Acoustoelectric effects in a gaseous medium

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The well-known acoustoelectric coupling effect in semiconductors, whereby a sound wave is amplified if a charge carrier's drift velocity exceeds the speed of sound, also exists for ions and electrons in a gaseous medium, but theoretical analysis so far has been limited to simplified collisional-exchange models [G. M. Sessler, Phys. Fluids **7**, 90 (1964); U. Ingard and M. Schultz, Phys. Rev. **158**, 106 (1967); T. D. Mantei and M. Fitaire, in *Proceedings of the 10th International Conference on Phenomena in Ionized Gases* (Oxford University Press, Oxford, 1971), p. 309]. The present paper is based upon more accurate considerations of collisional phenomena, leading to more realistic predictions of the qualitative and quantitative nature of the effect.

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It has long been known [1] that an acoustic wave in a semiconductor may be amplified by a dc current applied along the direction of propagation, if the drift velocity of the charge carriers exceeds the sound speed. The possibility of manufacturing devices, such as acoustoelectric amplifiers, based on an understanding of the coupling between electrons and lattice vibrations was recognized [2] at a early stage. Theoretical and experimental developments have continued [3] up until the present date. The phenomenon is explained in detail in textbooks [4]. It is also well known that there is often a one-to-one correspondence between hot-charge-carrier transport in semiconductors and electron and ion transport in gaseous media. Anisotropic diffusion in an electric field and negative differential conductivity are just two cases in point. An effect analogous to the semiconductor acoustoelectric coupling also exists for gases [5], although it may be anticipated that it will become manifest in experiment only as a *reduction* in attenuation due to viscous damping and heat conduction, rather than straight out amplification. As far as we are aware, little theoretical investigation has been carried out in this direction [5], apart from some earlier work in which either only momentum transfer between fluid species is considered [6], the charged-particle motion is treated as a small perturbation [7], or otherwise nonzero charged-particle drift is not accounted for correctly in considerations of energy transfer [8]. It is emphasized that the effect we consider has nothing to do with ion acoustic waves or plasma effects *per se*.

The mechanism we propose below involves both collisional-momentum and energy-exchange effects and is, for that reason, a more soundly based representation of the true physical situation, in both a qualitative and quantitative sense. Consider ions of mass m in a gas of neutral atoms of mass m_0. (Subscripts 0 pertain to neutral-gas quantities.) If an electric field E is present, it will pump momentum and energy into the ion system. The ions, through their collisions with the neutral species, will transfer momentum and energy to the neutral species. The ions will acquire an average drift velocity $\mathbf{v}$ and an average energy ε, which will exceed the thermal value $\frac{3}{2}kT_0$, where T_0 is gas temperature. The number density n of ions is assumed to be much less than the number density n_0 for neutral species so that the parameter

$$\delta = n/n_0 \tag{1}$$

is small, allowing neglect of ion-ion interactions in comparison with ion–neutral-atom collisions. The latter are assumed to be elastic and governed by the Maxwell model of interaction [9], in which an inverse fourth-power law potential operates with a corresponding momentum-transfer cross section $Q_m(g)$ inversely proportional to relative speed g. Thus the quantity

$$K_m = gQ_m(g) \tag{2}$$

is a constant, independent of g. Under these circumstances exact equations of continuity, momentum, and energy balance can be obtained by forming appropriate moments of Boltzmann's equation. For the neutral species, these are [10]

$$\frac{1}{n_0}\frac{dn_0}{dt} + \nabla\cdot\mathbf{v}_0 = 0 , \tag{3}$$

$$m_0\frac{d\mathbf{v}_0}{dt} + \frac{1}{n_0}\nabla\cdot\mathsf{P}_0 = -nK_m\mu(\mathbf{v}_0 - \mathbf{v}) , \tag{4}$$

and

$$\frac{d\varepsilon_0}{dt} + \frac{1}{n_0}\nabla\cdot(\mathbf{q}_0 + \mathsf{P}_0\cdot\mathbf{v}_0) = -nK_e[\varepsilon_0 - \varepsilon + \tfrac{1}{2}(m - m_0)\mathbf{v}\cdot\mathbf{v}_0] , \tag{5}$$

respectively, where $\mu = mm_0/(m+m_0)$ is the reduced mass,

$$K_e \equiv 2\mu K_m/(m+m_0) , \tag{6}$$

and $\mathbf{v}_0$ and ε_0 denote the mean velocity and energy of the neutral species, respectively. The convective time derivative is defined by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_0 \cdot \nabla . \tag{7}$$

Similar equations for ionic number density, mean velocity, and energy, $\mathbf{n}$, $\mathbf{v}$, and ε, respectively, could be written down but they are not required at present. If viscous effects are ignored in the gas, then the pressure tensor is isotropic,

$$\mathsf{P}_0 = p_0 \mathbb{1} , \tag{8}$$

where

$$p_0 = \tfrac{2}{3}(n_0\varepsilon_0 - \tfrac{1}{2}m_0 v_0^2) \tag{9}$$

is the hydrostatic pressure and $\mathbb{1}$ is the unit tensor. We also ignore heat conduction in this work, i.e., we set $\mathbf{q}_0 = 0$ in (5).

