PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Preparation and detection of macroscopic quantum superpositions by two-photon field-atom interactions

B. Sherman and G. Kurizki
Department of Chemical Physics, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 5 March 1992)

We put forward a simple, feasible scheme for the preparation and subsequent detection of macroscopic quantum superposition (MQS) states. It is based on the two-photon model which obtains when a cascade of two atomic transitions is resonant with twice the field frequency. The initial conditions amount to a field in a mixed state characteristic of lasers or masers and an excited atom. The MQS is generated by a conditional measurement of the atomic excitation after an interaction time that determines the relative phase of the MQS components. Remarkably, the MQS is subsequently detected and its phase is inferred by measuring the excitation probability of a second, "probe," atom, as a function of its interaction time. The realization of the scheme in the optical domain, using dielectric microspheres, is discussed.

PACS number(s): 42.50.−p, 32.80.−t, 42.52.+x

Superposed macroscopically distinguishable quantum states of the electromagnetic field, hereafter referred to as macroscopic quantum superpositions (MQS), have aroused considerable interest in recent years [1–9]. Their main appeal is that they constitute potentially realizable "Schrödinger cats" that embody the well-known paradoxical aspect of quantum mechanics [4]. Attention has been focused on a single-mode field in a MQS of two coherent quasiclassical states, with identical mean amplitudes and a fixed relative phase [1,3]. Various rather intricate mechanisms have been proposed for the preparation of such MQS, but none of them has been realized thus far. These mechanisms include the following:

(a) Nonlinear evolution of the field from an initially coherent state in amplitude-dispersive [5,6], and, particularly, bistably dispersive [7] media. The generation of MQS states in such media is contingent on dissipative losses being negligible.

(b) Quantum measurements (photon counting [8] or quadrature detection [9]) of the field, following its preparation in a nonclassical state, e.g., in a parametric amplifier. Such schemes are hampered not only by dissipation, but also by the performance of photon detectors that falls short of ideal (unit) efficiency.

(c) Interactions of a two-level atom with a quantized single-mode field, describable by the Jaynes-Cummings model (JCM). This model is of fundamental importance in quantum optics [10,11] and is realizable to a very good approximation in high-quality resonators [12]. Certain JCM schemes for the generation of MQS rely on the initial preparation of the atom in a polarized state, i.e., a coherent superposition of the ground and excited states [13,14]. A more recent scheme [15] exploits a remarkable inherent property of the resonant JCM [16], whereby initial preparation of the atom in the ground or excited state and of the field in a quasiclassical coherent state results in the spontaneous disentanglement of the atomic and field states, with the field forming a MQS of two quasiclassical states. The limitations of this scheme stem from the complexity of the field evolution in the JCM: (i) It is difficult to exactly characterize the superposed states in the spontaneously-formed MQS. (ii) Likewise, it is difficult to account for changes in the MQS due to initial deviations of the field from a perfectly coherent state, although such deviations are inevitable in real systems. (iii) The field-atom interaction time is restricted to exactly half the time between the "collapse" and "revival" of the oscillations of atomic population inversion. Correspondingly, the MQS relative phase is restricted to 180°.

The detection of MQS, particularly the "signature" of its relative phase, poses yet another difficulty. The only viable detection scheme that has been proposed thus far is based on the interference of a MQS with an intense coherent field of the same frequency at a homodyne detector, which results in sensitivity of the output current to the MQS phase [5]. As noted above, the losses or inefficiency of the detector will severely degrade this MQS signature.

Our purpose here is to put forward a simple, feasible scheme for the preparation and subsequent detection of MQS *in the same setup.* It is based on the *two-photon resonant JCM* with due account for Stark shifts [17,18], which obtains when a cascade of atomic transitions $|e\rangle \rightarrow |i\rangle \rightarrow |g\rangle$ is resonant with twice the field frequency, $\omega_{eg}=2\omega$, whereas the intermediate transition frequencies ω_{ei} and ω_{ig} are strongly detuned from ω. In Ref. [19], the authors have studied the splitting of the initial quasiprobability distribution of the field into two identical coherently superposed parts via a *resonant two-photon interaction* in a degenerate Raman model which is isomorphic to the present scheme. However, whereas the degenerate Raman model involves two *indistinguishable* atomic states, the present scheme allows the MQS preparation by projecting the entangled field-atom system on one of the *distinguishable* atomic states $|e\rangle$ or $|g\rangle$. The present scheme allows the generation of MQS with *any relative phase,* starting from a field in a narrow statistical distribution of coherent states characteristic of lasers or masers and from an excited or ground state of the atom. The MQS is generated by a conditional measurement in which the excited atom is registered after an interaction time corresponding to the chosen relative phase, provided this time is much shorter than the dissipation time. Remarkably, the MQS is subsequently detected and its phase is inferred by monitoring the excitation probability of a second, "probe," atom, as a function of its interaction time (still well within the dissipation time). The realistic initial conditions on the field and atom, the flexible choice of the interaction time, and the simplicity of the phase detection, all stem from the advantageous property of the two-photon resonant JCM, whereby each of the superposed field states evolves periodically in time [19], in contrast to the complicated evolution of their counterparts in the one-photon resonant JCM. Another advantage of the present scheme is the high efficiency of atomic excitation measurements, as compared to photon detection employed in other schemes [5,8,9]. Finally, it will be argued that the present scheme is realizable not only in high-Q microwave cavities [11,12,17], but also in dielectric microspheres that can serve as high-Q optical resonators [20,21].

