Phase properties and atomic coherent trapping in the system of a three-level atom interacting with a bimodal field

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We have studied the time evolution of the phase operators of the radiation field in the system containing a V-type three-level atom interacting with the bimodal field by means of the phase formalism given by Pegg and Barnett [J. Mod. Opt. **36**, 7 (1989); Phys. Rev. A **39**, 1665 (1989)]. That the atomic coherent trapping will happen in an appropriate initial state of the system has also been verified.

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I. INTRODUCTION

The Jaynes-Cummings (JC) model [1] and its extension [2,3] have received considerable interest over recent years. This interest stems from the fact that these models can be exactly solvable in the rotating-wave approximation and yield nonclassical results such as the collapses and revivals of the atomic inversion [4], the squeezing of the field [5] and the atomic dipole [6], and the sub-Poissonian photon-counting statistics [7]. Recent advances in experiments using micromasers have led to the observation of some of these effects [8].

One of the most fundamental features of quantum mechanics is the linear superposition principle. The correlated-emission laser [9] and the laser without atomic inversion [10] are examples of phenomena resulting from this principle. Recently, some authors have discussed the relations among the amount of squeezing of the field [11], the atomic dipole [12], and the different superposition state preparations of an atom in the atom-field coupling system, respectively. Zaheer and Zubairy *et al.* [13,14] have shown that a two-level atom coupled to a single-mode field, initially prepared in a coherent superposition of its two states, is possible to obtain coherent trapping for a particular choice. Slosser and co-workers [15] have studied the evolution of a single-mode field driven by a current of two-level atoms, each interacting with the mode for a time τ, showing that under certain trapping conditions it evolves towards a new class of pure states. So it is necessary to study in depth the property of the field-atom coupling system.

As we know, the phase property of the radiation field is very important [16]. Recently, Pegg and Barnett [17,18] introduced a formalism based on a Hermitian phase operator that has properties coincident with those normally associated with phase. Also, Pegg and Barnett [19] rectified three minor errors in their previous paper [18] following a suggestion by Ma and Rhodes [20]. Some authors investigated the phase properties of the radiation field in the JC model [21,22] and the nonlinear JC model [23,24]. However, they all assumed that the two-level atom is initially in one of its two states and did not pay attention to the influence of the initial atomic superposition state on the phase properties of the radiation field.

In the present paper, we focus our attention on the effect of the initial atomic superposition state preparations on the phase property of the radiation field. First, we give the time-dependent state of the system containing a V-type three-level atom interacting with the bimodal field. Then, the Pegg-Barnett phase formalism is employed to calculate the phase probability distribution, the mean value, and the variance of the phase operators by means of an analytical method. In Sec. IV, we analyze the role of the initial atomic superposition state and verify that the atomic coherent trapping will happen for a particular choice. Finally, we present a conclusion.

II. TIME EVOLUTION OF THE STATE VECTOR OF THE SYSTEM

The scheme of the V-type three-level atomic system, as shown in Fig. 1, consists of two allowed transitions $|a\rangle \leftrightarrow |c\rangle$ and $|b\rangle \leftrightarrow |c\rangle$. Each interaction has a different mode of the field. In the rotating-wave approximation, its Hamiltonian is described by

$$H = H_0 + V \ , \tag{1}$$

where

$$H_0 = \sum_{i=a,b,c} \omega_i |i\rangle\langle i| + \nu_1 a_1^\dagger a_1 + \nu_2 a_2^\dagger a_2 \quad (\hbar = 1) \ , \tag{2}$$

$$V = g_1 a_1 |a\rangle\langle c| + g_1 a_1^\dagger |c\rangle\langle a| + g_2 a_2 |b\rangle\langle c| + g_2 a_2^\dagger |c\rangle\langle b| \ . \tag{3}$$

Here $a_i^\dagger$ and a_i $(i=1,2)$ are, respectively, the creation and annihilation operators for the field of frequency ν_i. $|i\rangle$ $(i=a,b,c)$ is the eigenstate of the atom with eigenfrequency ω_i, and g_i is the corresponding coupling constant. We assume the coupling constants to be real throughout the paper.

