Regeneration and stopping of $(\alpha\mu)^+$ in a degenerate plasma

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The regeneration and stopping of $(\alpha\mu)^+$ following dt fusion is computed within the environment of a dense degenerate plasma. It is found that the final sticking fraction can be reduced to 10^{-5} of the final sticking fraction in molecular hydrogen.

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In the past decade deuterium-tritium (d,t) fusion catalyzed by the presence of a muon (μ) has been intensively investigated. Experiments performed in dt molecular hydrogen mixtures at liquid hydrogen densities (LHD, $\equiv 4.25\times10^{22}\,\mathrm{cm}^{-3}$) have reported that one muon is able to catalyze as many as 150 dt fusions [1, 2]. One of the two chief limiting factors is the "sticking" probability, viz., the probability that the muon will remain stuck to the α particle in the reaction:

$$dt\mu \rightarrow \begin{cases} \alpha+n+\mu & \text{(unstuck states)} \\ (\alpha\mu)+n & \text{(stuck states).} \end{cases} \tag{1}$$

The probability ω_s^0 that the muon will take the second branch of the above reaction is referred to as the initial sticking fraction, and has been computed by several authors to be about 0.9% [3–5]. Even if the muon sticks, however, there is still a significant probability that the muon will be stripped from the α following a collision with a hydrogen nucleus in the medium, before the $\alpha\mu$ is brought to rest. This process is referred to as regeneration and the final sticking fraction ω_s is conventionally related to the initial sticking fraction by $\omega_s=(1-R)\omega_s^0$. The regeneration coefficient R depends upon the stopping power of the medium and several important cross sections, and in molecular hydrogen it is found to be about 0.3 [6–8].

The significance of ω_s is that it places a fundamental limit of $1/\omega_s$ on the number of fusions a single muon can catalyze. Since the sticking fraction is currently the major bottleneck in the muon catalyzed fusion cycle, it has been the focus of some attention and several schemes have been proposed to reduce its value (see, for example, Ref. [9]). An alternative approach is to seek an environment in which the stopping power of the $\alpha\mu$ ion is fundamentally different. The stopping power $S(E)$ is defined by

$$-\frac{dE}{dt}=S(E)\,v, \tag{2}$$

where E is the time-dependent kinetic energy of the $\alpha\mu$ ion and v is its velocity. To a first approximation the regeneration coefficient is exponentially related to the stopping power by $1-R=\exp[-\rho\int_0^{E_0}dE\,\sigma_{\rm strip}(E)/S(E)]$, where ρ is the atomic density of the hydrogen medium, $\sigma_{\rm strip}$ is the stripping cross section from the $\alpha\mu$ bound states, and $E_0=3.5$ MeV is the initial $\alpha\mu$ kinetic energy. Thus even a small change in the stopping power has an important impact on ω_s. In molecular hydrogen there is little density dependence, as the stopping power scales linearly with density. Following the observation by Menshikov [10] that the stopping power of a plasma is less than that of molecular hydrogen, regeneration in a tepid plasma environment ($T<2000$ eV, $\rho<10$ LHD) has been investigated [11]. It has been found that the regeneration coefficient is generally confined to below 0.9, increasing the potential number of fusions by at most a factor of 7.

In this work I examine a more extreme environment, that of a very dense, degenerate plasma with $\rho>10^3$ LHD, $T\sim100$ eV. The degenerate plasma is fundamentally different from a conventional plasma as Fermi blocking prevents most of the electrons from participating in the stopping process, and the plasma becomes transparent to charged particles [12, 13]. Indeed, the stopping power begins to behave only as $\ln\rho$ at sufficiently high densities. The stopping power for a degenerate electron gas has been computed following Dar *et al.* [12], in the limit for slow ions, and is given by

$$S=\frac{4E_0y_0^2}{3a_0}\beta\left(\ln(1+y_0^{-2})-\frac{1}{1+y_0^2}\right), \tag{3}$$

where $E_0=p_0^2/2m_e$, $p_0=(3\pi^2n_e)^{1/3}$, $y_0=1/\sqrt{\pi a_0p_0}$, and $a_0=1/(\alpha m_e)$. n_e is the electron density, and is equal to the atomic density ρ for hydrogen. The above result is valid in the limit $\beta\equiv v/v_f\rightarrow0$, where $v_f=p_0/m_e$ is the Fermi velocity. Equation (3) can actually be extended to about $\beta=2$ before a significant deviation from the exact result occurs. The initial velocity of the $\alpha\mu$ ion is 5.9 αc, where α is the fine-structure constant, so (3) is generally applicable when the density of the electrons is greater than about 500 LHD.

