Quantum-nondemolition-measurement scheme using a Kerr medium

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In this paper, we present a theoretical study of the quantum-nondemolition properties of a scheme that uses a Kerr medium in a double-ended cavity with both the signal and the probe inputs being coherent fields incident from the two sides. We find that with a suitable choice of parameters this system satisfies almost perfectly the criteria for quantum-nondemolition measurements.

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I. INTRODUCTION

The aim of a quantum-nondemolition- (QND) measurement scheme is to measure the value of a signal observable as accurately as possible while adding minimum possible noise to it and hence not significantly degrading it during the measurement. In addition, one may also require the scheme to be useful as a state preparation device for the output signal. These criteria are expressed quantitatively in terms of a set of correlation coefficients and a related conditional variance. A number of possible QND-measurement schemes in optics have been proposed where one employs a probe beam of light to measure the properties of a signal beam [1–6]. Some of these schemes have been realized experimentally, for example, four-wave mixing [7] in optical fibers and a scheme involving nonlinear mixing in a $\chi^{(2)}$ medium [8]. A recent experiment has demonstrated a QND measurement using three-level atoms [9].

Consider making a good QND measurement of the amplitude quadrature of the input signal field by measuring the amplitude quadrature of the output probe field. In a general scheme, the amplitude quadrature of the signal field interacts strongly with that of the probe field, producing a strong correlation between them and thus the information on the signal quadrature gets imprinted on the probe quadrature. Thus by measuring the amplitude quadrature of the output probe field, one can find the value of the input signal quadrature. In order that there is no appreciable degradation of the signal (i.e., there is an addition of minimum possible noise or uncertainty to the signal output quadrature) during the measurement, a lot of noise or uncertainty will be added to the conjugate variable, i.e., the phase quadrature of the signal output, in order to satisfy the Heinsenberg's uncertainty principle.

The measurement correlation coefficient gives the precision with which the input signal amplitude quadrature can be measured. The back-action evasion correlation coefficient gives the ability of the scheme to avoid degradation of the signal. The ability of the scheme as a state preparation device is given by a conditional variance. For our system these quantities are defined later. For the ideal case, each of the above two correlation coefficients should be unity and the conditional variance should be zero.

An ideal QND-measurement scheme does not exist. Therefore, the interest naturally lies in devising practical schemes whose performance approaches that of the ideal one as closely as possible. In this paper, we study a QND-measurement scheme that uses a Kerr medium, with a third-order nonlinear susceptibility, in a double-ended cavity with both the signal and the probe inputs being coherent-field incident from the two sides. This system has been shown to exhibit bistable behavior [10]. For a single-ended cavity Collett and Walls [11] have shown that good squeezing is possible in the vicinity of the bistable turning points. The double-ended cavity has been analyzed by Collett and Walls [12] who investigated its properties as a nonlinear beam splitter. They showed that it is possible to superpose squeezed vacuum fluctuations onto coherent light thereby producing a bright squeezed light beam. One might expect that by suitably choosing the input phases and other parameters the squeezing exhibited by this device could be made use of for the purpose of a good QND measurement.

II. THE SYSTEM

Our system is shown in Fig. 1. It consists of a double-ended cavity with a Kerr medium having a third-order nonlinear susceptibility χ. The signal is an amplitude quadrature, $X_+^{L\,\mathrm{in}}$, of a coherent input field from the left with a boson annihilation operator $a_{L\,\mathrm{in}}$. The probe is also an amplitude quadrature, $X_+^{R\,\mathrm{in}}$, also of the coherent input field from the right and with a boson annihilation operator $a_{R\,\mathrm{in}}$. The corresponding output amplitude quadratures for the signal and the probe are $X_+^{L\,\mathrm{out}}$ and $X_+^{R\,\mathrm{out}}$ with the annihilation operators $a_{L\,\mathrm{out}}$ and $a_{R\,\mathrm{out}}$, respectively. If a is the intracavity annihilation operator for a single mode, the Hamiltonian of the system can be written [10] as

$$H_{\mathrm{sys}} = \hbar\omega_c a^\dagger a + \hbar\chi a^{\dagger 2} a^2 , \qquad (1)$$

where ω_c is the frequency of the cavity mode. This system shows a bistable behavior as shown by Drummond and Walls [10] who gave a full quantum analysis. Its frequency spectrum has been calculated by Collett and Walls [11] who have shown that perfect squeezing occurs

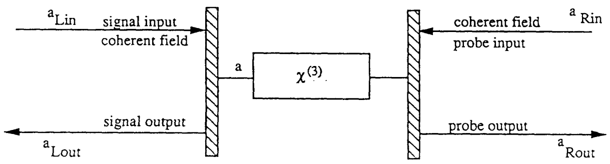

FIG. 1. A double-ended cavity with a Kerr medium as a quantum-nondemolition-measurement device. Amplitude quadratures of the coherent input fields from the left- and the right-hand sides serve as the signal and the probe, respectively.

at the critical point at the resonance frequency. The spectrum has a single peak or two peaks depending upon the range of parameters. We wish to study this system as a QND measurement scheme.

III. CALCULATIONS OF THE OUTPUT QUADRATURES

In order to evaluate the effectiveness of this scheme as a QND device we need to calculate the correlations between the signal input and the probe output and also the correlation between the signal input and output. It is, therefore, necessary for us to obtain expressions relating the signal and probe outputs to their inputs. For this we proceed as follows.