Suppose that the unperturbed situation (i.e., in the absence of the sound wave) is characterized by spatially uniform moments, designated by an overhead bar, satisfying

$$\frac{\partial \bar{n}_0}{\partial t} = 0 , \tag{10}$$

$$m_0 \frac{\partial \bar{\mathbf{v}}_0}{\partial t} = -n\,\mu K_m (\bar{\mathbf{v}}_0 - \bar{\mathbf{v}}) , \tag{11}$$

$$\frac{\partial \bar{\varepsilon}_0}{\partial t} = -\bar{n} K_e [\bar{\varepsilon}_0 - \bar{\varepsilon} + \tfrac{1}{2}(m - m_0)\bar{\mathbf{v}}_0 \cdot \bar{\mathbf{v}}] . \tag{12}$$

Here, and in what follows, it is assumed that the ionic properties, n, $\mathbf{v}$, and ε, are constants $\bar{n}$, $\bar{\mathbf{v}}$, and $\bar{\varepsilon}$, respectively, independent of time, whose values may be determined independently from corresponding moment equations [10]. This point will be discussed further below.

The sound wave appears as a small perturbation, as designated by the superscript (1):

$$\begin{aligned} n_0 &= \bar{n}_0 + n_0^{(1)} , \\ \mathbf{v}_0 &= \bar{\mathbf{v}}_0 + \mathbf{v}_0^{(1)} , \\ \varepsilon_0 &= \bar{\varepsilon}_0 + \varepsilon_0^{(1)} . \end{aligned} \tag{13}$$

Equations (3)–(5) are then linearized in small quantities

$$\frac{\partial n_0^{(1)}}{\partial t} + \bar{n}_0 \nabla \cdot \mathbf{v}_0^{(1)} = 0 , \tag{14}$$

$$m_0 \frac{\partial \mathbf{v}_0^{(1)}}{\partial t} + \nabla p_0^{(1)} = -\bar{\delta}\mu\nu_m \mathbf{v}_0^{(1)} , \tag{15}$$

$$\frac{\partial \varepsilon_0^{(1)}}{\partial t} + \frac{\bar{p}_0}{\bar{n}_0} \nabla \cdot \mathbf{v}_0^{(1)} = -\bar{\delta}\nu_e [\varepsilon_0^{(1)} + \tfrac{1}{2}(m - m_0)\bar{\mathbf{v}} \cdot \mathbf{v}_0^{(1)}] , \tag{16}$$

where

$$\bar{p}_0 = \tfrac{2}{3}\bar{n}_0\bar{\varepsilon}_0, \quad p_0^{(1)} = \tfrac{2}{3}(\bar{n}_0\varepsilon_o^{(1)} + n_0^{(1)}\bar{\varepsilon}_0) , \tag{17}$$

$$\bar{\delta} \equiv \bar{n}/\bar{n}_0 , \tag{18}$$

and

$$\nu_m = \bar{n}_0 K_m, \quad \nu_e = \bar{n}_0 K_e = 2\mu\nu_m/(m+m_0) \tag{19}$$

are the collision frequencies for momentum- and energy-transfer, respectively.

In Ref. [7], collisional-transfer terms are derived on the basis of a constant cross section rather than constant collision frequency as in the present paper. Nevertheless, the mathematical form of the expressions should be similar, as explained in Ref. [10]. There is, however, no allocation for either thermal motion of the neutral species or drift of the charged species in the energy transfer expression in Ref. [7] and hence nothing corresponding to the last term in the right-hand side of (16) above. Mantei and Fitaire [8] have used the $\bar{v}=0$ energy equation of Ref. [7] in order to model a $\bar{v}\neq 0$ situation. Their results should be viewed in the light of this obvious internal inconsistency.

In writing down (14)–(16), we have set

$$\bar{\mathbf{v}}_0 \approx 0 , \tag{20}$$

an approximation which can be justified from (11) for time scales $t \ll (\bar{\delta}\mu\nu_m/m_0)^{-1}$. The sound wave also perturbs the ion swarm, but it can be shown that such perturbations appear in (14)–(16) through terms of order $(\bar{\delta})^2$, and they are therefore neglected.