In the interaction picture, the state vector of this atom-field coupling system at time t can be described by

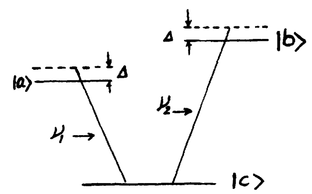

FIG. 1. Energy diagram of a three-level atom in the V-type configuration interacting with two quantized cavity modes.

$$|\Psi^I(t)\rangle=\sum_{n_1,n_2}(C_{a,n_1,n_2}|a,n_1,n_2\rangle+C_{b,n_1,n_2}|b,n_1,n_2\rangle+C_{c,n_1,n_2}|c,n_1,n_2\rangle)\ . \tag{4}$$

Substituting Eq. (4) into the Schrödinger equation in the interaction picture,

$$i\frac{d}{dt}|\Psi^I(t)\rangle=V^I(t)|\Psi^I(t)\rangle\ , \tag{5}$$

we obtain

$$\begin{aligned}\dot{C}_{a,n_1-1,n_2}&=-ig_1\sqrt{n_1}\exp(i\Delta t)C_{c,n_1,n_2}\ ,\\ \dot{C}_{b,n_1,n_2-1}&=-ig_2\sqrt{n_2}\exp(i\Delta t)C_{c,n_1,n_2}\ ,\\ \dot{C}_{c,n_1,n_2}&=-i(g_1\sqrt{n_1}C_{a,n_1-1,n_2}\\&\quad+g_2\sqrt{n_2}C_{c,n_1,n_2-1})\exp(-i\Delta t)\ ,\end{aligned} \tag{6}$$

where $\Delta=\omega_a-\omega_c-\nu_1=\omega_b-\omega_c-\nu_2$.

If the atom is initially in the state $|\Psi_A(0)\rangle$,

$$|\Psi_A(0)\rangle=\cos\left[\frac{\alpha}{2}\right]|a\rangle+\sin\left[\frac{\alpha}{2}\right]\exp(-i\psi)|b\rangle\ , \tag{7}$$

which means that the atom is in the coherent superposition state of its eigenkets $|a\rangle$ and $|b\rangle$, and the field is in the superposition of the photon number states at time $t=0$

$$|\Psi_f(0)\rangle=\sum_{n_1,n_2}F_{n_1,n_2}|n_1,n_2\rangle\ , \tag{8}$$

where $\sum_{n_1,n_2}|F_{n_1,n_2}|^2=1$, then the state vector of the total system at $t=0$ can be described as

$$|\Psi(0)\rangle=\sum_{n_1,n_2}\left[\cos\left[\frac{\alpha}{2}\right]F_{n_1-1,n_2}|a,n_1-1,n_2\rangle+\sin\left[\frac{\alpha}{2}\right]\exp(-i\psi)F_{n_1,n_2-1}\times|b,n_1,n_2-1\rangle\right]\ . \tag{9}$$

By using the initial condition [Eq. (9)], we obtain the solution of Eq. (6) as

$$\begin{aligned}C_a(t)&=-A_1g_1\sqrt{n_1}\times\left[\frac{e^{i(\Delta/2+\beta)t}-1}{\Delta/2+\beta}-\frac{e^{i(\Delta/2-\beta)t}-1}{\Delta/2-\beta}\right]+\cos\left[\frac{\alpha}{2}\right]F_{n_1-1,n_2}\ ,\\ C_b(t)&=-A_1g_2\sqrt{n_2}\times\left[\frac{e^{i(\Delta/2+\beta)t}-1}{\Delta/2+\beta}-\frac{e^{i(\Delta/2-\beta)t}-1}{\Delta/2-\beta}\right]+\sin\left[\frac{\alpha}{2}\right]\exp(-i\psi)F_{n_1,n_2-1}\ ,\\ C_c(t)&=-A_1\left\{\exp\left[-i\left[\frac{\Delta}{2}-\beta\right]t\right]-\exp\left[-i\left[\frac{\Delta}{2}+\beta\right]t\right]\right\}\ .\end{aligned} \tag{10}$$