Using the input-output formulation of Collett and Gardiner [13,14], we can write the equation of motion of a as

$$\frac{da}{dt}=-(i/\hbar)[a,H_{\rm sys}]-\gamma a-\sqrt{2\gamma}_{\rm in}a_{\rm in}$$
$$=-i\omega_c a-2i\chi a^\dagger a^2-\gamma a-\sqrt{2\gamma}_{\rm in}a_{\rm in}\ , \tag{2}$$

where $\gamma=\gamma_L+\gamma_R$ and

$$\sqrt{2\gamma}_{\rm in}a_{\rm in}=\sqrt{2\gamma_L}a_{L\,\rm in}+\sqrt{2\gamma_R}a_{R\,\rm in}\ , \tag{3}$$

γ_L and γ_R being the cavity damping rates at the left-hand and the right-hand mirrors, respectively. In the frame rotating with the frequency ω_1 of the coherent signal and the probe inputs, Eq. (2) can be written as

$$\frac{da}{dt}=-i\delta a-2i\chi a^\dagger a^2-\gamma a-\sqrt{2\gamma}_{\rm in}a_{\rm in}\ , \tag{4}$$

where $\delta=\omega_c-\omega_1$ is the detuning. The equation of motion for the classical mean value $\alpha=\langle a\rangle$ of the intracavity mode is given by

$$\frac{d\alpha}{dt}=-i\delta\alpha-2i\chi|\alpha|^2\alpha-\gamma\alpha-\sqrt{2\gamma}_{\rm in}\alpha_{\rm in}\ . \tag{5}$$

In the steady state, α is given by

$$\alpha=-(\sqrt{2\gamma_L}\alpha_{L\,\rm in}+\sqrt{2\gamma_R}\alpha_{R\,\rm in})/[\gamma+i(\delta+\epsilon)]\ , \tag{6a}$$

where

$$\epsilon=2\chi|\alpha|^2\ . \tag{6b}$$

If we call the phases of α, $\alpha_{L\,\rm in}$, and $\alpha_{R\,\rm in}$ as θ, $\theta_{L\,\rm in}$, and $\theta_{R\,\rm in}$, respectively, and define η_L and η_R as

$$\eta_L=\sqrt{2\gamma_L}|\alpha_{L\,\rm in}|/(\gamma|\alpha|),\quad \eta_R=\sqrt{2\gamma_R}|\alpha_{R\,\rm in}|/(\gamma|\alpha|)\ , \tag{7}$$

Eq. (6) leads to the following relations:

$$\eta_R\cos(\theta-\theta_{R\,\rm in})=-1-\eta_L\cos(\theta-\theta_{L\,\rm in})\ , \tag{8a}$$

$$\eta_R\sin(\theta-\theta_{R\,\rm in})=(\delta+\epsilon)/\gamma-\eta_L\sin(\theta-\theta_{L\,\rm in})\ . \tag{8b}$$

By defining the fluctuations around the semiclassical mean values by the fluctuation operators $\Delta a(t)$ defined by $a(t)=\alpha+\Delta a(t)$ and linearizing Eq. (4), we can write the equations of motion for $\Delta a(t)$ and $\Delta a^\dagger(t)$ together as

$$\frac{d}{dt}\begin{bmatrix}\Delta a(t)\\ \Delta a^\dagger(t)\end{bmatrix}=\begin{bmatrix}-\gamma-i(\delta+2\epsilon) & -i\epsilon\exp(2i\theta)\\ i\epsilon\exp(-2i\theta) & -\gamma+i(\delta+2\epsilon)\end{bmatrix}\begin{bmatrix}\Delta a(t)\\ \Delta a^\dagger(t)\end{bmatrix}-\sqrt{2\gamma}_{\rm in}\begin{bmatrix}\Delta a_{\rm in}(t)\\ \Delta a^\dagger_{\rm in}(t)\end{bmatrix}. \tag{9}$$

Taking the Fourier transform of Eq. (9) we obtain

$$\begin{bmatrix}\Delta a(\omega)\\ \Delta a^\dagger(\omega)\end{bmatrix}=\frac{\sqrt{2\gamma}_{\rm in}}{\lambda(\omega)}\begin{bmatrix}-\gamma+i(\omega+\delta+2\epsilon) & i\epsilon\exp(2i\theta)\\ -i\epsilon\exp(-2i\theta) & -\gamma+i(\omega-\delta-2\epsilon)\end{bmatrix}\begin{bmatrix}\Delta a_{\rm in}(\omega)\\ \Delta a^\dagger_{\rm in}(\omega)\end{bmatrix}, \tag{10}$$

where the Fourier components are defined as

$$\Delta a(\omega)=\frac{1}{\sqrt{2\pi}}\int\Delta a(t)\exp(i\omega t)dt\ , \tag{11a}$$

$$\Delta a^\dagger(\omega)=\frac{1}{\sqrt{2\pi}}\int\Delta a^\dagger(t)\exp(i\omega t)dt\ , \tag{11b}$$

and

$$\lambda(\omega)=(\gamma-i\omega)^2+(\delta+\epsilon)(\delta+3\epsilon)\ . \tag{12}$$

The boundary condition at the left-hand mirror is

$$a_{L\,\rm out}=\sqrt{2\gamma_L}a+a_{L\,\rm in}\ . \tag{13}$$

Using this for the steady-state semiclassical mean values in the form

$$\alpha_{L\,\rm out}=\sqrt{2\gamma_L}\alpha+\alpha_{L\,\rm in}\ , \tag{14}$$

we get the following expressions for the phase difference $(\theta-\theta_{L\,\rm out})$ and the magnitude of output amplitude $|\alpha_{L\,\rm out}|$ in terms of the corresponding input quantities:

$$\cos(\theta-\theta_{L\,\text{out}})=[\sqrt{2\gamma_L}|\alpha|+|\alpha_{L\,\text{in}}|\cos(\theta-\theta_{L\,\text{in}})]/|\alpha_{L\,\text{out}}| , \tag{15a}$$

