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Generation of intensity-correlated twin beams using series-coupled semiconductor lasers

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The low-frequency intensity noise of high-quantum-efficiency lasers or light-emitting diodes (LED's) is dominated by the pump noise. By coupling two or more semiconductor lasers (or LED's) in series with a current source, the pump noise of the two lasers will be equal, and the intensity noise of the lasers will be highly correlated. It is shown that one may generate sub-Poissonian light by measuring the intensity fluctuations of one of the beams and feeding back (or forward) the information to regulate the other photon beam. The generation scheme is shown to have much in common with intensity-correlated twin beam generation from nondegenerate optical parametric oscillators.

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I. INTRODUCTION

Intensity-correlated photon beams are useful to reduce the influence of intensity noise and thus increase the sensitivity of measurements involving photoabsorption, such as absorption spectroscopy. If the correlation is sufficiently strong, the measurement accuracy will surpass that achievable with photons prepared in a coherent state. Alternatively, the correlation can be used to reduce the intensity noise in one of the beams to levels below the standard quantum limit [1–3].

The most commonly used generation process of correlated photon pairs is nondegenerate optical parametric frequency down-conversion. With no optical feedback, the nondegenerate parametric amplification process will produce photon pairs highly correlated in space and in time [4–7]. However, the wide correlation bandwidth (fundamentally about 100 GHz, but in present realizations only a few hundred MHz) is traded off against a low average power and, in general, a poor quantum efficiency of the pump process.

With optical feedback, parametric oscillation may be accomplished. The oscillator can generate photon beams with reasonable power, but at the cost of limited correlation bandwidth. The low nonlinear coefficients in optical crystals require relatively long and high-finesse cavities to give reasonable oscillation threshold pump power. The best parametric oscillators for correlated twin beam generation today have a cavity bandwidth of about 20 MHz [3, 8].

For quite some time now it has been known that the low-frequency intensity noise of high-quantum-efficiency lasers and light emitting diodes (LED's) is caused by pump noise, whatever the pump mechanism may be [9, 10]. It has also been proposed, and demonstrated, that it is possible to reduce the intensity noise of semiconductor lasers and LED's to levels below the standard quantum limit by reducing the noise of the injection current (pump noise) [10–13].

In this paper it is suggested that an alternative, probably simpler, and possibly better way to generate intensity-correlated photon beams is to let two lasers be subject to the same pump noise. With optically pumped lasers this is difficult, or maybe even impossible, but with semiconductor lasers (or LED's) it is easy, one can simply couple them in series with the injection-current generator. As will be shown in the paper, the correlation between the intensity noise of the lasers at frequencies below the cold-cavity bandwidth is limited only by the overall quantum efficiency of the system.

The paper is organized as follows. In Sec. II the equivalent electrical circuits for a semiconductor laser are presented. From the electrical circuit models it is rather easy to deduce the rate equations for the free-carrier population and for the photon number in the cavity mode. The quantum noise processes in the laser are modeled by Langevin noise sources that are included in the rate equations. In Sec. III the intensity-correlation spectra are calculated and the possible noise reduction is delineated. In Sec. IV the correlation spectra and noise reduction of series-coupled LED's are calculated. In Sec. V we compare the suggested scheme with other noise reduction schemes in terms of bandwidth, ultimate noise reduction limits and experimental realization. Finally, in Sec. VI some of the conclusions are summarized.

II. LASER MODEL

A. Equivalent electrical circuit

The equivalent electrical circuit of a semiconductor laser is shown in Fig. 1. The current I_p is noisy; for the moment we can assume that it can be described by a Poisson process (shot noise). The resistors $R_{\rm nr}$, $R_{\rm sp}$, $R_{\rm gen}$ represent nonradiative recombination, spontaneous emission into nonlasing modes, and photon emission into the lasing mode. Please note that in general the respective resistors are pump-current dependent. The resistors are also associated with pump-current-dependent noise

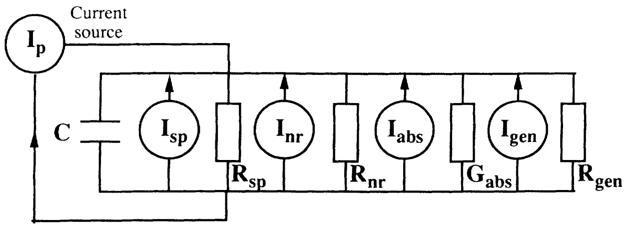

FIG. 1. The equivalent electrical circuit of a semiconductor laser.

currents $I_{\rm nr}$, $I_{\rm sp}$ and $I_{\rm gen}$, respectively. The negative conductance $G_{\rm abs} < 0$ represents stimulated absorption and is associated with the noise current $I_{\rm abs}$. The charge Q over the capacitance C represents the free-carrier population in the active medium. It is clear from Fig. 1 that when coupling two or more lasers in series, the pump current I_p will be identical for every laser. (This follows if we neglect circuit time delays and impedance mismatch, but both these effects have negligible influence if the circuit is designed properly.) The rate of change for Q can be written

$$\frac{dQ}{dt} = I_p - \frac{Q}{CR_{\rm sp}} - \frac{Q}{CR_{\rm nr}} - \frac{Q}{CR_{\rm gen}} - \frac{QG_{\rm abs}}{C} + I_{\rm sp} + I_{\rm nr} + I_{\rm gen} + I_{\rm abs} \ . \qquad (2.1)$$

The photon field of the lasing mode can be described by a similar electrical circuit, Fig. 2. The driving current $I_{\rm net} = Q/CR_{\rm gen} + QG_{\rm abs}/C$ can be found from Fig. 1. The resistors R_m and R_i represent the laser mirror losses and the internal losses. The associated noise sources are $i_{\rm net}$, i_m, and i_i, respectively. The charge Q_p over capacitor C_p represents the photons stored in the resonator. The rate of change of Q_p can be written

$$\frac{dQ_p}{dt} = I_{\rm net} - \frac{Q_p}{C_pR_m} - \frac{Q_p}{C_pR_i} + i_{\rm net} + i_m + i_i \ . \qquad (2.2)$$

With these equations we are ready to express the rate equations in more physical terms.