Here we have used the symbols $C_a(t)$, $C_b(t)$, and $C_c(t)$ to replace the symbols $C_{a,n_1-1,n_2}(t)$, $C_{b,n_1,n_2-1}(t)$, and $C_{c,n_1,n_2}(t)$, respectively, and A_1 and β obey

$$A_1=-\left[g_1\sqrt{n_1}\cos\left[\frac{\alpha}{2}\right]F_{n_1-1,n_2}+\sin\left[\frac{\alpha}{2}\right]\exp(-i\psi)F_{n_1,n_2-1}\right]\Big/(2\beta)\ , \tag{11}$$

$$\beta=(\Delta^2/4+g_1^2n_1+g_2^2n_2)^{1/2}\ ,$$

where β is associated with the frequency of the atomic Rabi oscillation. Substituting Eq. (10) into Eq. (4), we can obtain the state vector of the system at time t in the interaction picture. Then transferring it to the Schrödinger picture, we have

$$|\Psi(t)\rangle=\sum_{n_1,n_2}(C_a(t)\exp\{-i[(n_1-1)\nu_1+n_2\nu_2+\omega_a]t\}|a,n_1-1,n_2\rangle+C_b(t)\exp\{-i[n_1\nu_1+(n_2-1)\nu_2+\omega_b]t\}|b,n_1,n_2-1\rangle+C_c(t)\exp\{-i[n_1\nu_1+n_2\nu_2+\omega_c]t\}|c,n_1,n_2\rangle)\ . \tag{12}$$

Starting from Eq. (12), we can discuss the time evolution of the phase operators of the bimodal field, and analyze the influence of the different α,β on the properties of the system.

III. TIME EVOLUTION OF THE PHASE OPERATORS IN THE ATOM-FIELD COUPLING SYSTEM

To study the phase properties of the atom-field coupling system we use the Hermitian phase formalism introduced by Pegg and Barnett [17,18]. The phase operator of the one-mode field operates on an $(s+1)$-dimensional subspace Ψ spanned by the number states $|0\rangle, |1\rangle, \ldots, |s\rangle$. The value of s can be made to tend to infinity after all necessary expectation values have been calculated. A complete orthonormal basis of the $(s+1)$ phase state is defined as

$$|\theta_m\rangle=(s+1)^{-1/2}\sum_{n=0}^{s}\exp(in\theta_m)|n\rangle \tag{13}$$

with

$$\theta_m=\theta_0+2\pi m/(s+1)\ ,\quad m=0,1,2,\ldots,s \tag{14}$$

where the value of θ_0 is arbitrary. These states are eigenstates of the Hermitian phase operator

$$\Phi_\theta=\sum_{m=0}^{s}\theta_m|\theta_m\rangle\langle\theta_m|\ . \tag{15}$$

For a two-mode field, the phase states are defined by [18]

$$|\theta_{m_1},\theta_{m_2}\rangle=[(s_1+1)(s_2+1)]^{-1/2}\times\sum_{n_1=0}^{s_1}\sum_{n_2=0}^{s_2}\exp[i(n_1\theta_{m_1}+n_2\theta_{m_2}\}]|n_1,n_2\rangle\ , \tag{16}$$

where the limits $s_1\rightarrow\infty$ and $s_2\rightarrow\infty$ are taken at a suitable point in the calculations. For simplicity, we here take $s_1=s_2=s$, and the phase Hermitian operator of the ith mode in the two-mode field can be described as

$$\Phi_i=\sum_{m_1=0}^{s}\sum_{m_2=0}^{s}\theta_{m_i}|\theta_{m_1},\theta_{m_2}\rangle\langle\theta_{m_1},\theta_{m_2}|\quad(i=1,2)\ . \tag{17}$$

