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# Maximum microwave conversion efficiency from a modulated intense relativistic electron beam

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We calculate the power transfer from an intense modulated electron beam to an external load. The simple model fully incorporates various effects such as nonlinear beam loading, finite transit time, ac and dc space charges, and the beam's initial velocity modulation. We show that a fully modulated beam may deliver up to a maximum of 57% of its power to a load without forming a virtual cathode.

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A fundamental question that remains unanswered in ultrahigh-power microwave generation [1] concerns the maximum rf power that a load can extract from a modulated intense electron beam. Although an almost complete understanding has been achieved in the case of a weak beam, as in a conventional klystron [2], the extension to an intense beam is an open question because of the substantial potential energy that always accompanies an intense beam [3]. The interaction between an rf circuit and the beam involves three types of energies: the energies of the circuit, the beam's kinetic energy, *and* its potential energy, all having the same order of magnitude. In this paper, we adopt a simple model (Fig. 1) that accurately accounts for the exchanges of these energies. We also determine the optimal conversion efficiency of a modulated intense electron beam, using this "standard" model, but pushing it to the new regime in which virtual cathodes are on the verge of being formed at the gap. It is in this high current regime where the relativistic klystron amplifiers at the Naval Research Laboratory operate [3]. Such devices are currently considered as one of the most promising in the production of coherent microwave power beyond the gigawatt range.

The beam's intense space charge may influence the beam-gap interactions in the following ways: (i) It may act as a dielectric, thereby loading the gap [3-5]. (ii) Too much space charge would lead to virtual cathode oscillation at the gap, destroying the stability of the device. (iii) Since the self-fields of the space charge are significant, an individual electron experiences not only the rf electric field of the gap, but also the space charge forces due to other beam electrons. Thus, the familiar "transit time factor" in beam-gap interaction needs to be substantially modified. (iv) Because of the above, the available ac current that drives the load is rather different from that entering the gap, and the optimal conversion efficiency between the beam and the load cannot be assessed, if one is to rely only on the experience with the conventional klystrons. To accurately account for these space-charge effects, and to unambiguously identify the current $I_A(t)$ that drives the load (after taking into account the various beam loading effects), we use a parallel plate model for the gap but calculate the gap voltage $V_g(t)$ and $I_A(t)$ that are consistent with the nonlinear, time-varying electron motion [6,7].

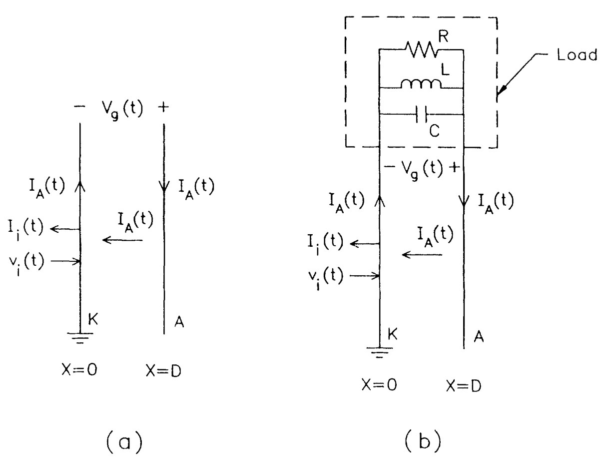


FIG. 1. (a) The gap and (b) its connection to the load.

The model consists of an electron beam impinging upon a gap formed by two parallel plates, $K$ and $A$, located at $x=0$ and $x=D$, respectively [Fig. 1(a)]. The electrons enter plate $K$, carrying a current $I_i(t)$ and a velocity $v_i(t)$, given by

$$I_i(t)=I_0[1+(I_1/I_0)\sin\omega t]\,, \tag{1}$$

$$v_i(t)=v_0[1+(v_1/v_0)\sin(\omega t+\theta)]\,, \tag{2}$$

where $I_0$ and $v_0$ are, respectively, the dc beam current and velocity, $I_1$ and $v_1$ are their respective ac components with frequency $\omega$, and $\theta$ is the relative phase between the current modulation and velocity modulation when the beam enters the gap. All of these quantities are assumed to be known, as they characterize the properties of the input beam. [In Fig. 1, $I_i(t)$ is positive in the direction shown, since the electron carries a negative charge]. In contrast with conventional klystron theory, we do not assume $I_1 \ll I_0$ nor $v_1 \ll v_0$.

