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Tunneling control in a two-level system

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The effect of external periodic driving forces in the tunneling process of a two-level system is studied within the Floquet formalism and perturbation theory. Coherent tunneling can be increased or decreased depending on the external force parameters. In particular, the model is able to account accurately for the tunneling suppression phenomenon recently discovered [Grossmann *et al.*, Phys. Rev. Lett. **67**, 516 (1991)] in a quartic bistable potential.

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External control of tunneling has been the subject of recent theoretical work stimulated by its numerous applications [1–4]. Lin and Ballentine [5] have found that a monochromatic external field acting on a quartic double-well oscillator can increase the rate of coherent tunneling to values several orders of magnitude higher than those for the undriven system. This behavior is obtained for field frequencies close to the harmonic frequency at the bottom of each well and relatively large values of the field strength. Classical mechanical analyses of the motion under these field conditions [5–7] show that the phase-space structure of the system is considerably perturbed with respect to that of the undriven system. Chaos and small stable islands coexist in the driven system, which determines its different tunneling behavior [7].

A completely opposite situation has been discovered by Grossmann *et al.* [8,9] in the same quartic, bistable oscillator. For specific values of field strength and frequency, tunneling is coherently destroyed, i.e., a localized packet can be built as a superposition of two degenerate Floquet states of the system, which remains localized forever in one well. For this behavior to occur the driving force frequency must be comparable to the splitting of the two lowest states in the unperturbed system, but much smaller than the harmonic frequency at the bottom of each well. The field strength for tunneling suppression is numerically found to fit a linear function of the frequency and its values are very small compared to those required in Lin and Ballentine's case [5].

We have performed an analysis of the classical phase space for this system in the range of driving force parameters containing those leading to tunneling suppression. The perturbation of the driving force on the classical motion of the system turns out to be negligible. Let us remember that in the case reported by Lin and Ballentine the driving term produced drastic changes in the classical phase space of the system and that these changes were responsible for most of the quantum tunneling features as coherence and rate enhancement [7]. The case reported by Grossmann *et al.* is therefore an intriguing example of intrinsic quantum control of tunneling.

In this Rapid Communication we will show that tunneling suppression under the conditions of Grossmann *et al.* is a very general behavior that can occur even in a two-level system. This model is also able to account accurately for the values of the parameters that give rise to the destruction of tunneling in the system studied by these authors.

We will therefore study the system described by the following general, two-level Hamiltonian ($\hbar=1$)

$$H=-(\Delta_0/2)(|1\rangle\langle 1|-|2\rangle\langle 2|)+V(t)(|1\rangle\langle 2|+|2\rangle\langle 1|)\,, \tag{1}$$

where Δ_0 is the energy splitting between the states $|1\rangle$ and $|2\rangle$ and $V(t)$ is the coupling between them induced by the external periodic driving force; we will assume a time dependence of the form $V(t)=V_0\sin(\omega t)$.

In this model the states $|r\rangle$ and $|l\rangle$ defined as

$$|r\rangle=(\tfrac{1}{2})^{1/2}(|1\rangle+|2\rangle)\,,\qquad |l\rangle=(\tfrac{1}{2})^{1/2}(|1\rangle-|2\rangle) \tag{2}$$

will be interpreted as wave packets localized in the "right" and "left" wells, respectively. Thus, the tunneling frequency for the undriven ($V=0$) system is given by Δ_0.

The Hamiltonian in Eq. (1) will properly describe processes involving a pair of eigenstates of a multilevel system whenever the driving frequency ω and the pair splitting Δ_0 are small compared to the energy of any other state with respect to the average energy of the pair states, not to mention the condition of small strength field V_0 values which is necessary for $|1\rangle$ and $|2\rangle$ to be identified as a pair of eigenstates from the unperturbed system. As already mentioned, the range of parameters chosen in Refs. [8,9] for the quartic oscillator satisfies these conditions.

Floquet theory reduces the problem of the solution of the periodically time-dependent Schrödinger equation to the determination of the propagator for just one period of the driving term. The Floquet states and quasienergies may be obtained by diagonalization of the one-period propagator, which for our model is expressed as a 2×2 anti-Hermitian matrix. Let us use the notation $|f_1\rangle$ and $|f_2\rangle$ for the two-level system Floquet states and ε_1 and ε_2 for the corresponding quasienergies.