Therefore the state vector $|\Psi(t)\rangle$ of the atom-field coupling system can be spanned by the phase eigenstates as

$$|\Psi(t)\rangle=\sum_{m_1,m_2}[\langle a,\theta_{m_1},\theta_{m_2}|\Psi(t)\rangle|a,\theta_{m_1},\theta_{m_2}\rangle+\langle b,\theta_{m_1},\theta_{m_2}|\Psi(t)\rangle|b,\theta_{m_1},\theta_{m_2}\rangle+\langle c,\theta_{m_1},\theta_{m_2}|\Psi(t)\rangle|c,\theta_{m_1},\theta_{m_2}\rangle]\ ; \tag{18}$$

and

$$P(\theta_{m_1},\theta_{m_2},t)=|\langle a,\theta_{m_1},\theta_{m_2}|\Psi(t)\rangle|^2+|\langle b,\theta_{m_1},\theta_{m_2}|\Psi(t)\rangle|^2+|\langle c,\theta_{m_1},\theta_{m_2}|\Psi(t)\rangle|^2 \tag{19}$$

represents the phase probability distribution function. Thus the expectation value of the phase operator is

$$\langle\Phi_1^n\Phi_2^k\rangle=\sum_{m_1,m_2}\theta_{m_1}^n\theta_{m_2}^kP(\theta_{m_1},\theta_{m_2},t)\quad(n,k=0,1,2)\ . \tag{20}$$

If the radiation field is initially in the two-mode uncorrelated coherent state, i.e.,

$$F_{n_1,n_2}=\exp(-\bar{n}_1/2)\exp(-\bar{n}_2/2)\alpha_1^n\alpha_2^n/\sqrt{n_1!n_2!}\ , \tag{21}$$

where

$$\alpha_i=\sqrt{\bar{n}_i}\exp(i\zeta_i)\quad(i=1,2)\ , \tag{22}$$

and $\bar{n}_i$ is the mean photon number of the field mode i, and ζ_i is the phase angle of α_i. For $\bar{n}_1,\bar{n}_2\gg1$, then the photon number of the field can be well approximated by Gaussian distribution [17,22]

$$F_{n_1,n_2}\approx(4\pi^2\bar{n}_1\bar{n}_2)^{-1/4}\exp[i(n_1\zeta_1+n_2\zeta_2)]\times\exp\left[\frac{(n_1-\bar{n}_1)^2}{4\bar{n}_1}-\frac{(n_2-\bar{n}_2)^2}{4\bar{n}_2}\right]\ . \tag{23}$$

Considering the property of the photon number distribution function, the approximation

$$F_{n_1-1,n_2}\approx F_{n_1,n_2-1}\approx F_{n_1,n_2}\ , \tag{24}$$

$$\beta(n_1,n_2)=\bar{\beta}+g_1^2(n_1-\bar{n}_1)/(2\bar{\beta})+g_2^2(n_2-\bar{n}_2)/(2\bar{\beta})$$

is reasonable when we sum with respect to n_1 and n_2 [22,25]. Here $\bar{\beta}=\beta(\bar{n}_1,\bar{n}_2)$. So the phase probability distribution $P(\theta_{m_1},\theta_{m_2},t)$ can be approximated as