In order to prepare the desired MQS, we start at $t=0$ from a single-mode field in a mixed state, characterized by a narrow quasiclassical distribution in phase and amplitude. This means that the initial density operator of the field, when written in the basis of coherent states $|\beta\rangle$

$$\rho_F(0)=\int d^2\beta P(\beta)|\beta\rangle\langle\beta| \tag{1}$$

has a distribution $P(\beta)$ that is localized around the state $|\bar{\beta}\rangle=||\bar{\beta}|\exp(i\chi)\rangle$, with $|\bar{\beta}|\gg 1$ and small normalized variances, $\Delta\beta/|\bar{\beta}|\ll 1$, and $\Delta\chi/|\bar{\chi}|\ll 1$. A laser (or maser) well above threshold (yet well below the saturation limit) are suitable examples. For the sake of definiteness, we impose the realizable condition that the atom is initially in the excited state $|e\rangle$, although we can equally well treat an atom initially in the ground state $|g\rangle$ or in a statistical mixture of the two.

Given these initial conditions, let the atom traverse the field region during $0\le t\le\tau$, and then measure whether it emerges in the $|e\rangle$ or $|g\rangle$ state. The density operator of the field $\rho_F(\tau)$ then corresponds to the projection of the entangled field-atom density operator $\rho_{A+F}(\tau)$ on either $|e\rangle$ or $|g\rangle$. The evolution of the field-atom system up to τ, under the two-photon resonant condition $2\omega=\omega_{eg}$, is analyzed on noting that the intermediate state $|i\rangle$ in the "ladder" $|e\rangle\rightarrow|i\rangle\rightarrow|g\rangle$ can be eliminated [17,18], provided that the detunings $\pm\Delta/2$ of ω_{ei} and ω_{ig} from ω are sufficiently large, as detailed below. Then, an n-photon component of the field at ω will cause two-photon Rabi nutation between $|e\rangle$ and $|g\rangle$ with the effective Rabi frequency [17]

$$\Omega_n=A+nB\,, \tag{2a}$$

where

$$A=(\Omega_{ei}^2+2\Omega_{ig}^2)/\Delta,\quad B=(\Omega_{ei}^2+\Omega_{ig}^2)/\Delta\,. \tag{2b}$$

Here Ω_{ei} and Ω_{ig} are the Rabi frequencies (per photon) for the respective transitions, giving rise to the Stark shifts $(n+1)\Omega_{ei}^2/\Delta$ for the first transition in the cascade and $(n+2)\Omega_{ig}^2/\Delta$ for the subsequent one. These Stark shifts can effectively counter the detuning. Therefore, in order to eliminate $|i\rangle$, these shifts should be small enough to preclude the buildup of population at $|i\rangle$ during the interaction time τ. This implies [18]

$$\frac{\Omega_n^2\tau}{\Delta}\ll\pi\,. \tag{2c}$$

By choosing Ω_n much smaller than Δ we can ignore the intermediate level and make the two-photon model valid over many nutation cycles.

The evolution in the two-photon resonant JCM is thus obtainable in complete analogy to the ordinary (one-photon) resonant JCM [10,11], on expanding the initial field state in terms of photon-number states $|n\rangle$ and writing the Rabi nutation solution for each $|n\rangle$ component, with Eq. (2a) replacing the ordinary Rabi frequency (which is proportional to $n^{1/2}$). For an atom measured to be in state $|e\rangle$, the corresponding density matrix of the field is

$$\rho_F(\tau)=\sum_{n,n'}\langle n|\rho_F(0)|n'\rangle\cos\left(\frac{\Omega_{n+2}\tau}{2}\right)\times\cos\left(\frac{\Omega_{n'+2}\tau}{2}\right)|n\rangle\langle n'|\,, \tag{3}$$

where τ is well within the dissipation time [1,3].

