PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

THIRD SERIES, VOLUME 45, NUMBER 9 — 1 MAY 1992

RAPID COMMUNICATIONS

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Lasing without inversion in a closed three-level system

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(Received 10 January 1992)

It is shown that a closed V-type, three-level system, which is incoherently pumped on the transition at which lasing occurs and coherently pumped on the other transition, exhibits lasing without population inversion in any atomic state basis. The same system can also be made to exhibit lasing without population inversion in the bare atomic state basis but with population inversion in the dressed-state basis.

PACS number(s): 42.50.Hz, 42.55.−f

There has been considerable interest recently in the study of lasing without the requirement of population inversion. Many schemes for lasing without population inversion have been proposed and the dependence of optical gain on various system parameters has been examined [1–7]. Lasers based on inversionless systems may have interesting statistical properties, such as narrower intrinsic linewidths due to reduced spontaneous emission noises [8]. From a practical point of view, the concept of lasing without population inversion may be quite useful in achieving laser actions in the spectral regions where lasing with population inversion is impractical with conventional pumping schemes. Among the proposed schemes of lasing without population inversion, some are based on the interference effects which result in different emission and absorption profiles in the atomic system; others depend on the utilization of external coherent fields which generate atomic coherences leading to optical gain in the absence of population inversion. In many of these schemes, although there is no population inversion in one state basis (often the bare atomic state basis), there is population inversion in another state basis. Recently Imamoğlu, Field, and Harris proposed a model consisting of a coherently pumped Λ-type, three-level system [9]. They showed that under certain conditions the system exhibits gain for a weak probe laser without the need of population inversion in any atomic state basis. Subsequently, Agarwal presented a dressed-state analysis and concluded that the optical gain in this system can be attributed to the coherence between the dressed states generated by the strong driving field [10].

In this Rapid Communication, I propose a closed V-type, three-level system. The system is pumped incoherently on the lasing transition while the other transition is coherently driven by a strong, external field. The uniqueness of this system is that it can be made to exhibit lasing without population inversion in any state basis, and also lasing without population inversion in one state basis but with population inversion in other state basis. Specifically, I show that if the external coherent field is resonant with the atomic transition, gain exists without population inversion in any state basis; on the other hand, if the external coherent field is off resonance with the atomic transition, gain exists without population inversion in the bare atomic state basis, but with population inversion in the dressed-state basis.

Consider a close V-type, three-level system with the ground state $|1\rangle$, and excited states $|2\rangle$ and $|3\rangle$ as illustrated in Fig. 1. The pumping scheme is similar to that in Ref. [9]. The transition $|1\rangle \leftrightarrow |2\rangle$ of frequency ω_{21} is driven by a strong coherent field of frequency ω_1 with Rabi frequency 2Ω. The transition $|1\rangle \leftrightarrow |3\rangle$ of frequency ω_{31} is incoherently pumped with a rate Λ by thermal radiations or electric discharges. $2\gamma_{31}$ $(2\gamma_{21})$ is the spontaneous decay rate from state $|3\rangle$ $(|2\rangle)$ to state $|1\rangle$. There is no direct coupling between states $|2\rangle$ and $|3\rangle$. A weak,

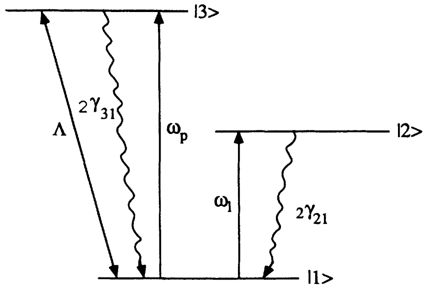

FIG. 1. V-type three-level system for lasing without population inversion.

coherent probe field of frequency ω_p with Rabi frequency $2g$ is applied to the transition $|1\rangle \leftrightarrow |3\rangle$. The optical gain experienced by the weak probe field and atomic population distributions will be examined. In the interaction representation, the semiclassical Hamiltonian under rotating-wave approximation is

$$H = \Omega(e^{i\Delta_1 t}\sigma_{21} + e^{-i\Delta_1 t}\sigma_{12}) + g(e^{i\Delta_2 t}\sigma_{31} + e^{-i\Delta_2 t}\sigma_{13}) . \quad (1)$$