$$P(\theta_{m_1},\theta_{m_2},t)=[2\pi/(s+1)]^2(2\bar{n}_1/\pi)^{1/2}(2\bar{n}_2/\pi)^{1/2}\times\left\{\exp[-2(\bar{n}_1z_1+\bar{n}_2z_2)]\frac{g_1^2\bar{n}_1\sin^2(\alpha/2)+g_2^2\bar{n}_2\cos^2(\alpha/2)-g_1g_2\sqrt{\bar{n}_1\bar{n}_2}\sin\alpha\cos\psi}{g_1^2\bar{n}_1+g_2^2\bar{n}_2}\right.$$
$$+\exp[-2(\bar{n}_1x_1+\bar{n}_2x_2)]\frac{g_1^2\bar{n}_1\cos^2(\alpha/2)+g_2^2\bar{n}_2\sin^2(\alpha/2)+g_1g_2\sqrt{\bar{n}_1\bar{n}_2}\sin\alpha\cos\psi}{2\bar{\beta}(\bar{\beta}+\Delta/2)}$$
$$\left.+\exp[-2(\bar{n}_1y_1+\bar{n}_2y_2)]\frac{g_1^2\bar{n}_1\cos^2(\alpha/2)+g_2^2\bar{n}_2\sin^2(\alpha/2)+g_1g_2\sqrt{\bar{n}_1\bar{n}_2}\sin\alpha\cos\psi}{2\bar{\beta}(\bar{\beta}-\Delta/2)}\right\}\ , \tag{25}$$

where

$$z_i=(\zeta_i-\theta_m-\nu_i t)^2\ ,\quad x_i=[\zeta_i-\theta_m-\nu_i t+g_i^2 t/(2\bar{\beta})]^2\quad (i=1,2)\ ,\quad y_i=[\zeta_i-\theta_m-\nu_i t-g_i^2 t/(2\bar{\beta})]^2\ . \tag{26}$$

In the continued limit, i.e., $s\to\infty$, θ_{m_1} and θ_{m_2} are two continued variables. The phase probability density is normalized according to Refs. [17,22]

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(\theta_1,\theta_2,t)\left[\frac{s+1}{2\pi}\right]^2 d\theta_1 d\theta_2=1\ , \tag{27}$$

where $[(s+1)/2\pi]^2$ is the density of the phase states.

Using Eqs. (20) and (25), we obtain the time evolution of the phase operators

$$\langle\Phi_1\rangle=\zeta_1-\nu_1 t-A_2 g_2^2\Delta t\ , \tag{28}$$

$$\langle\Phi_2\rangle=\zeta_2-\nu_2 t-A_2 g_2^2\Delta t\ , \tag{29}$$

$$\langle\Phi_1^2\rangle=\frac{1}{4\bar{n}_1}+(\zeta_1-\nu_1 t)^2+A_2 g_1^4 t-2A_2 g_1^2\Delta t(\zeta_1-\nu_1 t)\ , \tag{30}$$

$$\langle\Phi_2^2\rangle=\frac{1}{4\bar{n}_2}+(\zeta_2-\nu_2 t)^2+A_2 g_2^4 t-2A_2 g_2^2\Delta t(\zeta_2-\nu_2 t)\ , \tag{31}$$

$$\begin{aligned}\langle\Phi_1\Phi_2\rangle=(\zeta_1-\nu_1 t)(\zeta_2-\nu_2 t)+A_2&\left\{\frac{1}{\bar{\beta}+\Delta/2}\left[g_1^2 t(\zeta_2-\nu_2 t)+g_2^2 t(\zeta_1-\nu_1 t)+\frac{g_1^2 g_2^2 t^2}{2\bar{\beta}}\right]\right.\\&\left.+\frac{1}{\bar{\beta}-\Delta/2}\left[\frac{g_1^2 g_2^2 t^2}{2\bar{\beta}}-g_1^2 t(\zeta_2-\nu_2 t)-g_2^2 t(\zeta_1-\nu_1 t)\right]\right\}\ .\end{aligned} \tag{32}$$