By assuming plane-wave dependences $\sim\exp\{i(\mathbf{k}\cdot\mathbf{r} - \omega t)\}$ for perturbed quantities where $\mathbf{k}$ is along the direction of $\bar{\mathbf{v}}$ it follows, after some algebra, that (14)–(16) furnish the dispersion relation

$$\omega^2 = s^2k^2 - \frac{i\bar{\delta}\mu\nu_m\omega}{m_0}\left[1 + \frac{2}{3}\frac{(m-m_0)}{(m+m_0)}\frac{\bar{v}}{(\omega/k)} + \frac{4m_0 s^2}{5(m+m_0)(\omega/k)^2}\right] , \tag{21}$$

where

$$s^2 = \frac{10}{9}\frac{\bar{\varepsilon}_0}{m_0} \approx \frac{5}{3}\frac{kT_0}{m_0} \tag{22}$$

defines sound velocity s in the gas in the absence of any ions, i.e., for $\bar{\delta}=0$. To first order in $\bar{\delta}$, Eq. (21) gives

$$\omega = sk - \frac{i\bar{\delta}\mu\nu_m}{2m_0}\left[1 + \frac{4m_0}{5(m+m_0)} + \frac{2(m-m_0)}{3(m+m_0)}\frac{\bar{v}}{s}\right] , \tag{23}$$

from which we obtain the attenuation factor

$$\alpha = \frac{\bar{\delta}\mu\nu_m}{2m_0 s}\left[1 + \frac{4m_0}{5(m+m_0)} + \frac{2(m-m_0)}{3(m+m_0)}\frac{\bar{v}}{s}\right] . \tag{24}$$

Attenuation changes sign and becomes negative, i.e., amplification occurs, if

$$\frac{\bar{v}}{s} > \frac{3(m+9m_0/5)}{2(m_0-m)} . \tag{25}$$

For electrons, $m \ll m_0$, and the condition for amplification is

$$\bar{v} > 2.7s . \tag{26}$$

Clearly, no amplification is possible unless ions are lighter than the neutral species, $m < m_0$.

These results are quite different for the acoustoelectric effects observed in semiconductors [1]. There the condition for amplification is simply $\bar{v} > s$ and α is dependent upon wave number, whereas (24) has no k dependence. Differences would be expected, however, due to the qualitatively different nature of sound propagation in fluids and solids. Mass-dependent effects arise in the present situation because of the different, mass-dependent rates of collisional-energy- and momentum-exchange between ions and neutral species, as is apparent from the right-hand side of (4) and (5). Electron-phonon scattering in solids has no such counterpart.

As previously mentioned, the only previous discussion [8] of the effect of drift fails to treat energy transfer correctly and consequently gives quite different results. It is interesting to estimate the magnitude of the predicted effect and to this end we write (24) in terms of experimentally measured quantities using the electron parameters appropriate to this model [10]:

$$\bar{\mathbf{v}} = \frac{e\mathbf{E}}{\mu\nu_m} ,$$

$$\bar{\varepsilon} \approx \tfrac{1}{2} m_0 \bar{v}^2 , \tag{27}$$

and

$$D_T/\bar{v} \approx \frac{2}{3}\frac{\bar{\varepsilon}}{eE} \approx \frac{1}{3} m_0 \bar{v}^2/eE .$$

Thus we find from (24) and (27)

$$\alpha = \frac{J}{6e\bar{n}_0 D_T}\frac{\bar{v}}{s}\left[1+\frac{4m_0}{5(m+m_0)}+\frac{2(m-m_0)}{3(m+m_0)}\frac{\bar{v}}{s}\right] , \tag{28}$$

where $J=\bar{n}e\bar{v}$ is the current density. Consider as an example electrons in helium gas at room temperature, for which [12] $s \approx 10^3$ m/sec. For $E/n_0 = 3.5$ Td$=3.5\times10^{-21}$ V m^2, experiment shows [11] that electron drift velocity $\bar{v} \approx 10s$, well above the critical condition (26) for amplification. In addition [11], $\bar{n}_0 D_T \approx 4\times10^{24}$ m^{-1}sec^{-1}. With these values inserted into the right-hand side of (28), we find $|\alpha| \sim \frac{1}{4}J\times10^{-5}$ m^{-1}. Typically, $J \lesssim 10^{-7}$ A/m^2 in swarm experiments [11], so that $|\alpha| < 10^{-12}$ m^{-1}. On the other hand, attenuation due to heat flow and viscous effects in helium at STP is of the order [13] of $5\times10^{-12}\ f^2$ m^{-1}, where $f=2\pi/\omega$ is the frequency of the sound wave. Thus, for acoustoelectric amplification to exceed dissipative attenuation, current densities $J \gtrsim 2\times10^{-6}\ f^2$ A/m^2 would be required. Even at very low sound-wave frequencies, it therefore appears that current densities would have to be raised by several orders of magnitude above values currently employed in swarm experiments in order that the effect became significant and then perhaps it would only appear as a reduction in attenuation in viscous damping, rather than as an amplification *per se*. Space-charge effects may then become important and the estimates of α based upon the free-electron formulas (27) may have to be revised. However, the analysis up to (26) remains intact as does the prediction of the existence of the effect, regardless of the value of J. We look forward to experiments aimed at verifying these predictions.

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