We next assume that the load is represented by an $RLC$ circuit, connected across the gap [Fig. 1(b)]. In response to a driving current $I_A(t)$, whatever it may be, the terminal voltage $V_g(t)$ across the simple $RLC$ circuit [Fig. 1(b)] is governed by

$$\left[\frac{d^2}{dt^2}+\frac{\omega_0}{Q}\frac{d}{dt}+\omega_0^2\right]V_g(t)=-\frac{\omega_0 R}{Q}\frac{d}{dt}[I_A(t)-I_0]\,, \tag{3}$$

where $\omega_0$ is the natural frequency and $Q$ is the quality factor of that resonant circuit. Since $I_0$ is constant, it is attached to the right-hand side of Eq. (3) without loss of generality, so as to signify that it is the ac part of $I_A(t)$ which drives the load in Fig. 1(b).

The crucial step lies with the proper definition, and then the calculation, of the current $I_A(t)$. In general, it is not equal to the input current $I_i(t)$. An *exact* formulation is available for the one-dimensional model shown in Fig. 1. It is given by [2,4]

$$I_A(t)=I(x,t)-A\varepsilon_0\frac{\partial E(x,t)}{\partial t}\,, \tag{4}$$

which is the sum of the convection current $I(x,t)=A\rho v$ and the displacement current. Here, $A$ is the surface area of the plate, $-\rho$ and $v$ are, respectively, the volume charge density and the flow velocity within the gap.

Since Eq. (4) is independent of the position $x$, we may evaluate $I_A(t)$ right in front of plate $K$ at $x=0^+$, in which case $I(0^+,t)$ is simply the input current $I_i(t)$ defined in Eq. (1). The remaining quantity $-A\varepsilon_0\partial E(0^+,t)/\partial t$ in Eq. (4) is then the current that is shunted by the loaded gap. It may be further decomposed into two components: the shunted current $C_0\partial V_g(t)/\partial t$ due to the vacuum gap capacitor $C_0\equiv A\varepsilon_0/D$ and a remainder $I_{\rm NL}(t)$ which accounts for all of the space-charge and transit-time effects. In the limit of a weak beam and a low gap voltage, we recover [4] from $I_{\rm NL}(t)$ the small signal beam loading admittance that is formulated in the classical klystron theory [2].

The surface electric field $E(0^+,t)$ is obtained after we solve for $E(x,t)$, $\rho(x,t)$, and $v(x,t)$ from the force law, the equation of continuity, and the Poisson equation:

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)\gamma v=-\frac{eE}{m_0}\,, \tag{5}$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0\,, \tag{6}$$

$$\frac{\partial E}{\partial x}=-\rho/\varepsilon_0\,. \tag{7}$$

Equations (1) and (2) are the respective boundary conditions, at $x=0$, for the current $I\equiv A\rho v$ and for the velocity $v$. The electric field $E(x,t)$ satisfies the constraint

$$-\int_0^D dx\,E(x,t)=V_g(t)\,. \tag{8}$$

We have solved the system of equations (3)–(7), subject to the conditions just stipulated. The density and current distribution of electrons within the gap were solved by both a particle code and a Eulerian code that have been validated [4]. The following parameters need to be specified: $R$, $L$, $C$, $I_0$, $v_0$, $I_1/I_0$, $v_1/v_0$, $\omega$, and $\theta$. Our problem is to find suitable combinations of these parameters, so that the average rf power delivered to the load is maximized. There is one requirement, namely, that no virtual cathode is formed at the gap. We consider that requirement satisfied if no reflected particle is detected at $x=0^+$ in the particle code. To reduce the above parameter space, we fix the beam's dc kinetic energy at 511 keV and $\omega D/c=0.681$; all other parameters are allowed to vary.

Tune sensitivity is shown in Fig. 2, in which we arbitrarily set $I_A=I_i(t)$ [i.e., pretending the idealized situation where all the input current $I_i(t)$ can drive the load], and set $v_i(t)=v_0=0.866c$, $Q=300$, $\omega D/c=0.681$, $\bar{R}\equiv RI_s/(m_0c^2/e)=5$, $I_0/I_s=1.33$, and $I_1/I_0=0.56$. Here, $I_s=(A\varepsilon_0/D)(m_0c^2/e)/(D/c)$ is the current scale. Figure 2(a) shows the normalized power $\bar{P}_L\equiv -I_AV_g/(I_sm_0c^2/e)$ that can be delivered to the load when there is a perfect tune between the drive frequency in the current modulation and the resonant frequency. Figure 2(b) shows $\bar{P}_L$ when 1% of stray capacitance is added. In Fig. 2, $\bar{t}=\omega t$. Note the substantial reduction in the power $\bar{P}_L$ in Fig. 2(b). A similar degree of sensitivity to tuning has also been observed in experiments [8].

