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Anomalous interface roughening in porous media: Experiment and model

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We report measurements of the interface formed when a wet front propagates in paper by imbibition and we find anomalous roughening with exponent $\alpha = 0.63 \pm 0.04$. We also formulate an imbibition model that agrees with the experimental morphology. The main ingredient of the model is the propagation and pinning of a self-affine interface in the presence of quenched disorder, with erosion of overhangs. By relating our model to directed percolation, we find $\alpha \simeq 0.63$.

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I. INTRODUCTION

Recently, considerable progress has been made in understanding the dynamics of nonequilibrium interface growth in the context of a variety of models, analytical theories, and experiments [1]. Many recent investigations have concentrated on the dynamic scaling properties of the rms interface width

$$w(\ell,t) \equiv \langle [h(x,t) - \langle h(x,t)\rangle]^2\rangle^{1/2} \sim \ell^\alpha f(t/\ell^{\alpha/\beta}). \quad (1.1)$$

Here $h(x,t)$ is the surface height at time t, the angular brackets denote the average over x belonging to an interval of size ℓ; also, $f(u) \sim u^\beta$ for $u \ll 1$ and $f(u) \rightarrow$const for $u \gg 1$.

It has been widely believed that many such problems lie in the same universality class as the Kardar-Parisi-Zhang (KPZ) equation [2], which predicts scaling exponents $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ for (1+1)-dimensional systems. However, recent experiments on bacterial colony growth [3] and immiscible displacement of viscous fluids in porous media [4, 5] give quite different exponents, with α in the range 0.73–0.89. One possible explanation is based on the assumption that the noise in the system has power law distributed amplitudes [6], but the origin of such a noise in real systems remains unclear [7].

Here we report experiments in which ink, coffee, and other suspensions are absorbed by a hanging paper, forming a rough interface between wet and dry regions. We analyze this morphology and measure its roughness exponent α. Based on the experiment we propose a model for interface roughening. Both the model and the experiment produce interfaces with an anomalously large value of α. Our work is focused on the analysis of the height-height correlation function, defined by

$$c(\ell,t) \equiv \langle |\tilde{h}(\ell+x,t+\tau) - \tilde{h}(x,\tau)|^2\rangle^{1/2}. \quad (1.2)$$

Here $\tilde{h} \equiv h - \bar{h}$ and the angular brackets denote an average over x and τ, and $\bar{h}$ is the average height. For $\ell \ll L$ (where L is the system size), $c(\ell,0) \sim \ell^\alpha$, while for short times $c(0,t) \sim t^\beta$. Note that $c(\ell,t)$ scales similarly to the width $w(\ell,t)$.

II. EXPERIMENT

The imbibition experiment was performed by clipping paper to a ring stand, and allowing it to dip into a basin filled with suspensions of ink or coffee [Fig. 1(a)]. The suspension was absorbed into the paper, forming a rough interface between the wet and the dry regions. We allow the interface to rise until it stops and no change in either height or shape of the interface is observed. The stopping can be attributed to the evaporation of the fluid in the wet regions. Suspensions of coffee, ink, and various food colorings were used; also, various papers were experimented with. Parameters such as temperature, humidity, and concentration of coffee were varied. These changes affect the area and the speed of wetting, and the global width of the rough surface, but they do not affect the scaling properties of the surface. After drying, we digitize this rough interface [Fig. 1(b)]. We then calculate $c(l, t = 0)$ on different length scales l, averaging over 15 different interfaces. Figure 2(a) shows the data,

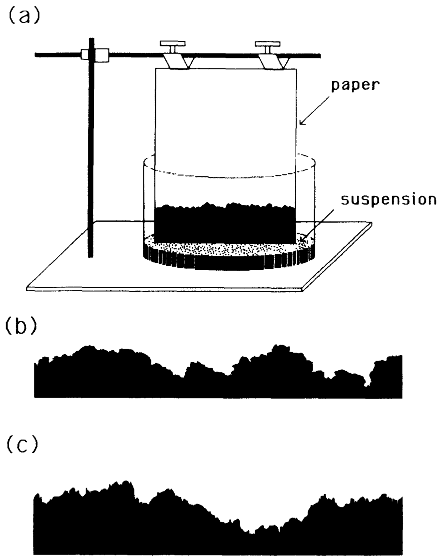

FIG. 1. (a) Schematic illustration of the experimental setup. (b) Digitized interface, using an Apple scanner with resolution 300 pixels per inch. The horizontal size of the paper is 20 cm. The function $h(x, t \to \infty)$ was obtained as the highest dark pixel in column x. (c) Typical result of the model with width $L = 400$ and $p = p_c \simeq 0.47$.