Using Eqs. (1)-(3) we can obtain the matrix elements of $\rho_F(\tau)$ between any two coherent states:

$$\langle\alpha|\rho_F(\tau)|\alpha'\rangle=\exp(-|\alpha|^2/2-|\alpha'|^2/2)\sum_{n,n'}\frac{\langle n|\rho_F(0)|n'\rangle}{4(n!n'!)^{1/2}}\times[e^{i\phi}(\alpha e^{iB\tau/2})^n+e^{-i\phi}(\alpha e^{-iB\tau/2})^n]^*[e^{i\phi}(\alpha' e^{iB\tau/2})^{n'}+e^{-i\phi}(\alpha' e^{-iB\tau/2})^{n'}]\,, \tag{4}$$

where $\phi=(A+2B)\tau/2$.

This expression can be rewritten as

$$\langle\alpha|\rho_F(\tau)|\alpha'\rangle=\langle\psi_F(\alpha,\tau)|\rho_F(0)|\psi_F(\alpha',\tau)\rangle\,, \tag{5}$$

where the ket vector

$$|\psi_F(\alpha,\tau)\rangle=N_\alpha(\tau)^{-1/2}(e^{i\phi}|\alpha e^{iB\tau/2}\rangle+e^{-i\phi}|\alpha e^{-iB\tau/2}\rangle) \tag{6a}$$

is a superposition of two coherent states with a relative phase of $B\tau$, normalized by [1]

$$N_\alpha(\tau)=2+2\cos[|\alpha|^2\sin(B\tau)+2\phi]\times\exp[-2|\alpha|^2\sin^2(B\tau/2)]\,. \tag{6b}$$

It is easily checked that the Q function $\langle\alpha|\rho_F(\tau)|\alpha\rangle$ consists of two diagonal parts peaked around the states $|\bar{\beta}\exp(iB\tau/2)\rangle$ and $|\bar{\beta}\exp(-iB\tau/2)\rangle$. Each part is described [Fig. 1(a)] by the initial Q function rotated either clockwise or counterclockwise by the phase $B\tau/2$.

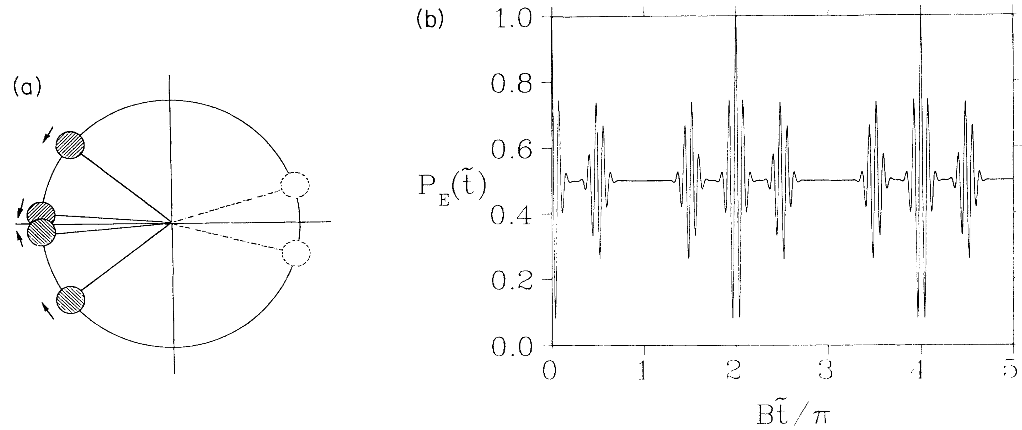

FIG. 1. Generation and detection of MQS in the two-photon Jaynes-Cummings model: (a) Q-function evolution in the phase plane. The dashed circles indicate a MQS generated after the passage of the first atom, by splitting the initial quasiclassical Q function into two identical parts and their counterrotation by phases $\pm B\tau/2$. The shaded circles depict the MQS generated by the second "probe" atom by splitting each dashed circle into two identical parts and their counterrotation at rate $B/2$. The overlap of the two foremost circles corresponds to one of the partial ("satellite") revivals. (b) The periodic temporal pattern of the excitation probability of the second probe atom following its interaction with the MQS generated by the first atom. The parameters used are $\bar{\beta}=6$, $B\tau=\pi/2$, and $B=A$. The satellite (half-amplitude) revivals are separated by $\pm\pi/4$ from the regular (full-amplitude) revivals.