Here $\Delta_1 = \omega_{21} - \omega_1$, and $\Delta_2 = \omega_{31} - \omega_p$. $\sigma_{ij} = |i\rangle\langle j|$ $(i,j = 1-3)$ are the atomic raising or lowering operators. Including the atomic population and phase decay terms, the density-matrix equations of motion can be written as

$$\begin{aligned} \dot\rho_{11} &= -\Lambda\rho_{11} + (\Lambda + 2\gamma_{31})\rho_{33} + 2\gamma_{21}\rho_{22} \\ &\quad + i\Omega(\rho_{21} - \rho_{12}) + ig(\rho_{31} - \rho_{13}) , \\ \dot\rho_{22} &= -2\gamma_{21}\rho_{22} + i\Omega(\rho_{12} - \rho_{21}) , \\ \dot\rho_{33} &= \Lambda\rho_{11} - (\Lambda + 2\gamma_{31})\rho_{33} + ig(\rho_{13} - \rho_{31}) , \\ \dot\rho_{12} &= i\Omega(\rho_{22} - \rho_{11}) - (\Lambda/2 + \gamma_{21} + i\Delta_1)\rho_{12} + ig\rho_{32} , \\ \dot\rho_{13} &= ig(\rho_{33} - \rho_{11}) - (\Lambda + \gamma_{31} + i\Delta_2)\rho_{13} + i\Omega\rho_{23} , \\ \dot\rho_{23} &= i\Omega\rho_{13} - ig\rho_{21} - [\Lambda/2 + \gamma_{31} + \gamma_{21} + i(\Delta_2 - \Delta_1)]\rho_{23} , \end{aligned} \quad (2)$$

along with the equations of their complex conjugates. The closeness of the system requires $\rho_{11} + \rho_{22} + \rho_{33} = 1$. For the convenience of the calculation, Ω and g are chosen to be real. The gain coefficient for the probe field coupled to the transition $|3\rangle \leftrightarrow |1\rangle$ is proportional to $\mathrm{Im}(\rho_{13})$. In my notation, if $\mathrm{Im}(\rho_{13}) > 0$, the system exhibits gain for the probe field. In the next section, $\Delta_1 = 0$ is considered; I obtain the analytical expression of ρ_{13} in the steady state, and derive the condition under which the system exhibits gain without population inversion in any state basis. In the section following, $\Delta_1 \neq 0$ is considered; I present numerical calculations which demonstrate the existence of gain without population inversion in the bare atomic state basis. Using a dressed-state analysis, I show that the probe gain can be attributed to the population inversion in the dressed-state basis.

$\Delta_1 = 0$: *Lasing without population inversion in any state basis.* For the resonant excitations, i.e., $\Delta_1 = \Delta_2 = 0$, I seek the steady-state solution of induced polarization ρ_{13}. Lasing without population inversion in any atomic state basis requires $\mathrm{Im}(\rho_{13}) > 0$, $\rho_{33} < \rho_{11}$, and $\rho_{33} < \rho_{22}$. In the limit of $\Omega \gg \Lambda$, γ_{ij} $(i,j = 1-3)$ and g, from (2) it is easy to obtain

$$\mathrm{Im}(\rho_{13}) = g\frac{\Lambda(\gamma_{21} - \gamma_{31}) - 2\gamma_{31}^2}{\Omega^2(3\Lambda + 4\gamma_{31})} . \quad (3)$$

$\mathrm{Re}(\rho_{13})$ is equal to zero. The weak probe field is amplified if the incoherent pumping rate Λ between states $|1\rangle$ and $|3\rangle$ satisfies

$$\Lambda > \frac{2\gamma_{31}^2}{\gamma_{21} - \gamma_{31}} . \quad (4)$$

The steady-state population inversion between states $|3\rangle$ and $|1\rangle$ is

$$\rho_{33} - \rho_{11} = -\frac{2\gamma_{31}}{3\Lambda + 4\gamma_{31}}\left[1 + \frac{\Lambda + 2\gamma_{21}}{2g}\mathrm{Im}(\rho_{13})\right] < 0 . \quad (5)$$

It is also easy to show that $\rho_{33} - \rho_{22} < 0$. Population inversion cannot happen in any atomic state basis. This is expected for the present pumping scheme since the system is closed. So if the single inequality (4) is satisfied, $\mathrm{Im}(\rho_{13}) > 0$, the system exhibits gain without population inversion in any atomic state basis. It is obvious that the inequality (4) can be satisfied by real atomic systems.