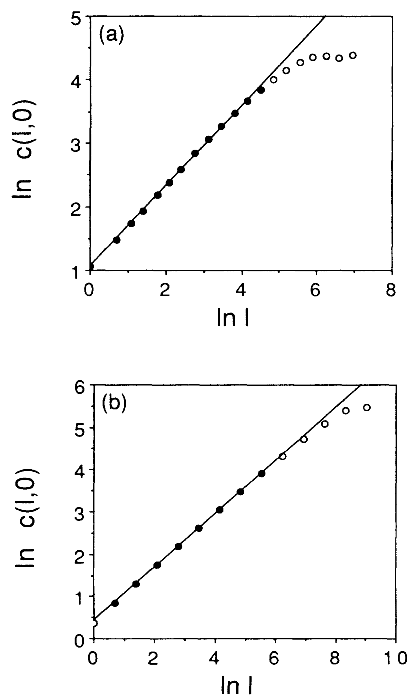

FIG. 2. Log-log plots showing the dependence on length scale ℓ of the height-height correlation function $c(\ell, 0)$ for (a) the experimental data (averaging over 15 different experiments), and (b) the numerical results (averaging over 1000 different realizations for system size $L = 16\,384$ and for $p = 0.469$, very close to p_c for the infinite system). The slope for the set of experimental points indicated by solid circles (two decades) is 0.63 ± 0.04, while the slope for the simulation points indicated by solid circles (three decades) is 0.63 ± 0.02.

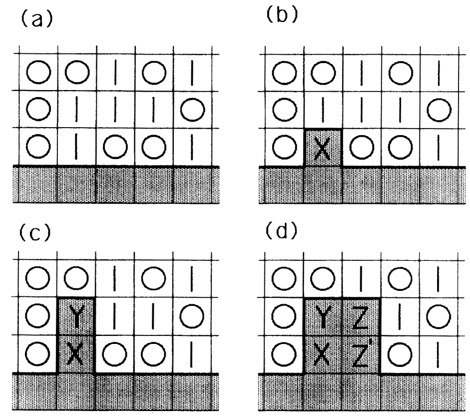

FIG. 3. Explanation of the model for interface growth with erosion of overhangs. Wet cells are indicated by shaded cells. Dry cells are randomly blocked with probability p (indicated by 0) or unblocked with probability $1 - p$ (indicated by 1). The interfaces between wet and dry cells are shown by a heavy line. (a) $t = 0$, (b) $t = 1$, (c) $t = 2$ and (d) $t = 3$.

which support a scaling of the form $c(l, 0) \sim l^{\alpha}$ with $\alpha = 0.63 \pm 0.04$.

III. MODEL

The model we propose is defined as follows: on a square lattice of edge L (with periodic boundary conditions) we block a fraction p of the cells to correspond to the inhomogeneous nature of the paper towel. At $t = 0$, we regard the "interface" to be the bold horizontal line shown in Fig. 3(a). At $t = 1$ we randomly choose a cell [labeled X in Fig. 3(b)] which is one of the unblocked dry cells that are nearest neighbors to the interface. We wet cell X and any cells that are below it in the same column. This process is then iterated. For example, Fig. 3(c) shows that at $t = 2$ we choose cell Y a second unblocked cell to wet, while Fig. 3(d) shows that at $t = 3$ we wet cell Z and also cell Z' below it [8].

We find that for p below a critical threshold $p_c = p_c(L)$ [9] the interface propagates without stopping, while for p above p_c the interface does not propagate. Figure 2(b) displays the scaling behavior of the model at criticality, and we find that $\alpha = 0.63 \pm 0.02$, a value identical to the experimental value of Fig. 2(a).

IV. DISCUSSION

Next we argue that the model presented above is connected to directed percolation [10], thereby providing a theoretical basis for the observed and calculated values of the anomalous roughening exponent α. The propagation of the interface will stop when it reaches for the first time a directed path of blocked cells leading from West to East—this path is such that one can walk on it from West to East without turning to the West (see

Fig. 4). Such a "directed path" is a path on the directed percolation cluster formed by the cells labeled 0. We assume that a single transverse length characterizes the directed percolation clusters so that the width w of this interface scales as the transverse correlation length $\xi_\perp$ of the directed percolation problem ($\xi_\perp$ is a rigorous upper bound). Thus we assume $w(\ell) \sim \xi_\perp$ and $\ell \sim \xi_{||}$, where $\xi_{||}$ is the longitudinal correlation length in the corresponding directed percolation problem. Since $\xi_\perp \sim \xi_{||}^{\nu_\perp/\nu_{||}}$ we identify $\alpha = \nu_{||}/\nu_\perp \simeq 0.63$ [11].