So the phase fluctuations scan be expressed as

$$\langle\Delta\Phi_1\rangle^2=\langle\Phi_1^2\rangle-\langle\Phi_1\rangle^2=\frac{1}{4\bar{n}_1}+g_1^4 t^2 A_2(1-A_2\Delta^2)\ , \tag{33}$$

$$\langle\Delta\Phi_2\rangle^2=\langle\Phi_2^2\rangle-\langle\Phi_2\rangle^2=\frac{1}{4\bar{n}_2}+g_2^4 t^2 A_2(1-A_2\Delta^2)\ , \tag{34}$$

with

$$A_2=\frac{g_1^2\bar{n}_1\cos^2(\alpha/2)+g_2^2\bar{n}_2\sin 2(\alpha/2)+g_1 g_2\sqrt{\bar{n}_1\bar{n}_2}\sin\alpha\cos\psi}{4\bar{\beta}^2(g_1^2\bar{n}_1+g_2^2\bar{n}_2)}\ . \tag{35}$$

From Eqs. (28)–(34) we can see that the mean values and the fluctuations of the phase operator [Eqs. (28), (30), and (33)] for frequency ν_1 contain the constants $\bar{n}_2$ and g_2 about the field mode for frequency ν_2 because of both of the field modes interacting with the atom. The case for the mode with frequency ν_2 is the same. When $\Delta\neq 0$, from Eqs. (28) and (29) we can easily find

$$\frac{d}{dt}\langle\Phi_i\rangle\neq-\nu_i\quad (i=1,2)\ . \tag{36}$$

This means that the frequencies of the two fields are shifted. If $\Delta=0$, the frequency shift is zero, and the changes of $d\langle\Phi_i\rangle/dt$ and $\langle\Delta\Phi_i\rangle^2$ according to the mismatch Δ are nonlinear [23].

IV. ATOMIC COHERENT TRAPPING

Zaheer and Zubairy [13] considered a two-level atom, initially prepared in a coherent superposition of the upper and lower levels, interacting with a single-mode coherent state of the field in an ideal cavity. They found that the population inversion is far from exhibiting revivals and remains constant for a certain initial condition, which is in sharp contrast with the result of Yoo and Eberly [26], who got that a pure two-level system cannot exhibit coherent trapping. On the other hand, Yoo and Eberly in Ref. [26] pointed out that the coherent trapping did not take place also in the V-type atom-field coupling system. However, our current calculations give a different result that seems reasonable. Here, we present our discussion as follows.

For simplicity, we let $g_1=g_2=g$ and $n_1=n_2=n$, thus Eqs. (26) and (27) give

$$\langle\Delta\Phi_1\rangle^2=\langle\Delta\Phi_2\rangle^2=\frac{1}{4\bar{n}}+\frac{g^4 t^2}{8\bar{\beta}^2}(1+\sin\alpha\cos\psi)\times\left[1-\frac{\Delta^2(1+\sin\alpha\cos\psi)}{8\bar{\beta}^2}\right]\ . \tag{37}$$

It is clear that the second term of the right-hand side in Eq. (37) is zero when $\alpha=\pi/2$, $\psi=\pi$. (In fact the implication of $\alpha=-\pi/2$, $\psi=0$ is the same as that of $\alpha=\pi/2$, $\psi=\pi$.) Thus Eq. (37) becomes

$$\langle \Delta\Phi_1 \rangle^2 = \langle \Delta\Phi_2 \rangle^2 = \frac{1}{4\bar{n}} \ . \tag{38}$$

For $\bar{n} \gg 1$, we can regard the photon number distribution as a Poissonian function with the time development; we have [22]

$$\langle \Delta N_1 \rangle^2 = \langle \Delta N_2 \rangle^2 = \bar{n} \ , \tag{39}$$

so that the number-phase uncertainty products of the two-mode field are

$$\langle \Delta N_1 \rangle^2 \langle \Delta\Phi_1 \rangle^2 = \langle \Delta N_2 \rangle^2 \langle \Delta\Phi_2 \rangle^2 = \tfrac{1}{4} \ . \tag{40}$$

It shows that each mode of the uncorrelated two-mode field, which is initially in a coherent state, retains coherence with time development.