The effect of beam loading is shown in Fig. 3. We use the same parameters as in Fig. 2, except that we lower $Q$ to 5 (to reduce the rise time in the numerical computation) and reduce $R$ by a factor of 2.5. Figure 3(a) shows $\bar{P}_L$ in the idealized situation: All of the input current $I_i$ is delivered to the load (i.e., $I_A=I_i$, no loading due to the beam or to the vacuum gap capacitance). The circuit is perfectly tuned to the driving frequency in the current

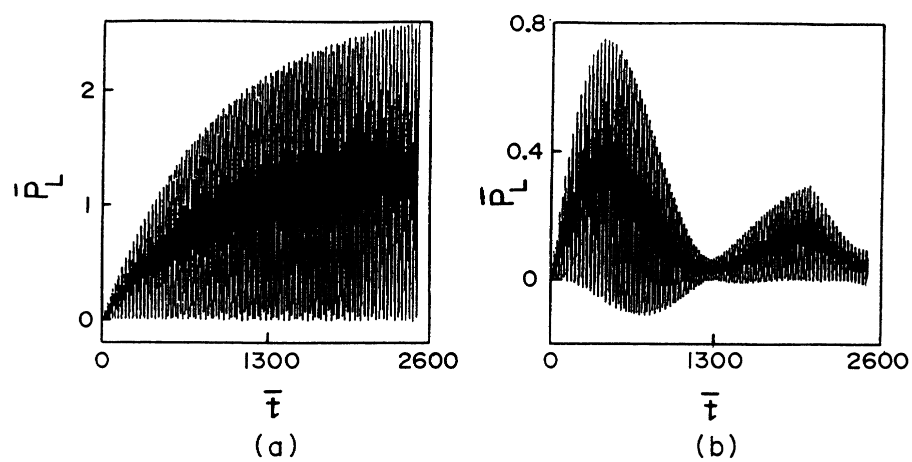


FIG. 2. The power $\bar{P}_L$ delivered (a) to a perfectly tuned load and (b) to a load including a 1% stray capacitance.

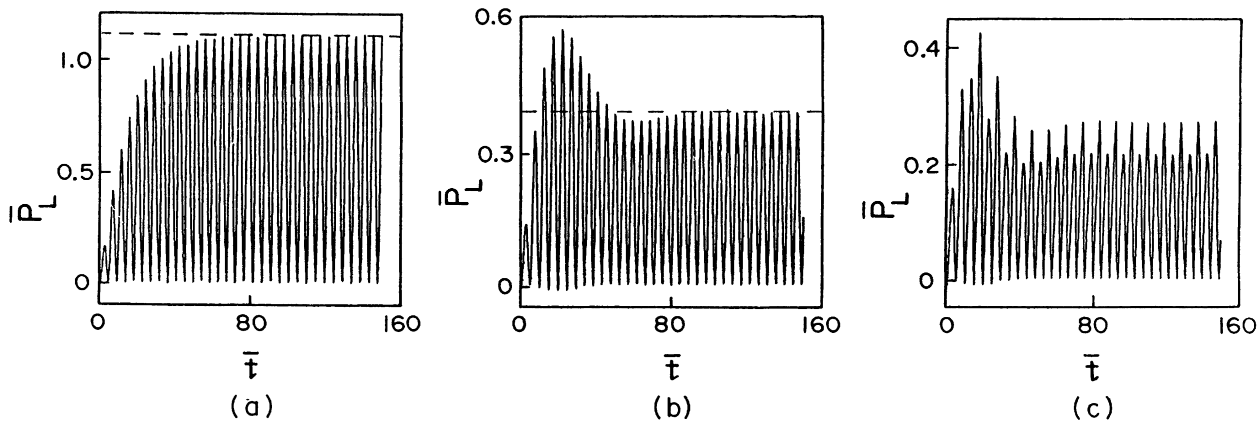


FIG. 3. The power $\bar{P}_L$ delivered to the load when (a) there is no gap loading nor beam loading, (b) there is only vacuum gap loading, and (c) both gap and beam loading are present. The dashed curves show the theoretical asymptotic values.

modulation. In Fig. 3(b), we keep the same frequency, but include in $I_A$ not only $I_i$ but also the quantity $C_0 dV_g/dt$, which is the current shunted by the vacuum gap capacitance $C_0$. We see that the asymptotic peak value of $\bar{P}_L$ is reduced from 1.1 in Fig. 3(a) to 0.39 in Fig. 3(b). This reduction is due to the loading of the *RLC* circuit by the vacuum gap through which the beam passes. In Fig. 3(b), beam loading is absent. In Fig. 3(c), we use the exact relation (4) for $I_A$ and thus include all loading effects: the loading by the vacuum gap and beam loading. We see that the asymptotic peak power is further reduced by beam loading.

