Lasing without inversion and coherence in dressed states

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The V and Ξ types of closed lifetime-broadened systems with lower-level-dressed-state preparation by a strong electromagnetic field are analyzed. It is found that the coherence in dressed states enhances the gain in the former system but reduces gain in the latter. However, we show that, since the dressed states are prepared in lower levels, both systems can lase without population inversion in steady state.

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It has been suggested that many multilevel systems with split lasing levels can provide gain without population inversion [1–12]. The essential factor for some of these systems is the quantum-interference effect. Recently, Imamoğlu, Field, and Harris [12] proposed a closed lifetime-broadened Λ system where the quantum interference is induced by a strong electromagnetic (EM) field. Agarwal [13] analyzed the origin of gain in this model and showed that it may be understood in terms of the coherence in dressed states created by the strong EM field. He pointed out that the existence of gain in a noninversion environment is due to the fact that the coherence in dressed states contributes a negative quantity to the imaginary part of the polarization, which in turn enhances the gain.

In this Brief Report, we shall consider the V- and Ξ-types of closed lifetime-broadened systems with lower-level-dressed-state preparation by a strong EM field. We show that the creation of lower-level dressed states changes the noninversion conditions and thus both systems can lase without inversion, though in the Ξ-type system the coherence in dressed states reduces the gain.

The model systems considered here are shown in Fig. 1. The strong interference between levels $|2\rangle$ and $|3\rangle$ is driven by an external EM field. The population of level $|1\rangle$ is incoherently pumped from level $|2\rangle$ with rate λ and spontaneous decay rates from $|1\rangle$ to $|2\rangle$ and from $|3\rangle$ to $|2\rangle$ (from $|2\rangle$ to $|3\rangle$) are r_1 and r_3 (r_2), respectively. We first consider the V-type system shown in Fig. 1(a). The equation of motion for the density matrix in the rotating frame may be written as

$$\begin{aligned}\dot\rho=&-r_1(A_{11}\rho-2\rho_{11}A_{22}+\rho A_{11})\\&-r_3(A_{33}\rho-2\rho_{33}A_{22}+\rho A_{33})\\&-\lambda(A_{22}\rho-2\rho_{22}A_{11}+\rho A_{22})+iG[A_{23}+A_{32},\rho]\\&+ig[A_{12}+A_{21},\rho]\ ,\end{aligned}\tag{1}$$

where $A_{\alpha\beta}=|\alpha\rangle\langle\beta|$, $2G$ and $2g$ are the Rabi frequencies of the external EM field and weak probe field, respectively, and both are assumed to be real positive.

Following the line of Agarwal [13], we transfer into the dressed states

$$|0\rangle=|1\rangle\ ,\quad |\pm\rangle=\frac{1}{\sqrt2}(|3\rangle\pm|2\rangle)\ .\tag{2}$$

Then the equations of motion for the density matrix elements in dressed states are

$$\begin{aligned}\dot\rho_{\pm\mp}=&\left[\pm iG-r_3-\frac{\lambda}{2}\right](\rho_{+-}+\rho_{-+})\\&+\left[iG\mp\frac{r_3+\lambda}{2}\right](\rho_{+-}-\rho_{-+})-r_1\rho_{00}\\&-\left[r_3-\frac{\lambda}{2}\right](\rho_{++}+\rho_{--})\ ,\end{aligned}\tag{3}$$

$$\dot\rho_{\pm\pm}=r_1\rho_{00}-\left[\frac{r_3}{2}+\lambda\right]\rho_{\pm\pm}+\frac{r_3}{2}\rho_{\mp\mp}+\frac{\lambda}{2}(\rho_{+-}+\rho_{-+})\ ,\tag{4}$$

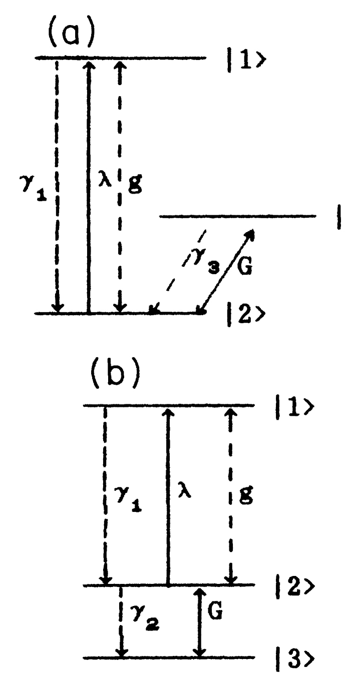

FIG. 1. Energy-level diagrams with dressed-state preparation in lower levels. (a) V-type system and (b) Ξ-type system.

$$\dot{\rho}_{0\pm}=\pm\frac{ig}{\sqrt{2}}(\rho_{\pm\pm}-\rho_{00})\mp\frac{ig}{\sqrt{2}}\rho_{\mp\pm}\mp iG\rho_{0\pm}-\left[r_1+\frac{r_3+\lambda}{2}\right]\rho_{0\pm}-\frac{r_3-\lambda}{2}\rho_{0\mp}\ . \tag{5}$$

In the equations of $\dot{\rho}_{\pm\mp}$ and $\dot{\rho}_{\pm\pm}$ the weak probe field has been omitted.