As we know, the atomic populations in the system containing a V-type atom interacting with the uncorrelated bimodal coherent field can exhibit the revivals and collapses with the time development when the atom is initially in one of its three states $|a\rangle$, $|b\rangle$, $|c\rangle$ [27]. But if the atom is initially in the state in which the field can retain its coherence, i.e., $\alpha=\pi/2$, $\psi=\pi$, we find that the atomic populations in the Hilbert space spanned by the field phase states and the atomic states obey

$$\begin{aligned} P_a(t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\frac{s+1}{2\pi}\right]^2 d\theta_1 d\theta_2 |\langle a,\theta_1,\theta_2|\Psi(t)\rangle|^2 \\ &= P_a(0) = \tfrac{1}{2} \ , \\ P_b(t) &= P_b(0) = \tfrac{1}{2} \ , \quad P_c(t) = P_c(0) = 0 \ . \end{aligned} \tag{41}$$

This means that the dynamics of the atom is the same as that for the initially excited case, and the field retains its coherence. So we can see that the coherent trapping occurs in the V-type atom. This result is quite surprising in view of the general belief that coherent trapping does not occur in a V-type three-level atom [26].

A possible explanation for such behavior can be as follows. When the mean photon number is very large, the probability of the transition between $|a\rangle \leftrightarrow |b\rangle$ is the same as that of the transition between $|b\rangle \leftrightarrow |c\rangle$, and the lifetime of the atom in state $|c\rangle$ is very short; its probability in state $|c\rangle$ is nearly zero. So we can regard the V-type atom as a two-level atom interaction with the bimodal field via a Raman two-photon process [28]; the atomic coherent trapping occurs in its initial asymmetric excitation. This initial condition is in agreement with the two-level atomic coherent trapping condition in the JC model [13]. Thus in this particular choice, the V-type three-level atom can undergo coherent trapping.

As we know, the system shown in Fig. (1) is the base of the correlated-emission laser. If we define an operator of phase difference

$$\Phi = \Phi_1 - \Phi_2 \ , \tag{42}$$

then the fluctuations of the phase difference satisfy

$$\langle \Delta\Phi \rangle^2 = \langle (\Phi_1 - \Phi_2)^2 \rangle - \langle \Phi_1 - \Phi_2 \rangle^2 = \frac{1}{2\bar{n}} \ . \tag{43}$$

It is evident that $\langle \Delta\Phi \rangle^2$ does not depend on time t, and is nearly the same as the fluctuations of phase difference in the correlated-emission laser. So as we can see, the phase property of the field in the V-type atom when the atomic coherent trapping occurs is the same as that of the correlated-emission laser [29].

V. CONCLUSIONS

We have investigated the phase properties of the two uncorrelated coherent field modes interacting with a V-type three-level atom, and have shown under certain conditions that the atom-field interaction has not given rise to the changes of the phase fluctuations; correspondingly, the phase-number uncertainty products have remained at a minimum. Our results have also exhibited that the coherent trapping can occur in this system for a particular choice of the initial atomic coherence, and in this case, the phase fluctuations are the same as that of the correlated-emission laser.

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[1] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).

[2] B. Buck and C. V. Sukumar, Phys. Lett. A **81**, 132 (1981); P. Alsing and M. S. Zubairy, J. Opt. Soc. Am B **5**, 177 (1987); M. S. Abdalla, M. M. A. Ahmed, and S. S. F. Obada, Physica A **170**, 393 (1991).

[3] D. L. Lin, X. S. Li, and Y. N. Peng, Phys. Rev. A **39**, 1933 (1989); G. Adam, J. Seke, and O. Hittmair, *ibid.* **42**, 5522 (1990).

[4] J. H. Ebelry, J. J. Sanchez-Mondragon, and N. B. Narozhy, Phys. Rev. Lett. **44**, 1323 (1980); P. L. Knight and P. M. Radmore, Phys. Rev. A **26**, 676 (1982); J. S. Peng, G. X. Li, and P. Zhou, Acta Phys. Sinica **40**, 1042 (1991).