We next performed an extensive search of the parameter space to maximize the rf power that can be delivered to the load without the formation of a virtual cathode. To reduce the sensitivity due to tuning, we considered the simple case, where the load in Fig. 1(b) consists only of a single resistor. After an extensive search, we found that to deliver maximum power to the load, the beam needs to be fully modulated and therefore the dc current is about half the ac limiting value [4]. The load resistance cannot be too high (to avoid virtual cathodes) or too low (to allow

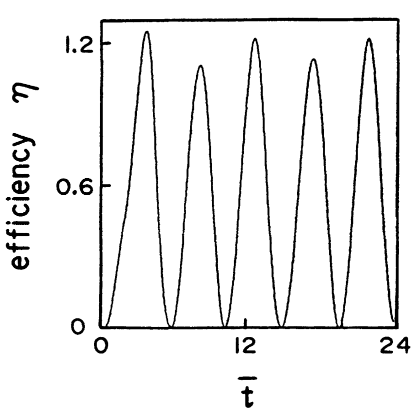


FIG. 4. Optimal efficiency in the power transfer to a resistive load, by an input beam with 10% velocity modulation. The dc beam energy is 511 keV.

appreciable power dissipation at the load). The efficiency can be improved somewhat (by about 20%) if the phase of velocity modulation lags that of current modulation by about 30°. When these conditions are satisfied, close to 60% of the beam power can be delivered to the load. Figure 4 shows the conversion efficiency $\eta$, defined to be $(V_g^2/R)/\langle P_b\rangle$, where $\langle P_b\rangle$ is the average beam power carried by the incoming beam. The parameters used for Fig. 4 are as follows: the dc beam energy is 511 kV, the dc beam current is $2.4I_s$, $I_1/I_0=1$, $v_1/v_0=0.1$, $R=0.8(m_0c^2/e)/I_s$, $\omega D/c=0.681$, and $\theta=-\pi/6$. The average conversion efficiency $\eta$ in this figure is 57%. If the beam is only partially modulated, e.g., $I_1/I_0=0.6$, the maximum efficiency is found to be only 35%, a value consistent with experimental observations [3].

In conclusion, we present here an analysis of beam-circuit interaction where the beam's instantaneous current may reach the limiting value. The study of this new regime strictly adheres to the formulation laid down in the classical klystron theory. Although the idealized gap model reveals many features observed in the Naval Research Laboratory relativistic klystron amplifier experiments [3], there is one aspect that is particularly puzzling to us: The increase in the rise time in the current modulation that was observed [8] in the experiments and in the earlier particle simulations of the real geometry. Our numerical results obtained thus far [using the model of Fig. 1(b)] failed to show a similar lengthening in the rise time. It remains to be determined whether this increase in the rise time is a result of the specific geometry used, namely, that of an annular beam interacting with coaxial cavities. We can think of two major differences between the present model and those in the two-dimensional particle CONDOR simulations [9] and experiments: the geometrical effects just mentioned and the inductive effects that were ignored here.

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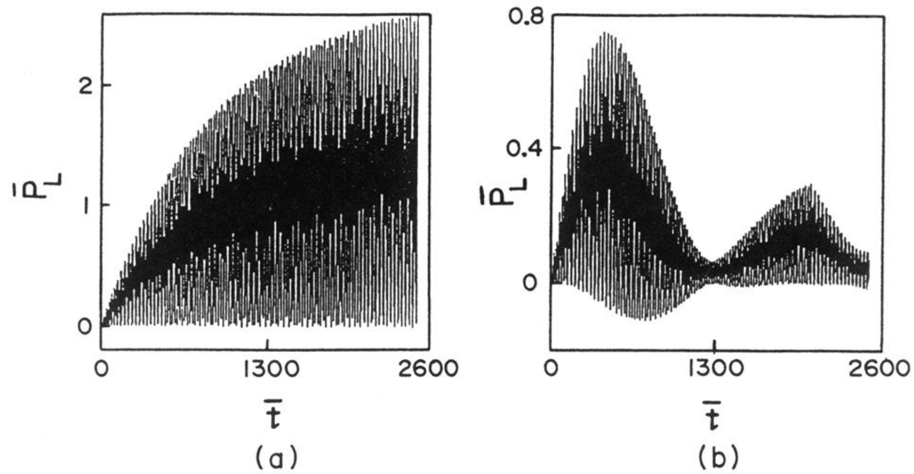


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