Equations (3)–(5) may be solved in the steady state. Here, we assume that the external EM field used to create the dressed states is strong enough (i.e., $G \gg r_1, r_2, \lambda$) that we may analyze their solutions by keeping the leading terms. We obtain

$$\rho_{+-}=-\rho_{-+}=\frac{1}{2iG}\left[\left[r_3-\frac{\lambda}{2}\right]+\left[r_1-r_3+\frac{\lambda}{2}\right]\rho_{00}\right], \tag{6}$$

$$\rho_{00}=\frac{\lambda}{\lambda+2r_1}\ ,\quad \rho_{++}=\rho_{--}=\frac{r_1}{\lambda+2r_1}\ , \tag{7}$$

$$\rho_{0\pm}=\frac{ig}{\sqrt{2}G^2}\left[\pm\frac{r_3-\lambda}{2}(\rho_{\mp\mp}-\rho_{00})\pm\left[r_1+\frac{r_3+\lambda}{2}\right](\rho_{\pm\pm}-\rho_{00})+iG\rho_{\mp\pm}-iG(\rho_{\pm\pm}-\rho_{00})\mp\left[r_1+\frac{r_3+\lambda}{2}\right]\rho_{\mp\pm}\mp\frac{r_3-\lambda}{2}\rho_{\pm\mp}\right]. \tag{8}$$

From Eq. (7), we find that the noninversion condition $\rho_{00}-\rho_{++}-\rho_{--}<0$, will be satisfied if

$$\lambda < 2r_1\ . \tag{9}$$

Furthermore, for the system to have gain, it is necessary to have

$$\mathrm{Im} d_{21}\rho_{12}=\frac{d_{21}}{\sqrt{2}}\mathrm{Im}(\rho_{0+}-\rho_{0-})<0\ . \tag{10}$$

Using Eq. (8), we get

$$\begin{aligned}\mathrm{Im}(\rho_{0+}-\rho_{0-})&=\frac{g}{\sqrt{2}G^2}[(r_1+r_3)(\rho_{++}+\rho_{--}-2\rho_{00})+\mathrm{Im}G(\rho_{+-}-\rho_{-+})]\\&=\frac{g}{\sqrt{2}G^2}\left[\frac{2(r_1+r_3)(r_1-\lambda)}{2r_1+\lambda}-\frac{2r_1r_3}{2r_1+\lambda}\right]<0\ .\end{aligned} \tag{11}$$

This amounts to

$$\frac{r_1^2}{r_1+r_3}<\lambda\ . \tag{12}$$

Thus the laser system shown in Fig. 1(a) can operate without inversion if λ fulfills the conditions $r_1^2/(r_1+r_3)<\lambda<2r_1$. Similar to the Λ-type system with upper-level-dressed-state preparation [12,13], the coherence in dressed states contributes a negative term to the imaginary part of polarization, which enhances the gain. However, from the gain and noninversion conditions we find that lasing without inversion is easier to realize in the V-type closed lifetime-broadened system with lower-level-dressed-state preparation than in the Λ-type system [12,13] (where a condition on the model atom, the spontaneous decay rate from one upper level to another exceeds that to the lower level, is required besides the condition on pumping rate).

We next consider the Ξ-type system shown in Fig. 1(b). The equation of motion for the density matrix is obtained by replacing the second term of the right-hand side of Eq. (1) by

$$-r_2(A_{22}\rho-2\rho_{22}A_{33}+\rho A_{22})\ .$$

Similar to the above analysis, we find that the noninversion condition is unchanged. The gain condition now becomes

$$\mathrm{Im}(\rho_{0+}-\rho_{0-})=\frac{g}{\sqrt{2}G^2}\left[\frac{2r_1(r_1-\lambda)}{2r_1+\lambda}+\frac{2r_1r_2}{2r_1+\lambda}\right]<0\ , \tag{13}$$

i.e.,

$$r_1+r_2<\lambda\ . \tag{14}$$

In Eq. (13), the first term in large parentheses comes from atomic populations and the second term from the coherence in dressed states.

It is clear in the Ξ-type system that the coherence in dressed states contributes a positive quantity to the imaginary part of polarization, which reduces the gain. However, if the spontaneous decay rate from level $|2\rangle$ to level $|3\rangle$ is less than that from level $|1\rangle$ to level $|2\rangle$, i.e., $r_2<r_1$, the system may realize lasing without inversion provided that the pumping rate satisfies $r_1+r_2<\lambda<2r_1$. This is because in the absence of external EM field, the noninversion condition is $\lambda<r_1$ in steady state (in this case, level $|3\rangle$ is separated out from the three-level laser system), while if a strong EM field is imposed to create the dressed states, it becomes $\lambda<2r_1$. The change of noninversion condition enables the simultaneous satisfaction of both noninversion and gain conditions, though the gain is reduced by the coherence term.

We note that in our model it is impossible for the Ξ-type system with $r_2>r_1$ to lase without inversion. In this case, if the pumping rate λ satisfies $r_1+r_2>\lambda>2r_1$, the system may inverse without lasing.

In conclusion, we have analyzed V and Ξ types of closed lifetime-broadened systems with dressed states prepared in lower levels by a strong EM field. We find that lasing without inversion is easier to realize in the V-type system with lower-level-dressed-state preparation than in the Λ-type system with upper-level-dressed-state preparation. We also show that the Ξ-type system may lase without inversion in the steady state, though the coherence in dressed states reduces the gain.

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