From Eq. (2), ρ_{13} in the steady state can be written as

$$\rho_{13} = \frac{ig(\rho_{33} - \rho_{11}) + i\Omega\rho_{23}}{\Lambda + \gamma_{31} + i\Delta_2} . \quad (6)$$

The induced polarization ρ_{13} at the probe frequency ω_p consists of two terms: The first term is proportional to the population inversion between states $|3\rangle$ and $|1\rangle$, and the second term is due to the induced coherence between states $|2\rangle$ and $|3\rangle$. If there is no population inversion, i.e., $\rho_{33} - \rho_{11} < 0$, the contribution to $\mathrm{Im}(\rho_{13})$ from the first term is always negative, indicating absorption. The probe gain can only be attributed to the induced optical coherence ρ_{23}. To appreciate how the probe gain and absorption is contributed by the induced coherence ρ_{23} and the population inversion $\rho_{33} - \rho_{11}$, respectively, I have calculated numerically the individual contributions to $\mathrm{Im}(\rho_{13})$ from $\rho_{33} - \rho_{11}$ and ρ_{23} as functions of Δ_2. The results are plotted in Fig. 2(b) with practically chosen parameters: $\Lambda = 4\gamma_{31}$, $\gamma_{21} = 6\gamma_{31}$, $\Omega = 20\gamma_{31}$, $g = 0.01\gamma_{31}$, and $\Delta_1 = 0$. For comparison, ρ_{13} is plotted in Fig. 2(a). The symmetric probe gain profile [proportional to $\mathrm{Im}(\rho_{13})$] shown in Fig. 2(a) is centered at $\Delta_2 = 0$, and the two absorption peaks located at about $\pm\Omega$ correspond to the transitions between state $|3\rangle$ and the Autler-Towne doublet (dressed states) generated by the strong coherent field. It is apparent from Fig. 2(b) that the probe gain is contributed by ρ_{23} only. The term from population inversion $\rho_{33} - \rho_{11}$ presents an absorptive Lorentzian line profile centered at $\Delta_2 = 0$. For a weak probe field, the effect of Δ_2 on the atomic population distributions is negligible. So as shown before, the system satisfies $\rho_{11} > \rho_{33}$ and $\rho_{22} > \rho_{33}$. No population inversion exists in any state basis.

$\Delta_1 \neq 0$: *Lasing with population inversion in the dressed-state basis.* For $\Delta_1 \neq 0$, the analytical expression of ρ_{13} is complicated. I have calculated numerically the steady-state response of ρ_{13} as functions of Δ_2, and found

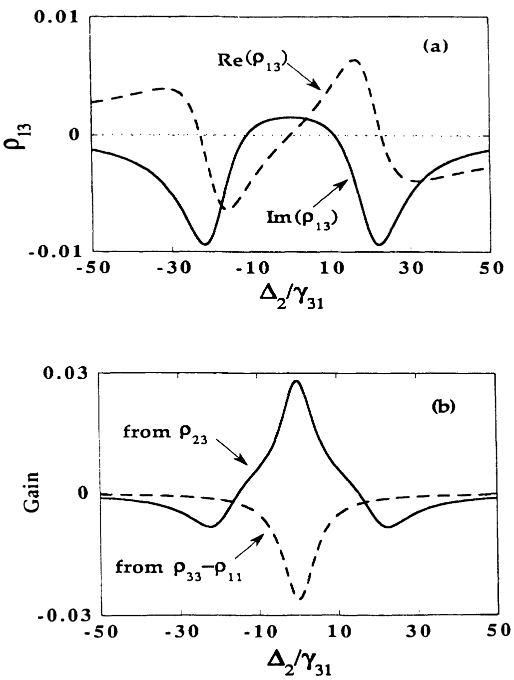

FIG. 2. Spectra for $\Lambda=4\gamma_{31}$, $\gamma_{21}=6\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.01\gamma_{31}$, and $\Delta_1=0$. (a) Calculated response of ρ_{13} as functions of Δ_2. Gain is present if $\mathrm{Im}(\rho_{13})$ (solid line) >0. $\mathrm{Re}(\rho_{13})$ represents atomic dispersion. (b) The gain contributions from ρ_{23} (solid line) and $\rho_{33}-\rho_{11}$ (dashed line). The dotted line in (a) is the zero gain (absorption) base line.

