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Observation of coherent transition radiation at millimeter and submillimeter wavelengths

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Coherent transition radiation has been observed in the wavelength range from 0.5 to 5.5 mm emitted from bunched electrons of 42 MeV passing through an Al foil in vacuum. The radiation intensity at $\lambda=4$ mm is enhanced by a factor of 3×10^6 in comparison with incoherent transition radiation. The intensity is proportional to the square of the beam current, and it increases with the emission length, or the length of the trajectory of the electrons between the foil and a mirror to observe the radiation. The divergence of radiation increases with the wavelength and it is qualitatively in agreement with theory. A sheet of Eccosorb is also used as a radiator, and a similar spectrum to that from the Al foil has been observed. The bunch shape is discussed on the basis of a bunch form factor derived from the spectrum.

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Transition radiation (TR) is emitted when electrons cross an interface between two media with different dielectric constants [1,2]. When the electrons are bunched, each electron emits TR in phase, and coherent TR is expected in the region where the wavelength is equal to or longer than the longitudinal bunch length. The intensity of coherent TR is given by [3]

$$P(\lambda)=N_e[1+N_e f(\lambda)]P_{\rm TR}(\lambda)\,, \tag{1}$$

where λ is the wavelength, $P_{\rm TR}(\lambda)$ is the TR intensity emitted from a single electron, N_e is the number of electrons in a bunch, and $f(\lambda)$ is the bunch form factor which is given by the Fourier transform of the distribution function of the electrons in the bunch [4–6].

Using 40-MeV electrons from a linear accelerator, coherent TR at millimeter wavelengths was observed in our previous experiment [7]; the radiation had a similar character as coherent synchrotron radiation [5–8], but it was erroneously assigned to wake-field radiation. In our later experiment, the geometrical shape of the vacuum chamber around the electron beam was changed so as to vary the impedance. However, no change of the intensity has been observed, and our initial assignment to the wake-field effect was found to be incorrect. The assignment to the Čerenkov effect is also not correct, because the radiation was emitted in vacuum. The observed radiation is due to either TR or bremsstrahlung. The intensity of the radiation has been found to be dependent on the emission length and independent of the atomic number of metals used as a radiator, as reported in the present paper. This is evidence for TR. In this Rapid Communication, experimental results of the spectrum, the effect of the emission length, and the angular distribution of coherent TR are reported. Quite recently, an observation of coherent TR was reported by Happek, Sievers, and Blum [9].

According to the theory [2,10], TR by high-energy electrons in the long-wavelength region is not localized at the interface of the two media, but it occurs over a formation zone, where the electromagnetic wave and the electrons exchange energy. When electrons pass through a metallic foil in vacuum, the formation zone of the forward TR is given by

$$Z=\frac{\beta\lambda}{2\pi(1-\beta\cos\theta)}\,, \tag{2}$$

where β is the ratio of the velocity of the electron to the light velocity in vacuum and θ is the direction angle from the beam axis. In the case of the electrons of 42 MeV, for example, $Z=1.1$ m at $\lambda=1$ mm and for $\theta=1/\gamma$, where $\gamma=(1-\beta^2)^{-1/2}$. If a mirror, which is used to observe the radiation, is placed at a downstream point within the formation zone, the emission length L, or the length of the trajectory of the emitting electrons, is limited by the foil and the mirror. When $L<Z$, the intensity of TR emitted by an electron is given by [11,12]

$$\frac{d^2P_{\rm TR}}{d\Omega\, d\lambda}=2\left(\frac{d^2P_{\rm TR}}{d\Omega\, d\lambda}\right)_0\left(1-\cos\frac{L}{Z}\right) \tag{3}$$

and

$$\left(\frac{d^2P_{\rm TR}}{d\Omega\, d\lambda}\right)_0=\frac{\alpha\beta^2\sin^2\theta\cos^2\theta}{\pi^2\lambda(1-\beta^2\cos^2\theta)^2}|\zeta|^2\,, \tag{4}$$

$$\zeta=\frac{(\epsilon-1)[1-\beta^2-\beta(\epsilon-\sin^2\theta)^{1/2}]}{[\epsilon\cos\theta+(\epsilon-\sin^2\theta)^{1/2}][1-\beta(\epsilon-\sin^2\theta)^{1/2}]}\,, \tag{5}$$

where α and ϵ are the fine-structure constant and dielectric constant of the metallic foil, and Ω is the solid angle directed to θ. When $L\ll Z$, the TR intensity given by Eq. (3) is proportional to L^2.