Up to now we have investigated only the *static* properties of the pinned interface, since experimentally we analyzed the interface only after it stopped propagating. To probe the *dynamics* of the growing interface in the model, we study the height-height correlation function $c(l,t)$. Our numerical results support an exponent $\beta = 0.68 \pm 0.04$ [Fig. 5(a)]. Numerical studies on the *moving* interface give α in the range $0.66 - 0.73$, larger than for the pinned interface. The usual exponent identity attributed to the Galilean invariance (which is known to be valid for the KPZ equation [2]) is violated; we find $\alpha + z$ smaller than 2; here $z \equiv \alpha/\beta$ is the dynamical exponent. This is a consequence of the strong anisotropy of the mechanism which excludes the overhangs: An infinitesimal tilting of the pinned interface will result in removing blocked cells, thus allowing the interface to propagate further.

Further support for the directed percolation model can be obtained if we consider a finite system at a fixed value $p_o < p_c$. If $\xi_{||}(p_o)$ is larger than the system size L, the interface may be stopped by the directed percolation path. Thus we identify two regimes: regime I where $\xi_{||} > L$ and regime II where $\xi_{||} < L$. In regime I, we observe only anomalous roughening ($\alpha \simeq 0.73$), while in Regime II we predict a crossover to behavior described by the KPZ exponent ($\alpha = 0.5$). Exactly such a crossover [12] is observed, both in our calculations [Fig. 5(b)] and even in some very recent experiments (Fig. 3 of Ref. 5).

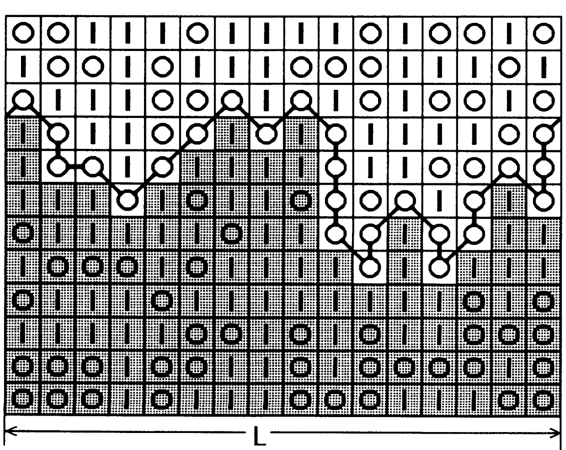

FIG. 4. Shown as a bold line is a *spanning* path formed by connected nearest-neighbor and next-nearest-neighbor blocked cells which pin the interface. Note that the various *nonspanning* clusters of blocked cells are insufficient to pin the interface.

In summary, we have presented an experimental study on the ink or coffee front propagating in a paper towel—a simple model of a randomly porous medium. The interface observed shows an anomalously large roughness exponent. Based on the physical ingredients of the experiment, a simple model has been proposed which describes the propagation and the pinning of an interface in porous media. The model reproduces the observed experimental behavior. In particular, we emphasized that the experiments correspond to the situation where the evolving interface is pinned by the random inhomogeneities, and scaling arguments based on directed percolation were presented to explain the value of the anomalously large roughness exponent α. [13]

