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Dynamical localization of atomic-beam deflection by a modulated standing light wave

R. Graham and M. Schlautmann
Fachbereich Physik, Universität Gesamthochschule Essen, D-4300 Essen 1, Germany

P. Zoller
Department of Physics and Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, Colorado 80309

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The deflection of an atomic beam passing a standing-wave laser field in front of an oscillating mirror occurs by chaotic diffusive momentum transfer in a classical description and, as we show, is limited by dynamical localization quantum mechanically. An experiment to observe this quantum effect in an atomic beam is proposed.

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The deflection of atomic beams by interaction with a standing light wave is an area of current theoretical [1,2] and experimental [3,4] research, because it provides the quantum-mechanical dual of diffraction of light waves by a matter grating—the diffraction of matter waves from a light grating. In the present paper we wish to investigate the system under special conditions where a new effect is observable, namely, dynamical localization. This effect has been discussed in model systems in "quantum chaos" such as the kicked rotator [5] and atomic [6] and molecular [7] models. Furthermore, there is experimental evidence [8] that it occurs in hydrogen atoms in Rydberg states driven by a strong microwave field. According to theory it should appear, under appropriate conditions, in periodically driven quantum systems which are chaotic in their classical limit. Proposals for observing the effect include periodically driven Josephson junctions [9] and optical fibers with periodically varying index of refraction [10]. Here we wish to point out the deflection of a beam of atoms passing a standing-wave laser field in front of an oscillating mirror as an additional type of experiment in which dynamical localization is predicted to appear. We shall demonstrate the effect theoretically and specify appropriate experimental conditions for its observation.

Let us consider a beam of two-level atoms (states $|g\rangle$, and $|e\rangle$ with energy difference $\hbar\omega_0$, dipole momentum d), which are initially in their ground state $|g\rangle$ and moving in the z direction with kinetic energy E_0 and are then passing through a single classical standing-wave light field $\mathcal{E}(x,t)=\mathbf{e}_y[\mathcal{E}_0\cos(k_Lx)e^{-i\omega_Lt}+\text{c.c.}]$. We assume that the x coordinate of the mirror, reflecting the incoming traveling light wave—and thereby determining the position of the nodes of the standing light wave—oscillates around its average with $\Delta L\sin\omega t$, so that the nodes are harmonically oscillating in the same way, i.e., neglecting retardation effects, $\mathcal{E}(x,t)$ passes into $\mathcal{E}(x-\Delta L\sin\omega t,t)$. This can be achieved, e.g., by appropriately driving a piezoelectrical crystal. The dipole and rotating-wave approximations then yield the Hamiltonian

$$H=\frac{p^2}{2M}+\hbar\omega_0|e\rangle\langle e|-\{d\mathcal{E}_0\cos[k_L(x-\Delta L\sin\omega t)]\times e^{i\omega_Lt}\sigma_+ +\text{H.c.}\}\,, \tag{1}$$

where p is the center-of-mass momentum of the atoms (with mass M) and $\sigma_\pm$ are Pauli spin operators. In a reference frame moving with $v=(2E_0/M)^{1/2}$ in the z direction there remains only the transverse atomic center-of-mass motion in the x direction and we can represent the atomic state as $\psi_g(x,t)|g\rangle+\psi_e(x,t)e^{-i\omega_Lt}|e\rangle$ with equations of motion

$$i\hbar\frac{\partial\psi_g}{\partial t}=-\frac{\hbar^2}{2M}\frac{\partial^2\psi_g}{\partial x^2}-\frac{\hbar\Omega}{2}\cos[k_L(x-\Delta L\sin\omega t)]\psi_e\,,$$

$$i\hbar\frac{\partial\psi_e}{\partial t}=-\frac{\hbar^2}{2M}\frac{\partial^2\psi_e}{\partial x^2}+\hbar\delta_L\psi_e-\frac{\hbar\Omega}{2}\cos[k_L(x-\Delta L\sin\omega t)]\psi_g\,.$$