The experimental setup is shown in Fig. 1. Electrons were accelerated by an L-band linear accelerator of the Research Reactor Institute, Kyoto University [7]. The rf frequency, the energy of electrons and the energy width were 1300 MHz, 42 MeV, and 11%, respectively. Dura-

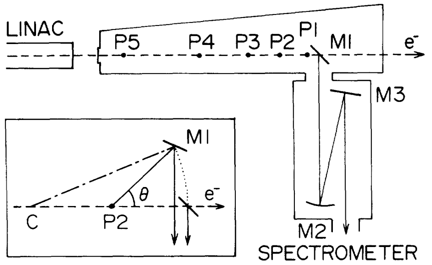

FIG. 1. Schematic diagram of the experimental setup. $M1$ and $M3$ are plane mirrors; $M2$ is a spherical mirror. The trajectory of electrons is shown by the dashed line. A transition radiator is located at $P1$–$P5$. The inset shows an arrangement to observe the angular distribution of TR intensity. The mirror $M1$ is rotated around a vertical axis C.

tion of a burst was 33 ns and its repetition rate was 55 Hz. The beam current was typically 2 μA, and hence N_e was 5.3×10^9. The pressure in the chamber was 0.4 Torr, which was far below the Čerenkov threshold of air of 208 Torr.

Two optical arrangements were used. In the first arrangement, a position of a transition radiator was changed from point $P1$ to $P5$ in Fig. 1 and the emission length L, i.e., the length between the radiator and a plane mirror $M1$, was varied from 36 to 756 mm. As the radiator, we used a 15-μm-thick Al foil or a sheet of Eccosorb AN72 (Emerson & Cuming Co.), an absorber of millimeter wave. TR was reflected by the mirror $M1$ (an Al evaporated silica glass). It was collected by a round spherical mirror $M2$, which had an acceptance angle of 100 mrad, and was led to a grating-type spectrometer by a plane mirror $M3$. The electron beam had a circular cross section with a diameter of about 20 mm at $P2$.

The second arrangement was for the measurement of the angular distribution of TR. The Al foil, the Eccosorb sheet, and foils of 20-μm-thick Ti and Cu were used as the radiator. It was located at point $P2$ ($L=156$ mm) as shown in the inset in Fig. 1, and the mirror $M1$ was rotated around axis C.

The spectrometer covered the wavelength range from 0.5 to 6 mm [6]. Radiation was detected by a liquid-He-cooled Si bolometer. The observational error of the absolute intensity was estimated to be within a factor of 2.

Observed spectra are shown in Fig. 2. The lower and upper solid curves are the spectra of TR emitted from the Al foil located at point $P2$ ($L=156$ mm) and at point $P5$ ($L=756$ mm), respectively. The structure of the two spectra is similar; it has a peak at about $\lambda=4$ mm, and decreases sharply towards shorter wavelengths. The intensity of the spectrum for $L=756$ mm is higher than that for 156 mm in the observed wavelength region.

The intensity of incoherent TR was calculated by Eq. (3) considering the geometry of the experiment. In the far-infrared region, the value of $|\epsilon|$ of the metallic foil is much larger than unity. Then, the factor $|\zeta|$ of Eq. (4) is nearly equal to unity for the 42-MeV electrons. The calculated intensity for $L=156$ mm is shown by the dashed curve in Fig. 2. At $\lambda=4$ mm, the observed intensity for $L=156$ mm is enhanced by a factor of 3×10^6 in compar-

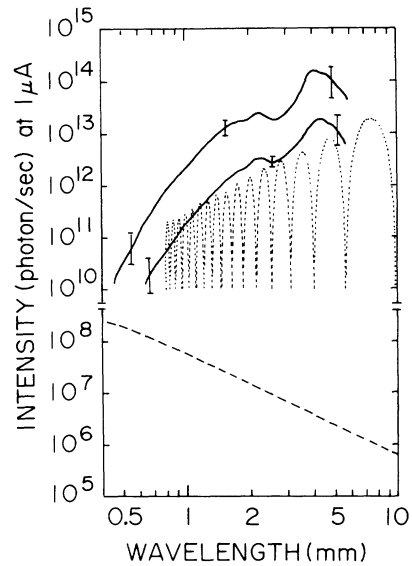

FIG. 2. Spectra of coherent TR observed for emission lengths of 756 mm (the upper solid curve) and of 156 mm (the lower solid curve). Intensity is shown in units of photon numbers per second per band width of 1%, i.e., per $\Delta\lambda/\lambda$ of 0.01. A spectrum of incoherent TR calculated for the emission length of 156 mm is shown by the dashed curve. The dotted curve shows a spectrum calculated by assuming a ellipsoidal bunch of 14 mm length.

ison with the calculation. Though the factor is less than the number of the electrons in a bunch, 5.3×10^9, it indicates an enormous enhancement due to the coherence effect.