$$\sin(\theta-\theta_{L\,\text{out}})=|\alpha_{L\,\text{in}}|\sin(\theta-\theta_{L\,\text{in}})/|\alpha_{L\,\text{out}}| , \tag{15b}$$

and

$$\begin{aligned}|\alpha_{L\,\text{out}}|^2&=|\alpha_{L\,\text{in}}|^2+2\gamma_L|\alpha|^2+2\sqrt{2\gamma_L}|\alpha|\,|\alpha_{L\,\text{in}}|\cos(\theta-\theta_{L\,\text{in}})\\&=(\gamma^2|\alpha|^2/2\gamma_L)[\eta_L^2+4(\gamma_L/\gamma)^2+4(\gamma_L/\gamma)\eta_L\cos(\theta-\theta_{L\,\text{in}})] ,\end{aligned} \tag{15c}$$

where we have used Eq. (7). Using the boundary condition at the mirror on the right-hand side, we can get similar relations with R replacing L everywhere, for example,

$$|\alpha_{R\,\text{out}}|^2=(\gamma^2|\alpha|^2/2\gamma_R)[\eta_R^2+4(\gamma_R/\gamma)^2+4(\gamma_R/\gamma)\eta_R\cos(\theta-\theta_{R\,\text{in}})] . \tag{15d}$$

Now by using the boundary condition, Eq. (13), for the fluctuation parts of the operators, i.e.,

$$\Delta a_{L\,\text{out}}(\omega)=\sqrt{2\gamma_L}\Delta a(\omega)+\Delta a_{L\,\text{in}}(\omega) , \tag{16}$$

we get the following expressions for $\Delta a_{L\,\text{out}}(\omega)$:

$$\begin{aligned}\Delta a_{L\,\text{out}}(\omega)=\lambda^{-1}(\omega)\{&[(\gamma-i\omega)(\gamma_R-\gamma_L-i\omega)+2i\gamma_L(\delta+2\epsilon)+(\delta+\epsilon)(\delta+3\epsilon)]\Delta a_{L\,\text{in}}(\omega)\\&+2i\epsilon\gamma_L\exp(2i\theta)\Delta a^\dagger_{L\,\text{in}}(\omega)-2\sqrt{\gamma_L\gamma_R}[\gamma-i(\omega+\delta+2\epsilon)]\Delta a_{R\,\text{in}}(\omega)\\&+2i\epsilon\sqrt{\gamma_L\gamma_R}\exp(2i\theta)\Delta a^\dagger_{R\,\text{in}}(\omega)\} ,\end{aligned} \tag{17}$$

which also leads to

$$\begin{aligned}\Delta a^\dagger_{L\,\text{out}}(\omega)=\lambda^{-1}(\omega)\{&-2i\epsilon\gamma_L\exp(-2i\theta)\Delta a_{L\,\text{in}}(\omega)\\&+[(\gamma-i\omega)(\gamma_R-\gamma_L-i\omega)-2i\gamma_L(\delta+2\epsilon)+(\delta+\epsilon)(\delta+3\epsilon)]\Delta a^\dagger_{L\,\text{in}}(\omega)\\&-2i\epsilon\sqrt{\gamma_L\gamma_R}\exp(-2i\theta)\Delta a_{R\,\text{in}}(\omega)-2\sqrt{\gamma_L\gamma_R}[\gamma-i(\omega-\delta-2\epsilon)]\Delta a^\dagger_{R\,\text{in}}(\omega)\} .\end{aligned} \tag{18}$$

We define the amplitude quadrature $\Delta X_+^{L\,\text{out}}(\omega)$ and the phase quadrature $\Delta X_-^{L\,\text{out}}(\omega)$ for the output from the left as

$$\Delta X_+^{L\,\text{out}}(\omega)=\Delta a_{L\,\text{out}}(\omega)\exp(-i\theta_{L\,\text{out}})+\Delta a^\dagger_{L\,\text{out}}(\omega)\exp(i\theta_{L\,\text{out}}) \tag{19a}$$

and

$$\Delta X_-^{L\,\text{out}}(\omega)=-i[\Delta a_{L\,\text{out}}(\omega)\exp(-i\theta_{L\,\text{out}})-\Delta a^\dagger_{L\,\text{out}}(\omega)\exp(i\theta_{L\,\text{out}})] , \tag{19b}$$

and similarly for the output from the right and the two inputs, with suitable superscripts and subscripts. We give the expressions for $\Delta X_+^{L\,\text{out}}(\omega)$ and $\Delta X_+^{R\,\text{out}}(\omega)$ in Appendix A.

IV. CHOICE OF PHASES AND EVALUATION OF CORRELATION COEFFICIENTS

Now we shall define the quantities needed for verifying the criteria for the QND measurement. The measurement correlation coefficient $C^2(\Delta X_+^{L\,\text{in}},\Delta X_+^{R\,\text{out}})$, is defined as

$$C_w^2(\Delta X_+^{L\,\text{in}},\Delta X_+^{R\,\text{out}})=\frac{|\text{cov}_w(\Delta X_+^{L\,\text{in}},\Delta X_+^{R\,\text{out}})|^2}{\text{var}_w(\Delta X_+^{L\,\text{in}})\text{var}_w(\Delta X_+^{R\,\text{out}})} , \tag{20}$$

where the covariance $\text{cov}_w(A,B)$ of A and B is defined as

$$\text{cov}_w(A,B)=\int dw'[\langle A(w),B(w')\rangle+\langle B(w'),A(w)\rangle]/2 , \tag{21}$$

and the variance $\text{var}_w(A)$ of A is defined as

$$\text{var}_w(A)=\int dw'[\langle A(w),A(w')\rangle+\langle A(w'),A(w)\rangle]/2 . \tag{22}$$

The value of the measurement correlation coefficient gives the precision with which the signal input amplitude quadrature can be determined by measuring the probe output amplitude quadrature. For an ideal device, this should be equal to unity.