[5] P. Meystre and M. S. Zubairy, Phys. Lett. A **89**, 390

(1982); G. Compagno, J. S. Peng, and F. Persico, Opt. Commun. **57**, 415 (1986).

[6] S. M. Barnett and P. L. Knight, Opt. Commun. **61**, 432 (1987); X. S. Li, D. L. Lin, and T. F. Geroge, Phys. Rev. A **40**, 2054 (1989); P.Zhou and J. S. Peng, Acta Phys. Sinica **38**, 2044 (1989).

[7] G. Rempe and H. Walther, Phys. Rev. A **42**, 1650 (1990).

[8] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987); G. Rempe, F. Schmidt-Kaler, and H. Walther, *ibid.* **64**, 2783 (1990).

[9] M. O. Scully and M. S. Zubairy, Phys. Rev. A **35**, 752 (1987); K. Zaheer and M. S. Zubairy, *ibid.* **38**, 5227 (1988).

[10] M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989).

[11] A. S. F. Obada and A. M. Abdel-Hafz, Phys. Rev. A **43**, 5161 (1991); P. Goldberg and L. C. Harrison, *ibid.* **43**, 376 (1991).

[12] K. Wodkiewicz, P. L. Knight, S. J. Buckle, and S. M. Barnett, Phys. Rev. A **35**, 2567 (1987); P. Zhou and J. S. Peng, *ibid.* **44**, 3331 (1991).

[13] K. Zaheer and M. S. Zubairy, Phys. Rev. A **39**, 2000 (1989).

[14] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3885 (1990); J. I. Cirac and L. L. Sanchez-Soto, Phys. Rev. A **42**, 2581 (1990); Opt. Commun. **80**, 67 (1990); P. Zhou, Z. L. Hu, and J. S. Peng, J. Mod. Opt. **39**, 49 (1992).

[15] J. J. Slosser, P. Meystre, and S. L. Braunstein, Phys. Rev. Lett. **63**, 934 (1989); J. J. Slosser and P. Meystre, Phys. Rev. A **41**, 3867 (1990).

[16] R. Loudon, *The Quantum Thoery of Light* (Clarendon, Oxford, 1983), p. 140.

[17] S. M. Barnett and D. T. Pegg, J. Mod. Opt. **36**, 7 (1989).

[18] D. T. Pegg and S. M. Barnett, Phys. Rev. A **39**, 1665 (1989).

[19] D. T. Pegg and S. M. Barnett, Phys. Rev. A **43**, 2579 (1991).

[20] X. Ma and W. Rhodes, Phys. Rev. A **43**, 2576 (1991).

[21] H. T. Dung, R. Tanas, and A. S. Shumovsky, Opt. Commun. **79**, 462 (1990).

[22] H. Meng and C. Chai, Phys. Lett. A **155**, 500 (1991); J. S. Peng and G. X. Li, Phys. Rev. A **45**, 3289 (1992).

[23] G. X. Li and J. S. Peng (unpublished).

[24] S. J. D. Phoenix and P. L. Knight, J. Opt. Soc. Am. B **7**, 116 (1990).

[25] S. Y. Zhu and X. S. Li, Phys. Rev. A **36**, 3889 (1987); L. Z. Wang and S. Y. Zhu, *ibid.* **43**, 2436 (1991).

[26] H. I. Yoo and J. H. Eberly, Phys. Rep. **118**, 232 (1985).

[27] X. S. Li, D. L. Lin,and C. D. Gong, Phys. Rev. A **36**, 5209 (1986).

[28] C. C. Gery and J. H. Eberly, Phys. Rev. A **42**, 6805 (1990).

[29] M. Orszag and C. Saavedra, Phys. Rev. A **43**, 2557 (1991).