Although these two parts are localized, the field is nonetheless in a *coherent* superposition of two quasiclassical distributions. This is evident from the fact that $\langle a|\rho_F(\tau)|a'\rangle$ is off diagonal in states localized near $|a\rangle=|\bar{\beta}\exp(\pm iB\tau/2)\rangle$ and correspondingly $|a'\rangle=|\bar{\beta}\exp(\mp iB\tau/2)\rangle$. These off-diagonal elements are multiplied by $\exp(\pm 2i\phi)$. For the idealized initial preparation of the field in a single coherent state $|\bar{\beta}\rangle$, the field-density matrix reduces to the pure state

$$\rho_F(\tau)=|\psi_F(\bar{\beta},\tau)\rangle\langle\psi_F(\bar{\beta},\tau)|\,. \tag{7}$$

Equations (5)–(7) demonstrate that in the two-photon resonant JCM, the projection of the field-atom system on an atomic-energy state creates a MQS by first splitting the initial quasiclassical Q function into two identical parts, and subsequently rotating them in opposite senses in the phase plane at a constant rate B. The advantageous properties of the model are linear dependence of the phases ϕ and $B\tau\,(\mathrm{mod}\,2\pi)$ on τ, as well as the preservation of each of the superposed Q-function parts in its initial form (at any τ well within the dissipation time). These properties allow the control of MQS by adjustment of τ.

The same advantageous properties are in the basis of our proposal for the *detection* of the MQS after its preparation. It consists in the injection of a probe atom in the state $|e\rangle$ into the resonator at $t_1>\tau$, still well within the dissipation time [1,3]. In the ensemble of excitation measurements, the fraction of probe atoms that remains in the $|e\rangle$ state upon exiting the resonator at time $\tilde{t}=t-t_1$ is obtained analogously to its counterpart in the one-photon resonant JCM [10,11]. This fraction, i.e., the probability of excitation is given by

$$P_e(t)=\sum_{n=0}^{\infty}\langle n|\rho_F(\tau)|n\rangle\cos^2\left[\frac{\Omega_{n+2}\tilde{t}}{2}\right]. \tag{8}$$

Here the initial-field preparation is given by $\rho_F(\tau)$, since the phases ϕ and $B\tau$ do not change during $t_1\geq t\geq\tau$. We substitute Eq. (3) and Eq. (1) into Eq. (8) and perform the summation over n again, utilizing the results of the one-photon resonant JCM [10,11]. The final result is

$$P_e(\tilde{t})=\tfrac{1}{2}+\sum_{j=0,\pm1}\int d^2\beta\,P(\beta)N_\beta^{-1}(\tau)e^{-|\beta|^2}\exp\{|\beta|^2\cos[B(\tilde{t}+j\tau)+\arg(\beta)]\}$$
$$\times\cos\{(A+2B)(\tilde{t}+j\tau)+|\beta|^2\sin[B(\tilde{t}+j\tau)+\arg(\beta)]\}\,. \tag{9}$$

Examination of Eq. (9) reveals that "revivals" of the oscillations of the population inversion occur when $B(\tilde{t}+j\tau)=2\pi n$, where n is any integer and $\tilde{t}>0$. Thus, a field initially prepared in a MQS with a relative phase of $B\tau$ gives rise to a pair of "satellites," shifted by $\pm\tau$, about each of the regular revivals which have a period of $T=2\pi/B$. The amplitude of each regular revival is twice that of its satellites. Hence, the temporal pattern $P_e(\tilde{t})$ can be readily used to detect a MQS and infer its relative phase [Fig. 1(b)].

We can obtain some insight into the temporal pattern $P_e(\tilde{t})$ in Eq. (9) by realizing that it reflects the evolution of the field distribution in the phase plane [3,16,19]. Each localized peak of the Q function in the initial MQS splits into two identical counter-rotating parts when the interaction with the probe atom is switched on, as we have seen. This splitting generates a four-peaked distribution [Fig. 1(a)], and, owing to the phase differences between these peaks, collapse of the atomic Rabi oscillations occurs. The two foremost peaks "collide" at $\tilde{t}=\tau$ or $\tilde{t}=T-\tau$

(whichever comes first), thereby restoring half of the initial distribution, and causing the first partial (satellite) revival of the Rabi oscillation. Likewise, at $\tilde{t}=T+\tau$ the other two peaks overlap, causing another satellite revival that marks the restoration of half the initial distribution. At $\tilde{t}=T$ the initial two-peaked distribution is completely restored, corresponding to a full revival, with twice the amplitude of the satellite revivals. The perfect periodicity of the evolution will produce the same series of revivals at $\tilde{t}+j\tau=nT$, with $n=1,2,3,\ldots$ and $j=0,\pm 1$ as long as dissipation is negligible.