The back-action evasion correlation coefficient $C_w^2(\Delta X_+^{L\,\text{in}},\Delta X_+^{L\,\text{out}})$ is defined as

$$C_w^2(\Delta X_+^{L\,\text{in}},\Delta X_+^{L\,\text{out}})=\frac{|\text{cov}_w(\Delta X_+^{L\,\text{in}},\Delta X_+^{L\,\text{out}})|^2}{\text{var}_w(\Delta X_+^{L\,\text{in}})\text{var}_w(\Delta X_+^{L\,\text{out}})} . \tag{23}$$

The back-action evasion correlation coefficient gives the ability of the device to avoid the degradation of the signal. If this is equal to unity, no noise or uncertainty is introduced to the signal during the process of measurement.

Now the correlation coefficient $C_w^2(\Delta X_+^{L\,\text{out}},\Delta X_+^{R\,\text{out}})$ is defined as

$$C_w^2(\Delta W_+^{L\,\mathrm{out}},\Delta X_+^{R\,\mathrm{out}})=\frac{|\mathrm{cov}_w(\Delta X_+^{L\,\mathrm{out}},\Delta X_+^{R\,\mathrm{out}})|^2}{\mathrm{var}_w(\Delta X_+^{L\,\mathrm{out}})\,\mathrm{var}_w(\Delta_+^{R\,\mathrm{out}})} . \quad (24)$$

In the linear approximation, the conditional variance $V_w(\Delta X_+^{L\,\mathrm{out}}/\Delta X_+^{R\,\mathrm{out}})$ is related to the correlation coefficient $C_w^2(\Delta X_+^{L\,\mathrm{out}},\Delta X_+^{R\,\mathrm{out}})$ by

$$V_w(\Delta X_+^{L\,\mathrm{out}}/\Delta X_+^{R\,\mathrm{out}}) =\mathrm{var}_w(\Delta X_+^{L\,\mathrm{out}})[1-C_w^2(\Delta X_+^{L\,\mathrm{out}},\Delta X_+^{R\,\mathrm{out}})] . \quad (25)$$

This conditional variance describes the effectiveness of the scheme as a state preparation device. For the ideal case this is equal to zero. In this case we know exactly the amplitude quadrature of the output signal on measuring the amplitude quadrature of the probe output.

We find that the best choice of the phase relations is the following:

$$\theta-\theta_{L\,\mathrm{in}}=\pi, \quad \theta-\theta_{R\,\mathrm{in}}=\pi/2 , \quad (26)$$

which, together with Eq. (8), leads also to

$$\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}}=\pi/2, \quad \eta_L=1, \text{ and } \eta_R=(\delta+\epsilon)/\gamma . \quad (27)$$

Under these conditions Eqs. (15c) and (15d) lead to

$$|\alpha|/|\alpha_{L\,\mathrm{out}}|=\sqrt{2\gamma_L}/(\gamma-2\gamma_L) ,$$
$$|\alpha|/|\alpha_{R\,\mathrm{out}}|=\sqrt{2\gamma_R}/[(\delta+\epsilon)^2+(2\gamma_R)^2]^{1/2} . \quad (28)$$

Now we define the dimensionless quantities

$$\gamma_L/\gamma=g, \quad \delta/\gamma=d, \quad \epsilon/\gamma=e, \text{ and } \omega/\gamma=w ,$$
$$l(w)=\lambda(\omega)/\gamma^2=(1-iw)^2+(d+e)(d+3e) , \quad (29)$$

and

$$l=l(0)+1+(d+e)(d+3e) . \quad (30)$$

In Appendix B we give the expressions for $\Delta X_+^{L\,\mathrm{out}}(w)$ and $\Delta X_+^{R\,\mathrm{out}}(w)$ in terms of these dimensionless quantities and under the phase relations given in Eq. (26).

Using Eqs. (21) and (22), the covariances and the variances occurring in Eqs. (20), (23), and (24) are calculated and given in Appendix B. Using Eqs. (20), (B4), (B7), and (B9), we obtain the following expression for the measurement correlation function:

$$C_w^2(\Delta X_+^{L\,\mathrm{in}},\Delta X_+^{R\,\mathrm{out}})=4g(1-g)[(l+1-2g)^2+4(1-g)^2w^2]$$
$$\times\{4(1-g)[(1-g)w^4+2(1-g)w^2+gl(l-4g+2)+(1-2g)^2]$$
$$+(d+e)^2[(w^2-l)^2-4w^2(2g^2-4g+1)+4(1-g)(1-2g)(l+1-2g)]\}^{-1} . \quad (31)$$

Using Eqs. (23), (B5), (B8), and (B9) the back-action evasion coefficient is

$$C_w^2(\Delta X_+^{L\,\mathrm{in}},\Delta X_+^{L\,\mathrm{out}})=\frac{(1-2g-w^2)^2+4(1-g)^2w^2}{(l-2-w^2)^2+4(1-g)(l-1)+4g(d+e)^2} . \quad (32)$$

On making use of Eqs. (24), (25), and (B6)–(B8), we find for the conditional variance

$$V_w(\Delta X_+^{L\,\mathrm{out}}/\Delta X_+^{R\,\mathrm{out}})=(\{(l-2-w^2)^2+4(1-g)(l-1)+4g(d+e)^2\}$$
$$\times\{4(1-g)[(1-g)w^4+2(1-g)w^2+gl(l-4g+2)+(1-2g)^2]$$
$$+(d+e)^2[(w^2-1)^2-4w^2(2g^2-4g+1)+4(1-g)(1-2g)(l+1-2g)]\}$$
$$-16g(1-g)e^2(d+e)^2(w^2l+4g-2)^2)$$
$$\times([(l-w^2)^2+4w^2]\{4(l-g)[(1-g)w^4+2(l-g)w^2+gl(l-4g+2)+(l-2g)^2]$$
$$+(d+e)^2[(w^2-l)^2-4w^2(2g^2-4g+1)$$
$$+4(1-g)(1-2g)(l+1-2g)]\})^{-1} . \quad (33)$$

A discussion of the properties of these correlation functions is given in the following section.