Here we neglect spontaneous emission from the upper atomic level, which is justified for sufficiently high detuning $\delta_L=\omega_0-\omega_L$; $\Omega/2=d\mathcal{E}_0/\hbar$ is the Rabi frequency. Adiabatic elimination of the excited-state amplitude with the assumption that the detuning δ_L is large compared to the Rabi frequency Ω and the excited-state kinetic energy term leads to

$$i\hbar\frac{\partial\psi_g}{\partial t}=-\frac{\hbar^2}{2M}\frac{\partial^2\psi_g}{\partial x^2}-\frac{\hbar\Omega_{\text{eff}}}{4}\cos^2[k_L(x-\Delta L\sin\omega t)]\psi_g\,,$$

where $\Omega_{\text{eff}}=\Omega^2/\delta_L$ is the effective Rabi frequency. (Note that here we are not concerned with the different problem of chaos on the Bloch sphere of internal states [11].) So the dynamic of the atoms in the ground state—with an energy shift of $\hbar\Omega_{\text{eff}}/8$—is governed by the Hamiltonian

$$H=\frac{p_x^2}{2M}-\frac{\hbar\Omega_{\text{eff}}}{8}\cos[2k_L(x-\Delta L\sin\omega t)]\,. \tag{2}$$

Since the probability to find an atom in the excited state is negligible in our case, the properties of the atoms are completely determined by the ground-state amplitude. After a rescaling $t'=\omega t$, $\phi=2k_Lx$, $p=(2k_L/M\omega)p_x$, and $H'=(4k_L^2/M\omega^2)H$, we get (rewriting $t'\to t$, $H'\to H$) the dimensionless Hamiltonian

$$H=\frac{p^2}{2}-k\cos(\phi-\lambda\sin t)\,, \tag{3}$$

with the (classical) parameters $k=\varepsilon_r\Omega_{\rm eff}/\omega^2$ and $\lambda=2k_L\Delta L$, where $\varepsilon_r=\hbar k_L^2/2M$ is the recoil shift. The quantized system in addition contains the rescaled Planck constant $\bar{k}=8\varepsilon_r/\omega$ via the commutator $[p,\phi]=-i\bar{k}$. Let us note that the classical and quantal properties of (3) recently have been discussed in connection with periodically driven Josephson junctions [9]. The following analysis closely follows the one given there. Our numerical examples are also taken from [9].

The classical Hamiltonian (3) is that of a periodically driven pendulum and was analyzed, e.g., in [12]. We briefly review the results of [9] relevant to our present purpose. The basic phenomenon described is the crossing, twice during each period $T=2\pi/\omega$, of the fundamental resonance at which $p=\dot{\phi}=\lambda\cos t$. It is useful to distinguish the cases of slow ($\lambda/k<1$) and fast crossing ($\lambda/k\gg 1$, i.e., $\Delta L\gg\varepsilon_r\Omega_{\rm eff}/2k_L\omega^2$) [12,13]. We will here only consider the case of fast crossing. It is then possible to neglect the rate of change ($\simeq k$) of the pendulum frequency ($\simeq\sqrt{k}$) compared to the rate of displacement ($\simeq\lambda$) of the fundamental resonance $\dot{\phi}=\lambda\cos t$ [13]. Outside each crossing the pendulum is effectively free, but each crossing acts like a sudden kick which randomizes the total phase $\varphi\equiv(\phi-\lambda\sin t)\mathrm{mod}(2\pi)$ and changes p by $\Delta p\simeq-\sqrt{2\pi}(k/\sqrt{\lambda})\sin(\varphi\pm\pi/4)$ where the sign depends on the direction of the passing of the resonance. The system can therefore be described, in reasonable approximation, by a standard map [12] of the form $\bar{p}=p-2\sqrt{\pi}(k/\sqrt{\lambda})\sin\varphi$, $\bar{\varphi}=\varphi+2\pi\bar{p}$ per period $T=2\pi$. From this description the classical chaos border is derived as $k>\sqrt{\pi\lambda}/40$. A crossing of the resonance occurs only for $|p|<\lambda$. Chaos is therefore essentially restricted to this domain. For our discussion of dynamical localization below we shall require a large chaotic domain $\lambda\gg 1$, i.e., $\Delta L\gg 1/2k_L$. In the chaotic domain p diffuses with the diffusion constant $D=2\langle\Delta p^2\rangle/(2\pi/\omega)$ where $\langle\Delta p^2\rangle$ is the mean square of the change of p per half period. The above description by a standard map yields roughly, for $k/\sqrt{\lambda}\gg 1$, $D\simeq k^2/\lambda$. A more quantitative estimate $D=(k^2/\lambda)F(4\pi^2k/\sqrt{\pi\lambda})$ with known function F of order 1 is available [14] if required. Thus, classically the normalized momentum p for sufficiently many crossings of the resonance ($t\gtrsim\lambda^3/k^2$) spreads diffusively over the entire chaotic domain $|p|\le\lambda$. Momentum fluctuations with root mean square $(\langle p_x^2\rangle)^{1/2}\simeq M\omega\Delta L/\sqrt{3}$ are therefore predicted classically.