In order to obtain the one-period propagator, perturbation theory will be used. In doing so, an adequate choice of the basic set to define the matrix representation of the propagator is fundamental for the perturbation ansatz to converge. Since we are interested in describing tunneling destruction then the proper choice is the set $\{|r\rangle,|l\rangle\}$. Expanding the solution $|\phi\rangle$ of the time-dependent

Schrödinger equation for the Hamiltonian (1) in this set we have

$$|\phi\rangle = |l\rangle c_l(t)\exp\left[-i\int_0^t dt\, V(t)\right] + |r\rangle c_r(t)\exp\left[i\int_0^t dt\, V(t)\right]. \quad (3)$$

The time-dependent phases in this equation have been extracted to simplify the following equations. The coefficients c_l and c_r must then satisfy the linear system

$$i\dot{c}_l = -(\Delta_0/2)c_r,\quad i\dot{c}_r = -(\Delta_0/2)c_l. \quad (4)$$

We first solve these equations for the initial condition $c_l(0)=1$, $c_r(0)=0$ assuming, as a zero-order approximation, that $c_r(t)\simeq 0$. Then in first order we have

$$c_l(t)=1,$$
$$c_r(t)=i(\Delta_0/2)\int_0^t dt\,\exp\left[-2i\int_0^t dt'\, V(t')\right]. \quad (5)$$

Particularizing for $t=\tau$ (the period of the perturbation) and $V(t)=V_0\sin(\omega t)$ we get the first row for the one-period propagator matrix U

$$U_{11}=c_l(\tau)\exp\left[-i\int_0^\tau dt\, V(t)\right]=1,$$
$$U_{12}=c_r(\tau)\exp\left[i\int_0^\tau dt\, V(t)\right] = i(\Delta_0\pi/\omega)\exp(-2iV_0/\omega)J_0(2V_0/\omega),$$

where $J_0(z)$ is the zero-order Bessel function.

Repetition of the process for the initial condition $c_l(0)=0$, $c_r(0)=1$, orthogonal to the previous one, gives the expected remaining elements of the U matrix

$$U_{21}=-U_{12}^*,\quad U_{22}=1.$$

Diagonalization of the matrix U gives in first order the following quasienergy splitting:

$$\Delta=\varepsilon_2-\varepsilon_1=\Delta_0 J_0(2V_0/\omega). \quad (6)$$

Note that the zero-order Floquet states coincide, except for a time-dependent phase of period τ, with the basis set states (the localized packets $|l\rangle$ and $|r\rangle$). This is a direct consequence of the use of perturbation theory.

Since the Bessel function $J_0(z)\le 1$, the tunneling frequency Δ for the perturbed system is, in this approximation, always smaller than the one corresponding to the unperturbed system Δ_0. For $V=0$, $J_0=1$ and Eq. (6) gives the correct result. If the value of $2V_0/\omega$ coincides with one of the zeros z_i of the Bessel function, tunneling is suppressed. The set of parameters satisfying this condition defines straight lines in the V_0-ω space. In Fig. 1 we give some of these curves for the driven quartic oscillator Hamiltonian [8,9]

$$H=p^2/2-x^2/4+x^4/64D+Sx\sin(\omega t). \quad (7)$$

In this case $V_0=S\langle 1|x|2\rangle$. We have obtained the lowest pair of states $\langle 1|$ and $\langle 2|$ for the undriven system and its energy splitting Δ_0 by diagonalization of the unperturbed Hamiltonian [Eq. (7) with $S=0$] in the basis set of the first 200 states of a harmonic oscillator whose frequency

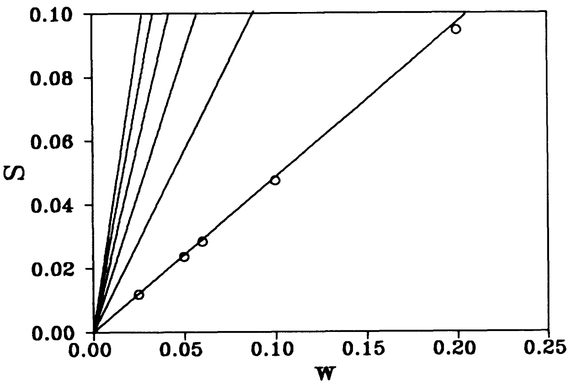

FIG. 1. S and ω values for the driven quartic oscillator of Eq. (7) ($D=1$) which lead to tunneling suppression as determined from Eq. (6). The circles are numerical results from Ref. [9].

was optimized. The values of Δ_0 and $\langle 1|x|2\rangle$ obtained for $D=1$ and $D=2$ are given in Table I.