V. RESULTS AND DISCUSSION

From Eqs. (31) and (32), we see that for $w=0$, $(d+e)^2\ll 1$, $(d+e)(d+3e)$ or $l\gg 1$, the measurement correlation coefficient and the back-action evasion coefficient approach unity and the conditional variance in Eq. (33) approaches zero. Thus, for parameters satisfying these conditions in addition to those imposed in Sec. IV, the device described here would serve as a perfect QND measurement scheme. Figure 2 shows the variation of the measurement correlation coefficient $C_w^2(\Delta X_+^{L\,\mathrm{in}},\Delta X_+^{R\,\mathrm{out}})$ versus the frequency for two values of $g=0.7$ and 0.9 and for $e=100.0$, $d=-99.9$. Figure 3 gives the variation of the back-action evasion correlation coefficient $C_w^2(\Delta X_+^{L\,\mathrm{in}},\Delta X_+^{L\,\mathrm{out}})$ versus frequency and Fig. 4 shows the variation of the conditional variance

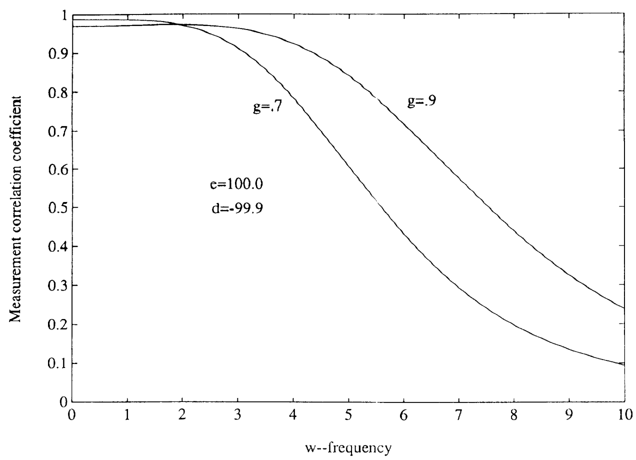

FIG. 2. Variation of the measurement correlation coefficient vs frequency for $e=100.0$, $d=-99.9$, and $g=0.7,0.9$.

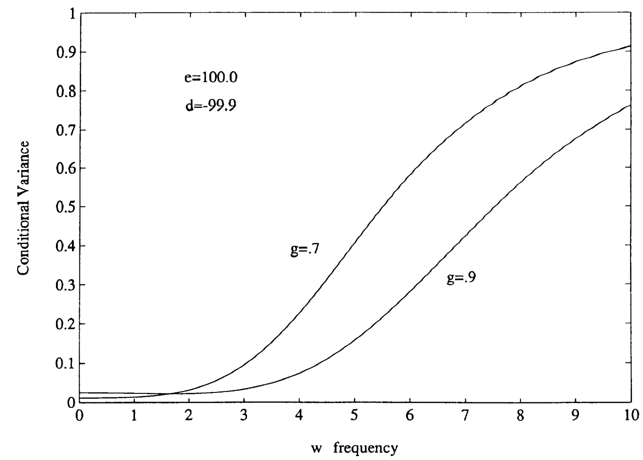

FIG. 4. Variation of the conditional variance vs frequency for $e=100.0$, $d=-99.9$, and $g=0.7,0.9$.

$V_w(\Delta X_+^{L\,\mathrm{out}}/\Delta X_+^{R\,\mathrm{out}})$ versus frequency for the same parameters. For $w=0$, all these quantities approach almost their ideal values for the QND measurement. For the larger values of g these quantities have larger width which would facilitate the measurement. These quantities all show slowly varying behavior with respect to the parameters for $g\geq 0.6$. For $g=0.5$ an instability in the system causes rapid variation in the correlation coefficients. This is shown in Figs. 5–7 where we have shown the variation of the correlation coefficients and conditional variance for $w=0$ versus the input phase difference divided by π, i.e., $(\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}})/\pi$ for $e=100.0$, $d=-99.9$, and $h=\eta_L=1.0$. These were obtained numerically from their expressions derived by making use of Eqs. (A1) and (A2) for $w=0$ from which $(\theta-\theta_{R\,\mathrm{in}})$ was eliminated in favor of $(\theta-\theta_{L\,\mathrm{in}})$ by making use of Eqs. (8a) and (8b) and then finding $(\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}})$ in terms of $(\theta-\theta_{L\,\mathrm{in}})$ from Eqs. (6) and (7).

From Eqs. (6), (7), and (29), we can express $|\alpha_{L\,\mathrm{in}}|^2$ and

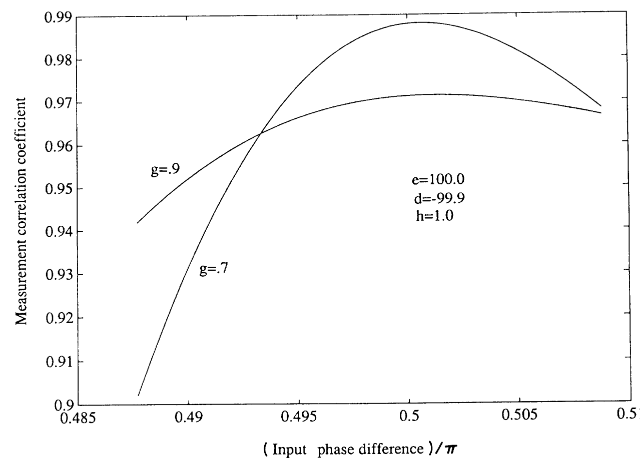

FIG. 5. Variation of the measurement correlation coefficient vs (input phase difference)/π for $e=100.0$, $d=-99.9$, $h=\eta_L=1.0$, and $g=0.7,0.9$.