In the quantized system (3) dynamical localization denotes the quantum-mechanical destructive interference of the transition amplitudes with large changes of the quantum number of p [15]. In particular the Floquet states ψ_ν, i.e., the quasienergy states of (3), in the $|n\rangle$ representation, with $p|n\rangle=\bar{k}n|n\rangle$, fall off exponentially as $|\psi_\nu|\sim\exp(-|n-n_\nu|/l)$ where l is the wave-function localization length. It is given in terms of the diffusion constant D_p of p over one period $D_p=2\pi D$ by $l=D_p/2\bar{k}^2$. Due to the 2π periodicity of (3) in ϕ the fractional part of the eigenvalues n is a conserved quasimomentum which we take equal to zero. Alternatively, one may leave the quasimomentum arbitrary but fixed in the interval [0,1) and take the average in the final results. An initial state, localized near $n=0$ and given by a linear superposition of about l Floquet states, first spreads by classical diffusion and then develops into an exponentially localized distribution $|\psi|\sim\exp(-|n|/l_D)$ with a localization length $l_D\simeq 2l$ [16]. Thus fluctuations of the transverse momentum are quantum mechanically reduced to $(\langle p^2\rangle)^{1/2}\simeq\bar{k}l_D/\sqrt{2}$ [9] or $(\langle p_x^2\rangle)^{1/2}\simeq(\sqrt{2}\pi\hbar\,\Omega_{\rm eff}^2/64\omega^2)/\Delta L$. For fixed external frequency, they decrease inversely proportional to ΔL, contrary to the classical case where they increase proportionally to ΔL. This finding provides us with a clear signature of the effect which should be observable, if the classical restriction of the fluctuations by the width of the chaotic domain ($\simeq\lambda/\sqrt{3}$) is larger than the quantum restriction due to dynamical localization ($\simeq\bar{k}l_D/\sqrt{2}$), i.e., $\Delta L>(\sqrt{6}\pi\hbar/64M\omega^3)^{1/2}\Omega_{\rm eff}$.

In order to demonstrate the effect we display the results of some numerical simulations [9]. For simplicity time was discretized and the Hamiltonian equations following from Eq. (3) were replaced by a discrete, periodically time-dependent, two-dimensional map, 300 iterations of which correspond to a single period $2\pi/\omega$. In the simulations the quantum version of this map was used. As a consequence of this discretization the spectrum of resonances of the continuous system is repeated on the frequency axis with a period 300ω. As the simulations were restricted to values $\lambda\le 130$ the chaotic domain $|p|<\lambda$ does not overlap with its repeated copies and the discretization, therefore, cannot significantly affect the results while leading to an enormous saving of computer time. In Fig. 1 we present $\langle\Delta n^2\rangle$ versus time as measured in periods of the mirror oscillations for $\lambda=85.0$, $k=15.0$, and $\bar{k}=1.58$ [9]. The initial sharp rise of $\langle\Delta n^2\rangle$ from the initial state at $n=0$ by classical diffusion is followed by a lo-