In Table I and Fig. 1 we also give the S and ω values for tunneling suppression which were determined in Refs. [8,9] from an extensive numerical study. Our simple model is able to account very accurately for these values and its functional dependence.

Note that expression (6) is completely general. If the conditions for the validity of the two-level approximation and those for the validity of the perturbation method used to solve the two-state model are met, tunneling control is solely determined by Eq. (6) and the parameters appearing in it.

The conditions that affect the validity of the two-level approximation have been established earlier in the description of the model. Let us discuss now the validity range of the perturbation approximation used to study the model system itself. The first-order solution given in Eq. (5) will be valid provided

$$|c_r(t)|\ll 1.$$

Evidently, this condition will not be satisfied for times close to the tunneling time $2\pi/\Delta$ for which $|c_r(t)|\simeq 1$. From Eq. (5) it is found that

$$c_r(t)=i(\Delta_0/2\omega)\exp(-2iV_0/\omega) \times[f(\omega t)+\omega t J_0(2V_0/\omega)], \quad (8)$$

where $f(t)$ is a bounded periodic function. Rough upper bounds for the magnitude of the real and imaginary parts

TABLE I. Parameters for the driven quartic bistable oscillator [Eq. (7)] and two sets of the parameters S and ω that lead to tunneling suppression from this work and Refs. [8,9].

				S	
D	Δ_0	$\langle 1\|x\|2\rangle$	ω	This work	Refs. [8,9]
1.0	0.0239	2.4783	0.06	0.029 11	0.028 39
2.0	1.89×10^{-4}	3.7910	0.01	3.172×10^{-3}	3.171×10^{-3}

of $f(t)$ can be estimated giving

$$|\mathrm{Re}[f(t)]| < \pi[1 - J_0(2V_0/\omega)]/2 ,$$

$$|\mathrm{Im}[f(t)]| < \pi .$$

Thus for $J_0(2V_0/\omega)=0$, $c_r(t)$ in Eq. (8) remains small for all times if $\Delta_0/2\omega \ll 1$. In other words, too small driving frequencies invalidate our treatment. For the case $J_0(2V_0/\omega)\neq 0$, the condition $\Delta_0/2\omega \ll 1$ is also necessary but not sufficient, since the linear term in time in Eq. (8) will increase and eventually produce the breakdown of the perturbation approximation. The limit time for this to happen is [$|c_r| \simeq 1$ in Eq. (8)]

$$t_{\mathrm{limit}} \simeq 2/J_0(2V_0/\omega)\Delta_0 . \tag{9}$$

However, if t_{limit} is still significantly larger than the tunneling time for the undriven system $(2\pi/\Delta_0)$, i.e., if $J_0 \ll 1/\pi$, Eq. (6) remains valid in predicting the decrease of the tunneling rate. If $J_0(2V_0/\omega)\simeq 1$, i.e., if $2V_0/\omega \simeq 0$ the approximation may fail and this can occur for either V_0 small or ω large. However, for $V_0 \simeq 0$ Eq. (6) gives the correct result, which can be explained as an error compensation in the diagonalization process of the one-period propagator matrix. For large ω values this matrix loses its physical sense since the period $\tau \to 0$ (note that in the limit $\omega \to \infty$ to get a finite time evolution one would have to apply infinitely many times the propagator).