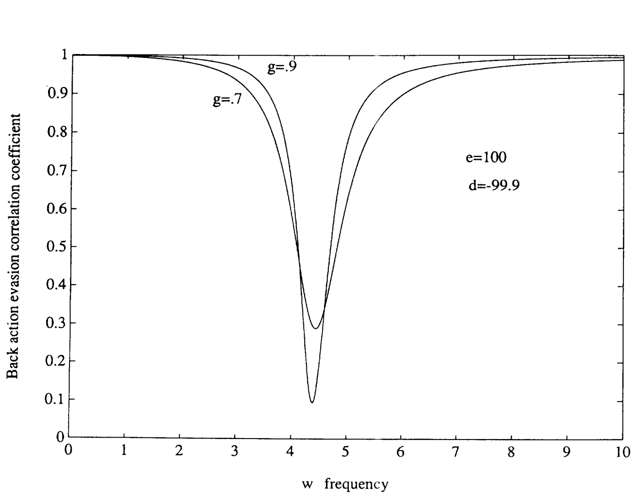

FIG. 3. Variation of the back-action evasion correlation vs frequency for $e=100.0$, $d=-99.9$, and $g=0.7,0.9$.

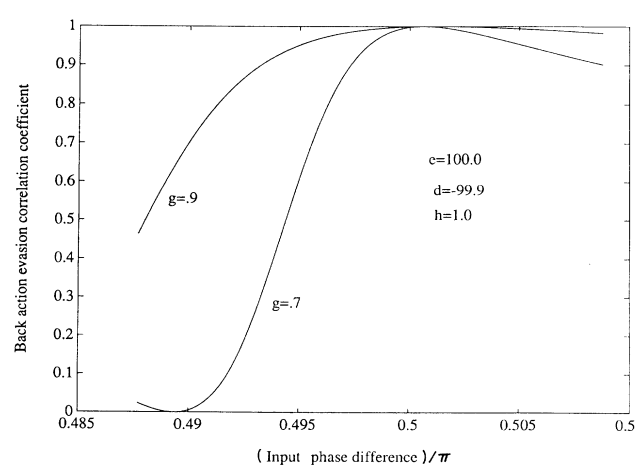

FIG. 6. Variation of the back-action evasion correlation coefficient vs (input phase difference)/π for $e=100.0$, $d=-99.9$, $h=\eta_L=1.0$, and $g=0.7,0.9$.

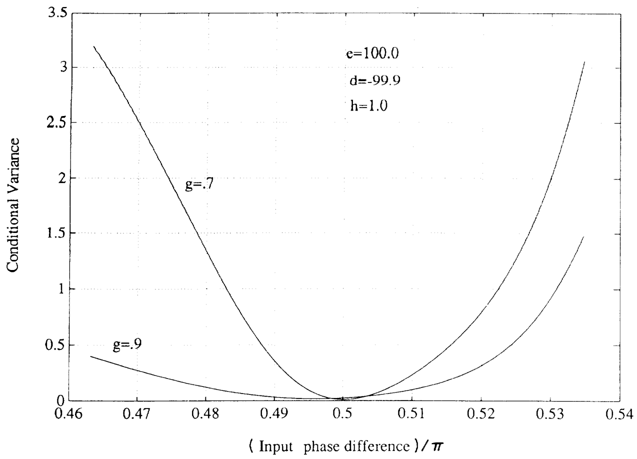

FIG. 7. Variation of the conditional variance vs (input phase difference)/π for $e=100.0$, $d=-99.9$, $h=\eta_L=1.0$, and $g=0.7,0.9$.

$|\alpha_{R\,\rm in}|^2$ as

$$|\alpha_{L\,\rm in}|^2=e\gamma^2/(4g\chi),$$
$$|\alpha_{R\,\rm in}|^2=e\gamma^2(d+e)^2/4\chi(1-g)\ . \tag{34}$$

For small values of the third-order nonlinear susceptibility, the two input intensities have to be large. From Eqs. (34) and (35), we also get

$$|\alpha_{R\,\rm in}|^2/|\alpha_{L\,\rm in}|^2=g(d+e)^2/(1-g)\ , \tag{35}$$

which gives the ratio of the two input intensities. For the chosen values of d and e given above and for $g=0.9$, the above ratio is 9% and for $g=0.5$, it is only 1%. Thus for the device to be a good QND-measurement scheme the optimum phase difference between the two inputs should be $\theta_{R\,\rm in}-\theta_{L\,\rm in}=\pi/2$ and the ratio of the two intensities $|\alpha_{R\,\rm in}|^2/|\alpha_{L\,\rm in}|^2\geq 5\%$.

VI. CONCLUSIONS

We have shown that the device consisting of a double-ended cavity with a Kerr medium having a third-order nonlinearity in the susceptibility offers a very good scheme for the QND measurement. The signal and the probe are, respectively, the amplitude quadratures of the inputs to the left- and right-hand cavity mirrors. Good QND-measurement correlations are predicted for a phase difference between the two inputs of $\pi/2$, and with a ratio of probe input intensity to signal input intensity in the range of 5–10%. These parameters would seem to be quite accessible to current experimental techniques.