The regeneration probability is computed by evolving the set of population equations describing the occupation of the states of $\alpha\mu$ as it slows down in the hydrogen medium. The regeneration process is described by (2) coupled to the equations

$$\frac{dP_n}{dt}=\lambda_n^+-\lambda_n^-\,P_n, \tag{4}$$

where the P_n are the occupation probabilities for the αn Coulomb states $1s$, $2s$, $2p$, $3s$,... and the continuum. λ_n^+ and λ_n^- are the transition rates to and from state n, respectively. They are given by

$$\lambda_n^+ = \sum_{n'(>n)} (\lambda_{nn'}^{\text{Auger}} + \lambda_{nn'}^{\text{rad}} + \lambda_{nn'}^{\text{dex}}) P_{n'} + \sum_{n'(<n)} \lambda_{nn'}^{\text{ex}} P_{n'} + \sum_{n(=n')} \lambda_{nn'}^{\text{Stark}} P_{n'}, \quad (5)$$

$$\lambda_n^- = \lambda_n^{\text{strip}} + \sum_{n'(<n)} (\lambda_{n'n}^{\text{Auger}} + \lambda_{n'n}^{\text{rad}} + \lambda_{n'n}^{\text{dex}}) + \sum_{n'(<n)} \lambda_{n'n} + \sum_{n(=n')} \lambda_{n'n}^{\text{Stark}}, \quad (6)$$

where the first index on each λ denotes "to," and the second index denotes "from." λ^{Auger}, λ^{rad}, λ^{dex}, λ^{ex}, λ^{Stark}, and λ^{strip} are the Auger, radiative, inelastic deexcitation, inelastic excitation, Stark mixing, and stripping rates, respectively. The sum over $n = n'$ for the Stark mixing term implies the summation over all angular-momentum states with the same principle quantum number.

All of the cross sections, with the exception of the Auger transitions, are identical in both atomic and plasma environments. The temperature of the medium is not relevant for $T < 1000$ eV as it is the kinetic motion of the $\alpha\mu$ that determines the interaction energy. The cross sections have been taken from the compilation of Cohen [14], which permits a direct comparison to the results of Ref. [11]. The Auger transitions require special attention as the electron density in the plasma is quite different from that in molecular matter. This calculation is technically difficult as the velocity of the $\alpha\mu$ is comparable to the velocity of the electrons in the plasma, and it is not possible to extract the thermal momenta distributions from the angular integrations to obtain the squared-averaged matrix element $\langle M^2 \rangle$. However, a rough estimate of the Auger rate indicates that it is always at least an order of magnitude less than the radiative deexcitation rate for $n \leq 3$, above which the stripping cross section is by far dominant [15]. Auger deexcitation has therefore been neglected.

The initial sticking fractions $P_n(t = 0)$ depends upon the process by which fusion occurs. Sticking fractions for in-flight fusion processes are unknown, and their computation would require the evaluation of a three-body continuum wave function. The only fractions that have been computed in detail are those for the rotational-vibrational molecular states $(J\nu) = (00)$ and (01) of $dt\mu$, and have been used in this calculation. Most of the stuck muons are initially in the $(\alpha\mu)_{1s}$ state, in which the the stripping rate is comparable to the excitation rate to higher states. The sticking fractions for other fusion processes are likely to be similar, up to an overall normalization. The initial sticking probabilities were taken from Jeziorski *et al.* [5]. Only the (01) sticking fractions

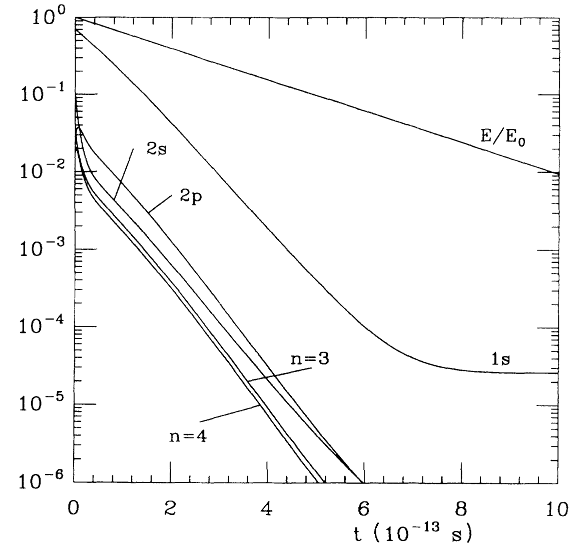

FIG. 1. Population of $\alpha\mu$ Coulomb states during regeneration (in %), and fractional energy loss, at $\rho = 10^3$ LHD.

have been used, as the (01) state is likely to be the dominant channel for fusion in the plasma environment [15]. Coulomb states up to $n = 6$ were included in the regeneration calculation, with each angular-momentum state individually represented. The occupation probabilities as a function of time are displayed in Fig. 1.

Ladder ionization processes are far more important in a dense plasma than in molecular hydrogen. The regeneration coefficient at 10^3 LHD is found to be $(1 - R) = 2.9 \times 10^{-5}$ for the full calculation, and 2.6×10^{-3} when just the $n = 1$ state is included. This difference of two orders of magnitude can be compared to regeneration in molecular hydrogen, where the difference in the two calculations is just a few percent. The qualitative difference is due to the ability of the stripping rate from the $n = 2$ states to compete with radiative deexcitation.

The degenerate plasma environment can potentially permit up to 10^7 fusions per muon. However, the densities are such that this environment can only exist in an inertially confined plasma with lifetimes of 0.1 μs or less. To take advantage of the low sticking, the catalytic cycle must proceed at a rate 10^5 that of the plasma lifetime. The higher density of the plasma does accelerate most rates (such as $d\mu \rightarrow t\mu$ transfer) by a factor of 10^3. However, the unique D_2-$dt\mu$ molecular resonance permitting fast $dt\mu$ molecular formation in liquid hydrogen is unavailable in the plasma environment, and all alternative fusion mechanisms computed thus far are generally of the order of $10^8\,\text{s}^{-1}$. A detailed analysis of the $dt\mu$ catalytic cycle in a plasma is planned to be presented in a future publication.

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