By changing the beam current from 0.1 to 3 μA, the TR intensity at $\lambda=1.4$, 3.0, and 4.9 mm was measured with the Al foil located at $L=756$ mm. The result is shown in Fig. 3. The gradients of the solid lines in the figure were determined by the least-squares method; they are 2.16 ± 0.21, 1.97 ± 0.07, and 2.26 ± 0.06 for $\lambda=1.4$, 3.0, and 4.9 mm, respectively, and the intensity is con-

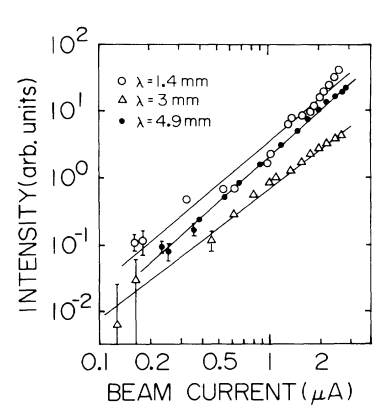

FIG. 3. Relation between the observed TR intensity and the beam current. The gradients of the straight lines are determined by the least-squares method, and are 2.16 ± 0.21, 1.97 ± 0.07, and 2.26 ± 0.06 for $\lambda=1.4$, 3.0, and 4.9 mm, respectively.

firmed to be proportional to the square of the current, or to the square of the number of the electrons in a bunch. This is an evidence for the coherent radiation.

Spectral intensities at $\lambda = 1.4$, 2.0, 3.0, 4.0, and 4.9 mm were observed by changing the emission length L. The intensities increased by about 2 orders of magnitude as L increased from 36 to 756 mm. Ratios of the observed intensities to that at $L = 156$ mm are plotted in Fig. 4; the solid circles show the ratios for the Al foil and the open circles those for the Eccosorb sheet.

The theoretical ratios of the TR intensities calculated by Eq. (3) are shown by the solid curves in Fig. 4. The calculated intensities are those received by the collecting mirror $M2$. The point $P2$ in Fig. 1 was at the focus of the collecting mirror $M2$ and of the optical system, but the other points were out of focus. The dashed curves in Fig. 4 show the corrected ratios for the defocusing effect in the following way. The intensity of a high-pressure mercury lamp with an aperture of 10 mm was measured at $\lambda = 1.4$ mm by moving the lamp from $P1$ to $P5$. The observed ratio of the intensity is shown by the dotted curve in Fig. 4. The theoretical ratios shown by the solid curves were multiplied by this ratio of the lamp to obtain the dashed curves.

The observed relation between the emission length and the intensity in Fig. 4 agrees fairly well with the calculation, especially at short wavelengths. The observed intensities at a long wavelength of $\lambda = 4.9$ mm, however, deviate considerably from the theoretical ones. The discrepancy indicates that further analysis is necessary by examining more rigorously the divergence of coherent TR in the optical system.

Angular distributions of the TR intensity at $\lambda = 1.4$, 2.0, 3.0, and 4.9 mm were observed for $L = 156$ mm and the results of the Al foil are shown in Fig. 5. In the figure, the horizontally polarized components are shown by the dashed curves, the vertical ones by the dotted curves, and the total intensities by the solid curves. Except for the distribution at $\lambda = 1.4$ mm, each of the distributions has two peaks at symmetric angles with respect to the beam trajectory, i.e., $\theta = 0$, and the intensity of the horizontally polarized component is much larger than the vertical one. The results are in accordance with the property of TR [2,10] that TR is emitted in a cone. The angle between the two peaks in Fig. 5 corresponds to the apex angle of the light cone of TR. As the angular resolution of the measuring system was as low as about 87 mrad, the conical distribution at $\lambda = 1.4$ mm was not resolved.