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APPENDIX A

Using the definitions in Eq. (19) and Eqs.(17) and (18), we get the following expression for $\Delta X_+^{L\,\rm out}(\omega)$:

$$\begin{aligned}
\Delta X_+^{L\,\rm out}(\omega)\Big/\left[\frac{|\alpha|}{\lambda(\omega)|\alpha_{L\,\rm out}|}\right]
&=\sqrt{2\gamma_L}\{[(\gamma\eta_L/2\gamma_L)+\cos(\theta-\theta_{L\,\rm in})][(\gamma-i\omega)(\gamma-2\gamma_L-i\omega)+(\delta+\epsilon)(\delta+3\epsilon)]\\
&\qquad+2\gamma_L(\delta+\epsilon)\sin(\theta-\theta_{L\,\rm in})-\eta_L\epsilon\gamma\sin2(\theta-\theta_{L\,\rm in})\}\Delta X_+^{L\,\rm in}(\omega)\\
&\quad-\sqrt{2\gamma_L}\{(\delta+\epsilon)[\eta_L\gamma+2\gamma_L\cos(\theta-\theta_{L\,\rm in})]-[(\gamma-i\omega)(\gamma-2\gamma_L-i\omega)+(\delta+\epsilon)(\delta+3\epsilon)]\sin(\theta-\theta_{L\,\rm in})\\
&\qquad+2\eta_L\epsilon\gamma\sin^2(\theta-\theta_{L\,\rm in})\}\Delta X_-^{L\,\rm in}(\omega)\\
&\quad-\sqrt{2\gamma_R}\{(\gamma-i\omega)[2\gamma_L\cos(\theta-\theta_{R\,\rm in})+\eta_L\gamma\cos(\theta_{R\,\rm in}-\theta_{L\,\rm in})]\\
&\qquad-(\delta+\epsilon)[2\gamma_L\sin(\theta-\theta_{R\,\rm in})-\eta_L\gamma\sin(\theta_{R\,\rm in}-\theta_{L\,\rm in})]+2\eta_L\gamma\epsilon\sin(\theta-\theta_{L\,\rm in})\cos(\theta-\theta_{R\,\rm in})\}\Delta X_+^{R\,\rm in}(\omega)\\
&\quad+\sqrt{2\gamma_R}\{(\gamma-i\omega)[\eta_L\gamma\sin(\theta_{R\,\rm in}-\theta_{L\,\rm in})-2\gamma_L\sin(\theta-\theta_{R\,\rm in})]\\
&\qquad-(\delta+\epsilon)[2\gamma_L\cos(\theta-\theta_{R\,\rm in})+\eta_L\gamma\cos(\theta_{R\,\rm in}-\theta_{L\,\rm in})]\\
&\qquad-[2\eta_L\gamma\epsilon\sin(\theta-\theta_{L\,\rm in})\sin(\theta-\theta_{R\,\rm in})]\}\Delta X_-^{R\,\rm in}(\omega)\ ,
\end{aligned}\tag{A1}$$

where we have also used Eqs. (15a) and (15b) to eliminate $(\theta-\theta_{L\,\rm out})$ in favor of $(\theta-\theta_{L\,\rm in})$ and $|\alpha_{L\,\rm out}|$ is given by Eq. (15c). Now interchanging L and $R\,(L\leftrightarrow R)$ in Eq. (A1), we also get the following expression for $\Delta X_+^{R\,\rm out}$:

$$\Delta X_{+}^{R\,\mathrm{out}}(\omega)\Bigg/\left[\frac{|\alpha|}{\lambda(\omega)|\alpha_{R\,\mathrm{out}}|}\right]$$
$$=-\sqrt{2\gamma_L}\{(\gamma-i\omega)[2\gamma_R\cos(\theta-\theta_{L\,\mathrm{in}})+\eta_R\gamma\cos(\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}})]$$
$$-(\delta+\epsilon)[2\gamma_R\sin(\theta-\theta_{L\,\mathrm{in}})+\eta_R\gamma\sin(\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}})]+2\eta_R\gamma\epsilon\sin(\theta-\theta_{R\,\mathrm{in}})\cos(\theta-\theta_{L\,\mathrm{in}})\}\Delta X_{+}^{L\,\mathrm{in}}(\omega)$$
$$-\sqrt{2\gamma_L}\{(\gamma-i\omega)[+\eta_R\gamma\sin(\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}})+2\gamma_R\sin(\theta-\theta_{L\,\mathrm{in}})]$$
$$+(\delta+\epsilon)[2\gamma_R\cos(\theta-\theta_{L\,\mathrm{in}})+\eta_R\gamma\cos(\theta_{R\,\mathrm{in}}-\theta_{L\,\mathrm{in}})]+2\eta_R\gamma\epsilon\sin(\theta-\theta_{L\,\mathrm{in}})\sin(\theta-\theta_{R\,\mathrm{in}})\}\Delta X_{-}^{L\,\mathrm{in}}(\omega)$$
$$+\sqrt{2\gamma_R}\{[(\gamma\eta_R/2\gamma_R)+\cos(\theta-\theta_{R\,\mathrm{in}})][(\gamma-i\omega)(\gamma-2\gamma_R-i\omega)+(\delta+\epsilon)(\delta+3\epsilon)]$$
$$+2\gamma_R(\delta+\epsilon)\sin(\theta-\theta_{R\,\mathrm{in}})-\eta_R\epsilon\gamma\sin2(\theta-\theta_{R\,\mathrm{in}})\}\Delta X_{+}^{R\,\mathrm{in}}(\omega)$$
$$-\sqrt{2\gamma_R}\{(\delta+\epsilon)[\eta_R\gamma+2\gamma_R\cos(\theta-\theta_{R\,\mathrm{in}})]-[(\gamma-i\omega)(\gamma-2\gamma_R-i\omega)+(\delta+\epsilon)(\delta+3\epsilon)]\sin(\theta-\theta_{R\,\mathrm{in}})$$
$$+2\eta_R\epsilon\gamma\sin^2(\theta-\theta_{R\,\mathrm{in}})\}\Delta X_{-}^{R\,\mathrm{in}}(\omega)\;, \tag{A2}$$

where $|\alpha_{R\,\mathrm{out}}|$ is given by Eq. (15d).