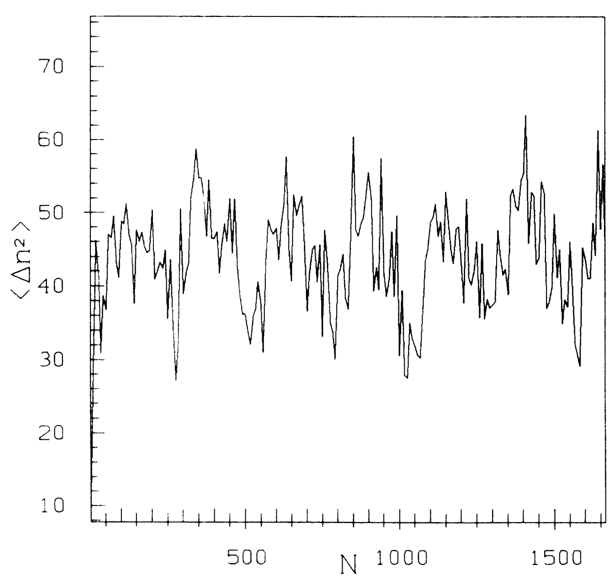

FIG. 1. Mean square of the number of occupied levels of the cosine potential vs the number N of periods of the mirror oscillations for $\lambda=85.0$, $k=15.0$, and $\bar{k}=1.58$.

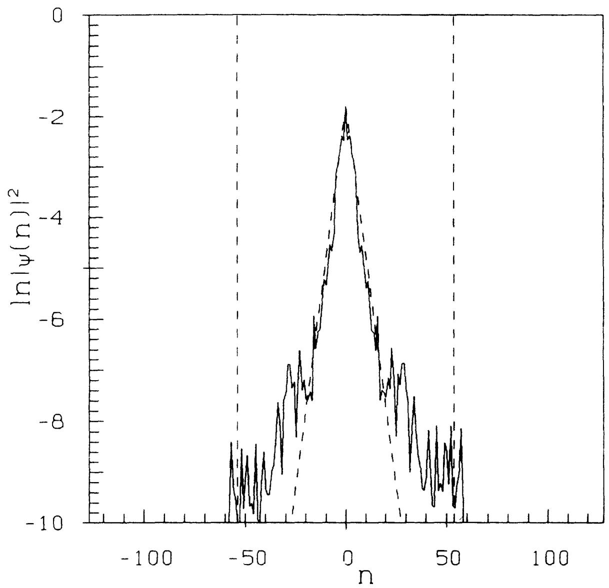

FIG. 2. Logarithm of the time-averaged occupation probability corresponding to Fig. 1. Dashed lines give the border $|n| = \lambda/k$ of the classical chaotic domain and the exponential falloff with the localization length l_D.

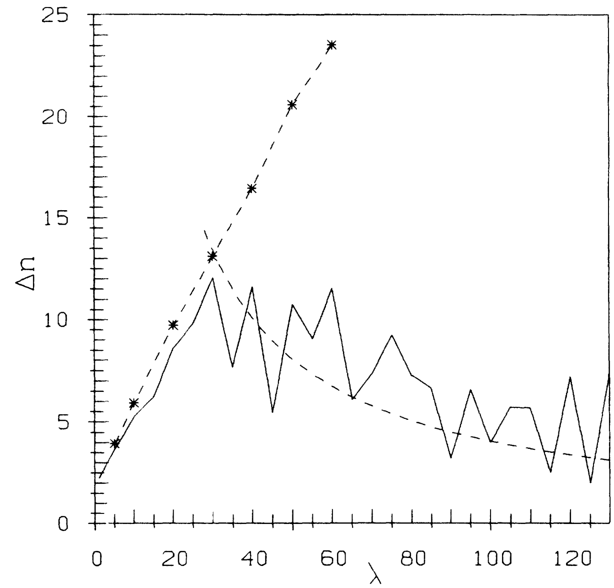

FIG. 3. Root mean square of the number of occupied levels vs the normalized amplitude λ of the mirror oscillations for the same values of the parameters k,$k\!\!\!/$ as in Fig. 1. Classical results, indicated by $*$, are joined by a dashed line. Another dashed line gives the analytical result for the quantum regime.