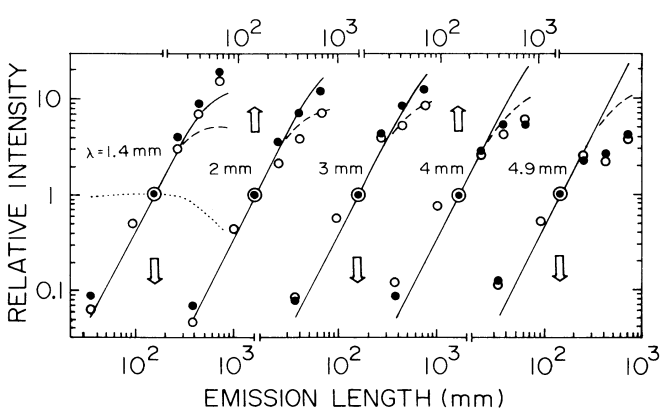

FIG. 4. Relation between the observed intensity and the emission length L. Intensities are normalized to that for $L = 156$ mm. Solid circles show intensities of an Al foil and open circles those of an Eccosorb sheet. Calculated intensities are shown by the solid curves and those corrected for the defocusing effect are shown by the dashed curves. The dotted curve shows the intensity of a high-pressure mercury lamp observed at $\lambda = 1.4$ mm.

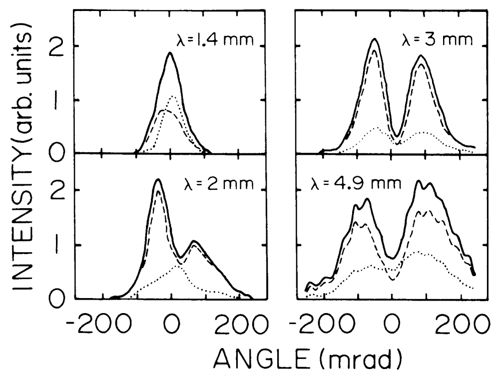

FIG. 5. Angular dependence of TR intensity of an Al foil. Polarized components are shown by the dashed curves (horizontal polarization) and by the dotted curves (vertical polarization).

Values of the peak angle θ_p observed in the angular distribution in Fig. 5 were 52, 69, and 90 mrad at $\lambda = 2.0$, 3.0, and 4.9 mm, respectively. The divergence of TR increases with the wavelength. From the theoretical intensity of Eq. (3), the peak angle is derived as $\theta_p = 0.86(\lambda/L)^{1/2}$. The observed dependence of θ_p on the wavelength is qualitatively in accordance with the theory, but the calculated peak angles are larger than the observed ones by a factor of about 1.6.

The observed distributions and intensities of the Cu and Ti foils were the same as those of the Al foil within an accuracy of 5%. This result is consistent with the theory of TR, because $|\epsilon|$ of these metals are larger than unity and no difference in the intensity is expected. The observed intensity and its angular distribution did not depend on the atomic number of the foil. This is inconsistent with the property of bremsstrahlung.

A bunch form factor was derived from the observed TR spectrum of the Al foil located at $L = 156$ mm. It resembled a spectrum obtained from coherent synchrotron radiation in our previous experiment [7]. After some trials, it has been confirmed that the form factor is not explained by a simple Gaussian function of the electron distribution in a bunch, nor by a combination of a few Gaussian functions. The dotted curve in Fig. 2 shows a spectrum calculated by assuming the uniform distribution in an ellipsoidal bunch [5] with the longitudinal bunch length of 14 mm, which explains the observed spectrum fairly well as in the case of coherent synchrotron radiation [7].

Backward TR [13] from the mirror $M1$ alone has not been considered in the analyses of the present experiment, because the observed intensity from the radiator located at $P1$, which included backward TR from $M1$, was much weaker than that observed from radiators placed at other points.

Finally, a remark is made that the angular distribution of TR is almost the same as that of Čerenkov radiation in air [14]. When $L < Z$, Eq. (3) is reduced to the following form, provided that $1-\beta \ll 1$, $|\epsilon| > 1$, and $\theta \ll 1$,

$$\frac{d^2 P_{\rm TR}}{d\Omega\, d\lambda} = \frac{\alpha}{\lambda} \left(\frac{L}{\lambda} \right)^2 \sin^2\theta \left(\frac{\sin(\pi L X/2\lambda)}{\pi L X/2\lambda} \right)^2 , \qquad (6)$$

$$X = 1/\gamma^2 + \theta^2 . \qquad (7)$$

With the replacement of $X = 1/\gamma^2 + \theta^2 - 2(n-1)$, the functional form is the same as Čerenkov radiation emitted from an electron moving through a distance L in gas with refractive index n [15].

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