After this work was completed, L.-H. Tang kindly sent

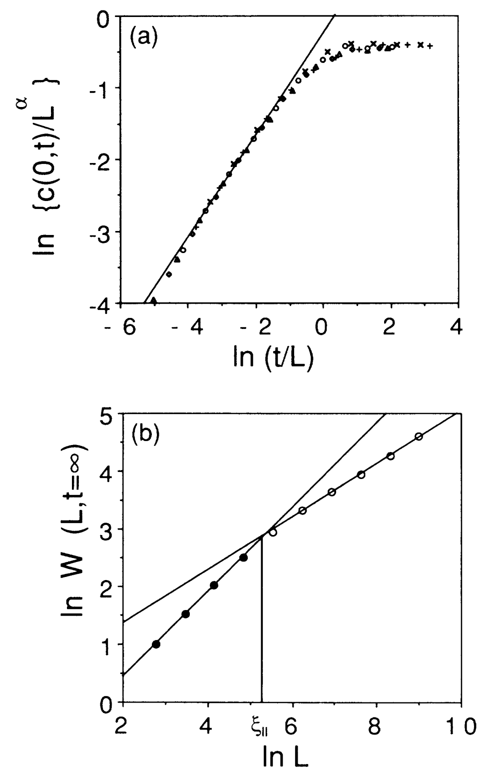

FIG. 5. (a) Scaling plot for the height-height correlation function $c(0,t)$ defined in (2), for $p = 0.47$ near the percolation threshold p_c. Shown are five different system sizes, $L = 512(\times), 1024(+), 2048(\circ), 4096(\diamond)$, and 8192 ($\triangle$). The time t is measured in terms of the number of Monte Carlo steps per average number of unblocked cells on the interface after the interface reached saturation. The data collapse was obtained using $\alpha = 0.68$ and $\beta = 0.68$ and $z = 1.00$. The slope of the straight line is 0.68. (b) Log-log plot of $w(L) \equiv w(L, t \to \infty)$, the global width of the system, as a function of system size L for a value of $p = 0.44$ well below p_c of an infinite system. If the system size L is larger than the correlation length $\xi_{||}$, then we observe an interface which is propagating in time without being stopped by the percolation cluster. If $L < \xi_{||}$ we can still obtain a propagating interface by removing randomly one cell of the percolation cluster which stops, at a given moment, the interface. This local change allows us to obtain a moving interface which is locally pinned by a critical percolation substrate.

us a copy of unpublished work which independently proposes the possible relevance of directed percolation as a mechanism for interface pinning in a medium with quenched disorder [14].

Note added in proof. After this manuscript was submitted, both the imbibition experiments and the imbibition model were extended to the case of three dimensions [15], and additional results were obtained on the dynamic growth in two dimensions [16].

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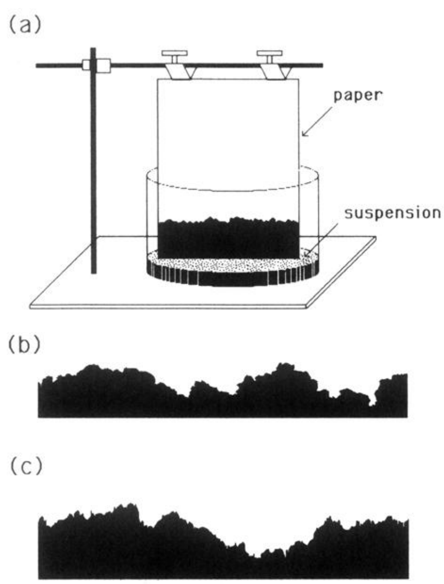

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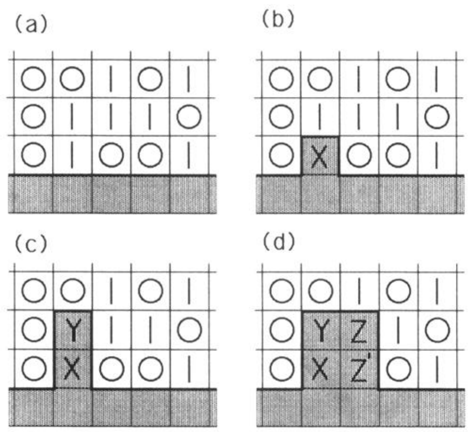

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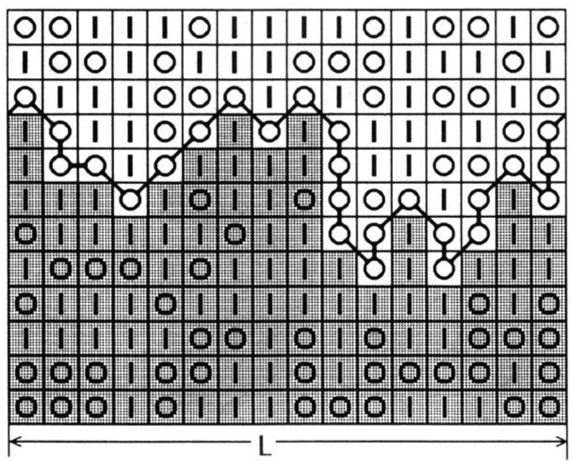

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