calized regime where $\langle \Delta n^2 \rangle$ changes due to random beatings of a finite number of Floquet states. Taking an average over the quasimomentum which was here taken as zero arbitrarily, will smoothen out these beatings. In Fig. 2 the time-averaged localized proability distribution over the eigenstates of p which has established itself towards the end of Fig. 1 is shown in a semilogarithmic plot [9]. The dashed lines give the classical border $|n| = \lambda/k\!\!\!/$ and the exponential falloff with the theoretically estimated localization length $l_D = 2\pi k^2/\lambda k\!\!\!/^2$.

In Fig. 3 a numerical example [9] of our experimentally accessible prediction is presented—the root-mean-square transverse momentum fluctuations, expressed in fluctuations of the quantum number n, versus the amplitude of the mirror oscillations, expressed in scaled form by λ. The values for k and $k\!\!\!/$ are the same as in Figs. 1 and 2. Also shown are results of a classical calculation for the same parameter values which are joined by a dashed line, for convenience, and a dashed curve giving the estimate $\Delta n = l_D/\sqrt{2} = \sqrt{2}\pi k^2/\lambda k\!\!\!/^2$ provided by localization theory. The transition from classical to quantum behavior takes place near $\lambda_q = (\sqrt{6}\pi)^{1/2} k/(k\!\!\!/)^{1/2} \simeq 33$.

The experimental conditions under which the effect shown in Fig. 3 should be observable can now be summarized. For example, for ytterbium atoms, optically pumped to a two-state system (atomic frequency $\omega_0/2\pi \simeq 5.40\times 10^{14}$ Hz) and passing orthogonally a modulated standing light wave with detuning $\delta_L/2\pi \simeq 4.0$ GHz (wave number $k_L \simeq 1.13\times 10^7\ \mathrm{m}^{-1}$), driving frequency of the mirror $\omega/2\pi \simeq 125$ kHz, and Rabi frequency $\Omega/2\pi \simeq 140$ MHz, we have $k \simeq 1.2$, $k\!\!\!/ \simeq 0.24$, and $\lambda \simeq 2.26 \times 10^7\ \mathrm{m}^{-1}\Delta L$. In order to observe the classical to quantum crossover the amplitude of the mirror oscillations should then be varied in the range 0.1–0.5 μm. The crossover occurs at about 0.3 μm. The localization needs about $l_D/2$ periods of the mirror oscillations to establish itself (cf. Fig. 1). The interaction time of the atoms with the standing-wave light field, t_{int}, therefore has to be large compared to $l_D\pi/\omega$. In this example, $l_D \simeq 23$ at the classical-quantum crossover, which amounts to $t_{\mathrm{int}} \gg 90\ \mu$s.

The predicted effect rests entirely on coherence, therefore dissipation and noise have to be kept sufficiently low [17]. This means that spontaneous decays of the upper atomic level must be suppressed, which is achieved by detuning sufficiently far from resonance. With the above given parameters and a spontaneous decay rate of $\gamma/2\pi \simeq 183$ kHz, the number of spontaneous decays of an atom during an interaction time $t_{\mathrm{int}} \simeq 300\ \mu$s (which is attained, for example, by ytterbium atoms of kinetic energy $E_0 \simeq 1.4\times 10^{-23}$ J passing an interaction region of about 3 mm) is $N = (\Omega/2)^2 \gamma t_{\mathrm{int}}/[(2\delta_L)^2 + \gamma^2] \simeq 0.03$ and therefore negligible [4]. The measurement of the transverse atomic momentum can be realized as in [3,4] without back action on the system.

In summary, we have shown that dynamical localization appears as a quantum effect in the classically chaotic deflection of two-state atoms in the standing-wave light field in front of an oscillating mirror, and we have described conditions under which this effect might be observed.

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