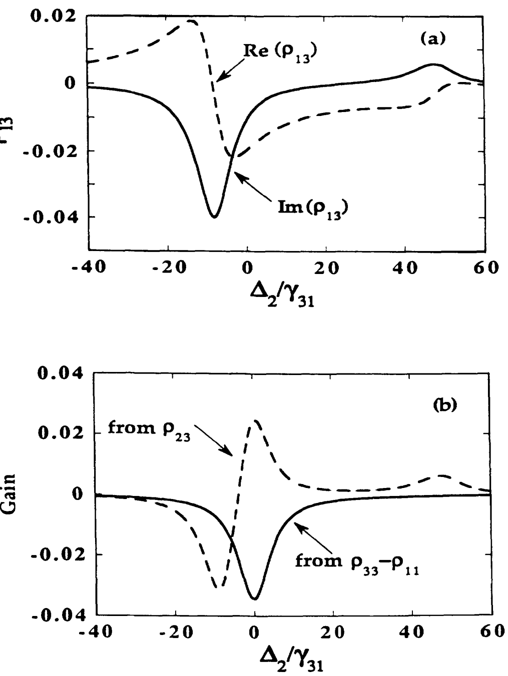

FIG. 3. Spectra for $\Lambda=4\gamma_{31}$, $\gamma_{21}=4\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.01\gamma_{31}$, and $\Delta_1=40\gamma_{31}$. (a) Calculated response of ρ_{13} as functions of Δ_2. As shown by $\mathrm{Im}(\rho_{13})$ (solid line, the absorption peak occurs at $\Delta_2=\Delta_1/2-(\Delta_1^2/4+\Omega^2)^{1/2}$, the gain is peaked at $\Delta_2=\Delta_1/2+(\Delta_1^2/4+\Omega^2)^{1/2}$. $\mathrm{Re}(\rho_{13})$, representing the dispersion, is shown as the dashed line. (b) The gain contributions from the population inversion, $\rho_{33}-\rho_{22}$ (solid line), and the coherence, ρ_{23} (dashed line). There is no population inversion in the bare atomic states basis, so $\rho_{33}-\rho_{11}$ presents an absorptive Lorenzian line profile centered at $\Delta_2=0$. Gain at $\Delta_2=\Delta_1/2+(\Delta_1^2/4+\Omega^2)^{1/2}$ is solely due to ρ_{23} while the combination of gain and absorption from ρ_{23} and $\rho_{33}-\rho_{11}$ results in absorption at $\Delta_2=\Delta_1/2-(\Delta_1^2/4+\Omega^2)^{1/2}$.

that the system exhibits gain for the probe field without population inversion between states $|1\rangle$ and $|3\rangle$. The response of ρ_{13} vs Δ_2 is plotted in Fig. 3(a) with chosen parameters: $\Lambda=4\gamma_{31}$, $\gamma_{21}=4\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.01\gamma_{31}$, and $\Delta_1=40\gamma_{31}$. The probe gain profile presents both a negative (absorption) and a positive (gain) peak; this is quite different from that shown in Fig. 2. The two peaks of $\mathrm{Im}(\rho_{13})$ in Fig. 3(a) correspond to the transitions from state $|3\rangle$ to the Autler-Towne doublet states. For $\Delta_1>0$, the absorption peak occurs at $\Delta_2=\Delta_1/2-(\Delta_1^2/4+\Omega^2)^{1/2}$ while the gain is peaked at $\Delta_2=\Delta_1/2+(\Delta_1^2/4+\Omega^2)^{1/2}$. If the sign of Δ_1 is reversed, the response spectrum of ρ_{13} is reversed; i.e., for $\Delta_1<0$, the gain appears at $\Delta_2=\Delta_1/2-(\Delta_1^2/4+\Omega^2)^{1/2}$ while the absorption occurs at $\Delta_2=\Delta_1/2+(\Delta_1^2/4+\Omega^2)^{1/2}$. Shown in Fig. 3(b) are contributions to $\mathrm{Im}(\rho_{13})$ from the population inversion $\rho_{33}-\rho_{11}$ and the coherence ρ_{23}. Again, the probe gain is solely generated by the atomic coherence ρ_{23}; the contribution from the population inversion term presents an absorptive Lorenzian line profile centered at $\Delta_2=0$. It is evident that $\rho_{33}-\rho_{11}<0$, and is independent of Δ_2 since the probe field is weak. The system exhibits gain without population inversion in the bare atomic state basis.

The probe gain and absorption in the Autler-Towne doublet transitions presented here are quite similar to those in a two-level system. A weak probe laser can be amplified in a detuned, driven two-level system [11]; lasers based on such atomic systems have been constructed [12]. It is known that the gain in a detuned driven two-level system can be attributed to the dressed-state inversion. To understand the essential physics, it is instructive to analyze the present system in the dressed-state basis. Assuming that Ω greatly exceeds all the spontaneous decay rates and the incoherent pumping rate, I diagonalize the part of the Hamiltonian (1) involving the interaction of the external coherent field with states $|1\rangle$ and

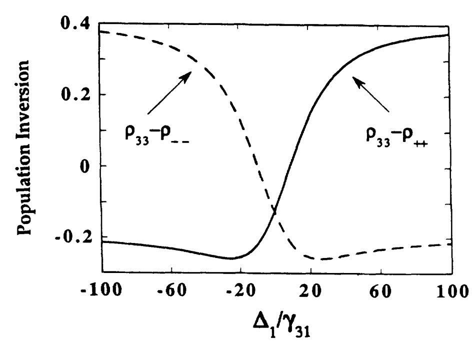

FIG. 4. Population inversions, $\rho_{33}-\rho_{++}$ and $\rho_{33}-\rho_{--}$, vs Δ_1 in the dressed-state basis with $\Lambda=4\gamma_{31}$, $\gamma_{21}=4\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.01\gamma_{31}$.