In conclusion, the preceding perturbation treatment is not appropriate in cases of driving forces with either large or small values of the frequency. However, these two frequency limits can also be solved by perturbation methods if the zero-order states are properly chosen. Let us discuss first the limit $\omega \to 0$. In this adiabatic limit the right zero-order states are the adiabatic states $|a_1(t)\rangle$ and $|a_2(t)\rangle$ which diagonalize the time-dependent Hamiltonian (7) at every time t. The zero-order Floquet states are then obtained by adding the corresponding adiabatic phase factors, i.e.,

$$|f_1\rangle = \exp\left[-i\int_0^t dt\, e_1(t)\right]|a_1(t)\rangle ,$$
$$|f_2\rangle = \exp\left[-i\int_0^t dt\, e_2(t)\right]|a_2(t)\rangle , \tag{10}$$

where

$$e_1 = (\Delta_0^2/4 + V^2)^{1/2}, \quad e_2 = -(\Delta_0^2/4 + V^2)^{1/2}$$

are the adiabatic eigenvalues. Particularizing for $t=\tau$ and $V=V_0\sin(\omega t)$, we get from the phases in Eq. (10) the quasienergies ε_1 and ε_2 and the tunneling frequency

$$\Delta = \varepsilon_2 - \varepsilon_1$$
$$= (2\Delta_0/\pi)(1+q^2)^{1/2} E(q/(1+q^2)^{1/2}) , \tag{11}$$

where $q = 2V_0/\Delta_0$ and $E(z)$ is the completer elliptic integral of the second kind. The tunneling frequency is now larger than that for the unperturbed system. This is a zero-order result independent of ω. First-order corrections can be calculated similarly to the previous case but these are exponentially small ($\sim e^{-1/\omega}$) since the adiabatic energies do not cross.

Formula (11) is very similar to the one found by Grossmann *et al.* [Eq. (28) in Ref. [9]] for the Hamiltonian (7) in the low-frequency regime. The parameter dependence, however, is slightly different. It turns out that Eq. (11) fits somewhat better the full numerical results of these authors [9] for the driven quartic oscillator.

Let us solve finally the high-frequency limit. By high frequency we mean $\omega \gg \Delta_0$ [Eq. (9)] but ω still smaller than the energy of any other possible state with respect to the energy of states $|1\rangle$ and $|2\rangle$ in order for the two-level approximation to be still valid. Due to the fast oscillations, in zero-order approximation the perturbation term averages out to zero. Thus the correct zero-order states in this case are precisely the states $|1\rangle$ and $|2\rangle$.

Trying for solutions of the form

$$|\phi\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle ,$$

with $c_1(0)=1$, $c_2(0)=0$ and $c_1(0)=0$, $c_2(0)=1$, we get from the first-order propagator matrix an energy splitting

$$\Delta = \varepsilon_2 - \varepsilon_1 = \Delta_0 + 2(V_0/\omega)^2\Delta_0 , \tag{12}$$

which indicates a tunneling enhancement also in this limit.

It can be easily shown that all the results obtained so far are independent of an arbitrary phase in the driving force. On the other hand, the introduction of additional terms with different frequencies alters some of the previous results. For incommensurable frequencies we cannot define the one-period propagator; however, expressions (3)–(5) are still valid. If we write

$$V(t) = \sum_i V_i \sin(\omega_i t + \varphi_i)$$

then from Eq. (5) we get

$$c_r(t) = i(\Delta_0/2)\exp\left[-2i\sum_i V_i/\omega_i\right]$$
$$\times \int_0^t dt \exp\left[2i\sum_i (V_i/\omega_i)\cos(\omega_i t + \varphi_i)\right] .$$

The integrand factor is a quasiperiodic function of time. Its Fourier zero-frequency component will give rise to a linear term in time after integration; the nonzero components will produce bounded oscillations in c_r. As before, if the amplitude of these oscillations is $\ll 1$, the tunneling behavior is determined solely by the linear term in time. In particular, if the zero-frequency component vanishes tunneling is suppressed. Of course, there is more freedom now to choose the parameter for that to happen.

Concluding, the two-state model is able to provide an accurate description of the tunneling between them in real multilevel systems if some conditions are met; namely, ω and V_0 should be small enough to disregard transitions to other states. In these circumstances only three parameters control the tunneling behavior; two of them are the external field strength V_0 and frequency ω, and the third one is the system-dependent energy splitting Δ_0. The simple expressions obtained [Eqs. (6), (11), and (12)], which relate these parameters, can be used for tunneling control in very different systems and situations. Tunneling can be easily enhanced, reduced, or totally suppressed.

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