APPENDIX B

In this appendix we give the expressions for $\Delta X_{+}^{L\,\mathrm{out}}(w)$ and $\Delta X_{+}^{R\,\mathrm{out}}(w)$ under the choice of the best boundary conditions given in Eq. (26). Also we give the expressions for the covariances and variances which we need for the calculations of the correlation coefficients and the conditional variance of interest.

By using Eqs. (26)–(30), Eqs. (A1) and (A2) reduce to

$$\Delta X_{+}^{L\,\mathrm{out}}(w)=[l(w)]^{-1}\{[l-2g-w^2-2(1-g)iw]\Delta X_{+}^{L\,\mathrm{in}}(w)-2g(d+e)\Delta X_{-}^{L\,\mathrm{in}}(w)$$
$$-2\sqrt{g(1-g)}(d+e)\Delta X_{+}^{R\,\mathrm{in}}(w)+2\sqrt{g(1-g)}(1-iw)\Delta X^{R\,\mathrm{in}}(w)\} \tag{B1}$$

and

$$\Delta X_{+}^{R\,\mathrm{out}}(w)=\{l(w)[(d+e)^2+4(1-g)^2]^{1/2}\}^{-1}\{2\sqrt{g(1-g)}[l+1-2g-2(1-g)iw]\Delta X_{+}^{L\,\mathrm{in}}(w)$$
$$+2\sqrt{g(1-g)}[(1-2g)(d+e)+iw(d+e)]\Delta X_{-}^{L\,\mathrm{in}}(w)$$
$$+(d+e)(-2giw-w^2+l+4g^2-6g+2)\Delta X_{+}^{R\,\mathrm{in}}(w)$$
$$+2(1-g)(-2giw-w^2+2g-1)\Delta X_{-}^{R\,\mathrm{in}}(w)\}\;. \tag{B2}$$

For calculating the variances and the covariances, defined in Eqs. (21) and (22), which occur in the definitions of the correlation coefficients, we shall use the following relations which hold for the coherent input:

$$\langle\Delta X_{\pm}^{L\,\mathrm{in}}(w),\Delta X_{\pm}^{L\,\mathrm{in}}(w')\rangle=\delta(w+w'),\quad\langle\Delta X_{+}^{L\,\mathrm{in}}(w),\Delta X_{-}^{L\,\mathrm{in}}(w')\rangle=i\delta(w+w')\;,$$
$$\langle\Delta X_{-}^{L\,\mathrm{in}}(w),\Delta X_{+}^{L\,\mathrm{in}}(w')\rangle=-i\delta(w+w')\;. \tag{B3}$$

Similar relations hold for the input from the right and we shall assume that there are no correlations between the two inputs. Using these relations and Eqs. (B1) and (B2) and for the best choice of the phase relations given in Eq. (26), we obtain the following expressions for the covariances and the variances needed for the calculations of the correlation coefficients of interest:

$$|\mathrm{cov}_w(\Delta X_{+}^{L\,\mathrm{in}},\Delta X_{+}^{R\,\mathrm{out}})|^2=\frac{4g(1-g)[(1+1-2g)^2+4(1-g)^2w^2]}{[(l-w^2)^2+4w^2][(d+e)^2+4(1-g)^2]}\;, \tag{B4}$$

$$|\mathrm{cov}_w(\Delta X_{+}^{L\,\mathrm{in}},\Delta X_{+}^{L\,\mathrm{out}})|^2=\frac{(l-2g-w^2)^2+4(1-g)^2w^2}{(l-w^2)^2+4w^2}\;, \tag{B5}$$

$$|\mathrm{cov}_w(\Delta X_{+}^{L\,\mathrm{out}},\Delta X_{+}^{R\,\mathrm{out}})|^2=\frac{16g(1-g)e^2(d+e)^2(w^2-l+4g-2)^2}{[(l-w^2)^2+4w^2]^2[(d+e)^2+4(1-g)^2]}\;, \tag{B6}$$

$$\mathrm{var}_w(\Delta X_{+}^{R\,\mathrm{out}})=\{4(1-g)[(1-g)w^4+2(1-g)w^2+gl(1-4g+2)+(1-g)^2]$$
$$+(d+e)^2[(w^2-l)^2-4w^2(2g^2-4g+1)+4(1-g)(1-2g)(l+1-2g)]\}$$
$$\times\{[(l-w^2)+4w^2][(d+e)^2+4(1-g)^2]\}^{-1}\;, \tag{B7}$$

$$\mathrm{var}_w(\Delta X_+^{L\,\mathrm{out}}) = \frac{(l-2-w^2)^2+4(1-g)(l-1)+4g(d+e)^2}{(l-w^2)^2+4w^2}\ , \tag{B8}$$

and

$$\mathrm{var}_w(\Delta X_+^{L\,\mathrm{in}}) = 1\ . \tag{B9}$$

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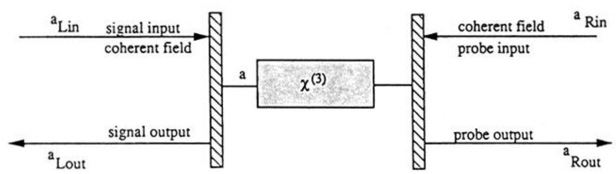

FIG. 1. A double-ended cavity with a Kerr medium as a quantum-nondemolition-measurement device. Amplitude quadratures of the coherent input fields from the left- and the right-hand sides serve as the signal and the probe, respectively.