$|2\rangle$ only. The energy eigenvalues are

$$E_\pm = \frac{\Delta_1}{2} \pm [(\Delta_1/2)^2 + \Omega^2]^{1/2}, \qquad (7)$$

and the corresponding eigenfunctions (dressed states) are

$$|\pm\rangle = a_\pm |1\rangle + b_\pm |2\rangle, \qquad (8)$$

where the coefficients are chosen such that

$$a_\pm = \frac{E_\pm}{(E_\pm^2 + \Omega^2)^{1/2}}, \quad b_\pm = \frac{\Omega}{(E_\pm^2 + \Omega^2)^{1/2}}.$$

For a weak probe field, $g \ll \Lambda$ and γ_{ij} $(i,j = 1-3)$, the system evolution in the dressed-state basis is well approximated by the rate equations for the diagonal density-matrix elements which represent population probabilities [13]. In the basis of dressed states $|+\rangle$ and $|-\rangle$, and state $|3\rangle$, the rate equations are

$$\begin{aligned}
\dot\rho_{33} &= -(\Lambda_{3+} + \Lambda_{3-} + \Gamma_{3+} + \Gamma_{3-})\rho_{33} + \Lambda_{3+}\rho_{++} + \Lambda_{3-}\rho_{--}, \\
\dot\rho_{++} &= -(\Lambda_{3+} + \Gamma_{+-})\rho_{++} + \Gamma_{-+}\rho_{--} + (\Lambda_{3+} + \Gamma_{3+})\rho_{33}, \\
\dot\rho_{--} &= -(\Lambda_{3-} + \Gamma_{-+})\rho_{--} + \Gamma_{+-}\rho_{++} + (\Lambda_{3-} + \Gamma_{3-})\rho_{33}.
\end{aligned} \qquad (9)$$

Here $\Lambda_{3+} = \Lambda(a_+)^2$ $[\Lambda_{3-} = \Lambda(a_-)^2]$ is the incoherent pumping rate between the dressed state $|+\rangle$ ($|-\rangle$) and the bare atomic state $|3\rangle$; Γ_{ij} $(i,j = \pm, 3)$ are the spontaneous decay rates from state $|i\rangle$ to state $|j\rangle$, and can be calculated using Fermi's "golden rule." Utilizing the normalization condition, $\rho_{++} + \rho_{--} + \rho_{33} = 1$, it is easy to obtain steady-state solutions of ρ_{++}, ρ_{--}, and ρ_{33}. The gain coefficients for the weak probe field on the transition from state $|3\rangle$ to dressed states $|+\rangle$ and $|-\rangle$ are proportional to $(a_+)^2(\rho_{33} - \rho_{++})$ and $(a_-)^2(\rho_{33} - \rho_{--})$, respectively. The population inversion between state $|3\rangle$ and dressed state $|+\rangle$ ($|-\rangle$), $\rho_{33} - \rho_{++}$ ($\rho_{33} - \rho_{--}$), is plotted in Fig. 4 with the same set of parameters as in Fig. 3. The system exhibits gain for the probe field whenever the condition of population inversion in the dressed state basis is satisfied; i.e., $\rho_{33} - \rho_{++} > 0$ or $\rho_{33} - \rho_{--} > 0$. It should be noted that for $\Delta_1 = 0$, the probe field experiences absorption in the dressed-state analysis. The probe gain for $\Delta_1 = 0$ cannot be attributed to the population inversion in the dressed-state basis.

In conclusion, I have shown that a closed V-type, three-level system exhibits optical gain without population inversion under certain conditions. Lasers can be built with such a system as an active medium. The system proposed here can serve as a model for lasing without population inversion in any atomic state basis, as well as lasing without population inversion in the bare atomic state basis but with population inversion in the dressed-state basis. A closed V-type, three-level structure exists in many real atomic systems and should be accessible to experimental investigation. This model system may also be useful in the realization of lasing actions in the spectral regions where population inversion is difficult to achieve by conventional pumping schemes.

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