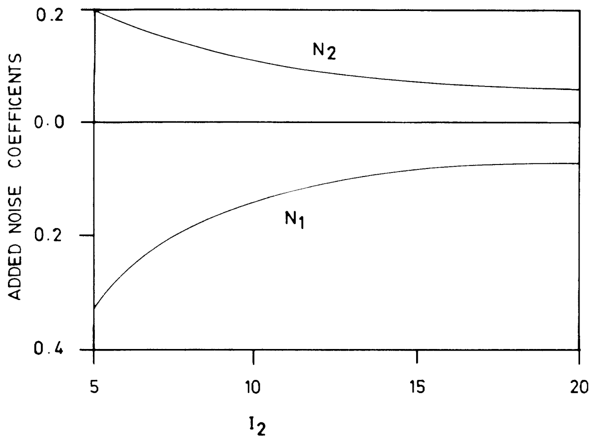

FIG. 3. Added noise in the two quadratures, N_1 and N_2, vs I_2. All parameters are the same as in Fig. 2.

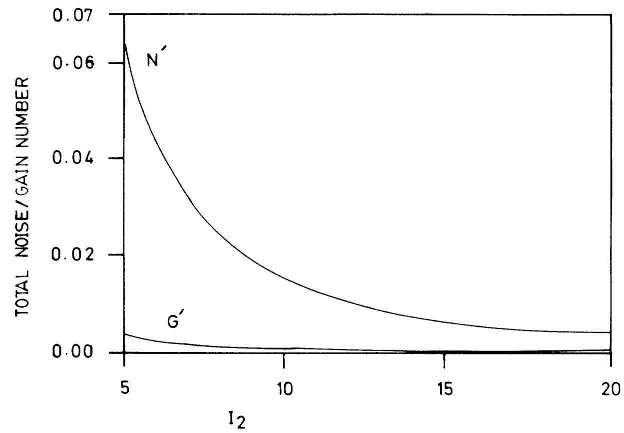

FIG. 4. Plot of total added noise number N' and total gain number G' vs dimensionless pump intensity I_2. All parameters are the same as in Fig. 2. Note that N' always remains larger than G', thus satisfying Caves's theorem.

and then for large values of I_2 both the noises go to zero (nondegenerate parametric amplifier limit). In Fig. 4 we have plotted both sides of Eq. (20) versus I_2. It can be seen from the graph that the amplifier uncertainty principle is satisfied for all values of I_2.

For nonzero side-mode detuning no condition is achieved under which Eqs. (13a), (13b), (18a), and (18b) are satisfied.

IV. DISCUSSION

In this paper we have developed a theory of the two-mode phase-sensitive linear amplifier. We consider a three-level atomic system in cascade configuration. Two photons of intense pump mode of frequency ν_2 are responsible for bottom-level–to–top-level transition. The top-level–to–bottom-level transition via the middle level results in the production of two modes of frequencies ν_1 and ν_3 and the exact resonance condition $2\nu_2=\nu_1+\nu_3$ is satisfied. Treating the ν_1 and ν_3 frequencies quantum mechanically up to second order in coupling constant and pump-mode frequency up to all orders, we discuss conditions under which real gain and added noise can be factored out for the coupled field-mode Hermitian operators. We have predicted that these quadratures are amplified with unequal gain and an unequal amount of noise is added to them.

In Ref. [9], a theory of the two-photon phase-sensitive amplifier was developed, where the system behaves as a degenerate parametric amplifier under certain conditions. In the present case, we have predicted that the three-level cascade atomic system for two field modes also reduces to a nondegenerate parametric amplifier for the large values of dimensionless pump intensity.

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