Superconducting microwave resonators [12] can satisfy the requirements of the present scheme, namely, above-threshold stimulated emission at $\omega=\omega_{eg}/2$, in a single high-Q mode ($Q\sim 10^{12}$), with a much longer dissipation time than the atomic time of flight through the resonator. The point we wish to make here is that these requirements may also be satisfied at optical frequencies by dielectric low-loss (transparent) microspheres with radii $\leq 10^2$ wavelengths. Optical pumping of the active medium in such a microsphere can yield lasing at ω in a spherical mode with a Q value as high as 10^{10}, associated with a low-order Mie resonance [20,21]. In the current state of the art, it is more practical to settle for spherical modes with $Q\sim 10^7$. The spatial amplitude of such a mode appreciably extends outside the surface over a radial region ΔR which is several wavelength large, a region in which this amplitude exceeds the amplitudes of other modes by a few orders of magnitude [21]. A beam of atoms with a two-photon resonance at $\omega_{eg}=2\omega$, which pass within a distance ΔR away from the sphere surface will therefore selectively couple to this mode. This requires the collimation of the atomic beam down to radial widths of $\sim 1\ \mu$m, which is quite feasible. The time of flight of an atom with velocity $v\geq 10^5$ cm/sec through the field region $\sim 10^{-9}$ sec $\geq \Delta R/v$, can correspond to appreciable $B\tau$, yet shorter than the dissipation time of a MQS in a mode with $Q\sim 10^7$.

To conclude, we have pointed out the conceptual simplicity and feasibility of macroscopic quantum superpositions generation and detection based on conditional measurements in the two-photon resonant Jaynes-Cummings model. The MQS prepared by the field interaction with a passing atom has been shown to be detectable by the periodically recurring satellite revivals of the Rabi oscillation, which are displayed by the population inversion of a subsequent probe atom.

This work has been supported in part by the U.S.-Israel BSF. One of us (B.S.) acknowledges the support of the Bat-Sheva Foundation.

[1] W. Schleich, M. Pernigo, and Fam Le Kien, Phys. Rev. A **44**, 2172 (1991); W. Zurek, Phys. Today **44** (10), 36 (1991).

[2] J. Janszky and A. V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990).

[3] J. Eiselt and H. Risken, Opt. Commun. **72**, 351 (1989).

[4] W. Schleich, B. Yurke, and D. F. Walls, Phys. Rev. A **42**, 1703 (1990).

[5] B. Yurke and D. Stoler, Phys. Rev. A **35**, 4876 (1987).

[6] G. J. Milburn and C. A. Holmes, Phys. Rev. Lett. **56**, 2237 (1986).

[7] C. M. Savage and W. A. Cheng, Opt. Commun. **70**, 439 (1989).

[8] T. Ogawa, M. Ueda, and N. Imoto, Phys. Rev. Lett. **66**, 1046 (1991).

[9] M. Wolinsky and H. Carmichael, Phys. Rev. Lett. **60**, 1836 (1988).

[10] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980).

[11] P. L. Knight and P. M. Radmore, Phys. Rev. A **26**, 676 (1982).

[12] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987); G. Rempe, F. Schmidt-Kaler, and H. Walther, *ibid.* **64**, 2783 (1990).

[13] P. Meystre, J. J. Slosser, and M. Wilkens, Opt. Commun. **79**, 300 (1990).

[14] C. M. Savage, S. L. Braunstein, and D. F. Walls, Opt. Lett. **15**, 628 (1990).

[15] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990); Phys. Rev. A **44**, 5913 (1991).

[16] S. J. D. Phoenix and P. L. Knight, Ann. Phys. (N.Y.) **186**, 381 (1988); Phys. Rev. Lett. **66**, 2833 (1991); Phys. Rev. A **44**, 6023 (1991).

[17] M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **35**, 154 (1987).

[18] A. H. Toor and M. S. Zubairy, Phys. Rev. A **45**, 4951 (1992).

[19] S. J. D. Phoenix and P. L. Knight, J. Opt. Soc. Am. B **7**, 116 (1990).

[20] S.-X. Qian, J. B. Snow, and R. K. Chang, Phys. Rev. Lett. **56**, 926 (1986); Science **231**, 486 (1986).

[21] S. C. Hill and R. E. Benner, J. Opt. Soc. Am. B **3**, 1509 (1991).