B. Langevin equations

The different circuit elements in the preceding section can now be identified so as to get self-consistent semiclassical equations for the time evolution of the free carriers and the cavity-photon number. We can immediately identify Q/e and Q_p/e, where e is the elementary charge, with the free-carrier number N and the cavity-photon number p. The different destruction and creation rates can be expressed as follows: The nonradiative lifetime $CR_{\rm nr}$ is denoted $\tau_{\rm nr}$. The spontaneous-emission lifetime is denoted $\tau_{\rm sp}$. Both these time constants will be assumed independent of the injection current of the laser, as the free-carrier number is clamped above threshold. The time constant $CR_{\rm sp}$ associated with spontaneous decay into nonlasing modes can thus be expressed $\tau_{\rm sp}/(1-\beta)$, where the dimensionless number β represents the fraction of the spontaneous emission that goes into the lasing mode. In conventional lasers, β is a small number, on the order of 10^{-5}. The stimulated-emission lifetime $CR_{\rm gen}$ can be expressed as $1/[g_0N(p+1)]$ where g_0 is the differential gain coefficient [14]. Similarly, the stimulated-absorption lifetime $G_{\rm abs}/C$ can be expressed $1/(g_0N_{\rm tr}p)$, where $N_{\rm tr}$ is the transparency free-carrier number. Using the fact that the spontaneous-emission rate into the lasing mode can be written either $\beta N/\tau_{\rm sp}$, or g_0N, one can eliminate g_0 using the identity $g_0 = \beta/\tau_{\rm sp}$ [14].

In a similar fashion the different photon-decay rates can be reexpressed in physical terms. $1/C_pR_m$ is identified as the photon-decay rate γ_m due to mirror-coupling loss, and the internal-loss rate of the cavity $1/C_pR_i$ is denoted γ_i.

The four free-carrier noise currents can be lumped into one free-carrier fluctuation operator, $F_N = (I_{\rm sp} + I_{\rm nr} + I_{\rm gen} + I_{\rm abs})/e$. In the same manner, the photon-circuit noise currents can be lumped into one fluctuation operator. However, for reasons of later convenience, we will keep two fluctuation operators for the photon field, $F_p = (i_{\rm net} + i_i)/e$ and $F_m = i_m/e$. Using these new noise operators we can translate the circuit equations (2.1) and (2.2) to ordinary Langevin noise-source-driven rate equations.

$$\frac{dN}{dt} = \frac{I_p}{e} - \frac{N}{\tau_{\rm sp}} - \frac{N}{\tau_{\rm nr}} - \frac{\beta(N-N_{\rm tr})p}{\tau_{\rm sp}} + F_N \qquad (2.3)$$

and

$$\frac{dp}{dt} = \left(\frac{\beta(N-N_{\rm tr})}{\tau_{\rm sp}} - \gamma_m - \gamma_i\right)p + \frac{N\beta}{\tau_{\rm sp}} + F_p + F_m \ . \qquad (2.4)$$

The steady-state solution of these equations (neglecting the fluctuation operators which all have a zero mean) can readily be solved, and the solutions will be denoted N_0 and p_0. Please note that N_0 and p_0 are both functions of the injection current. Making a small-signal expansion around the steady-state values for a given injection current, $N = N_0 + \Delta N$ and $p = p_0 + \Delta p$, and linearizing the equations, the Fourier transform of ΔN and Δp can be calculated. The expression for the Fourier transform (denoted with a tilde) of Δp is

$$\Delta\tilde{p}(\Omega) = \frac{\tilde{F}_N + D_1(\tilde{F}_p + \tilde{F}_m)}{D_2} \qquad (2.5)$$

where

$$D_1 = [1 + (1 - \beta + j\Omega\tau_{\rm sp})/\beta(p_0+1)] \qquad (2.6)$$

and

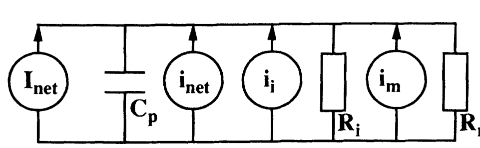

FIG. 2. The circuit equivalent for the photon field.

$$D_2 = [\gamma_i + \gamma_m - \beta(N_0 - N_{tr})/\tau_{sp} + j\Omega]D_1 + \beta(N_0 - N_{tr})/\tau_{sp} \ . \quad (2.7)$$

However, we are not interested in the correlation between the cavity-photon number of the series-coupled lasers, but in the correlation between the photon flux (intensity) of the respective lasers. The relation between the cavity-photon number and the photon flux I is

$$I = \gamma_m p - F_m \ . \quad (2.8)$$

It is convenient to define the mean and fluctuation of I too. It can be expressed $I = I_0 + \Delta I$, where $I_0 = \gamma_m p_0$.

III. LASER CORRELATION AND NOISE SPECTRA

A. Intensity-noise spectra

In this section the Langevin equations derived in Sec. II B are used to calculate the spectral density of the intensity noise and the correlation between the intensity noise of two (or more) lasers connected in series. We define the general cross spectral density $S_{A,B}(\Omega)$ between two fluctuating operators $A(t)$ and $B(t)$ by

$$S_{A,B}(\Omega) \equiv \int_{-\infty}^{\infty} \langle A(t)B(t+\tau)\rangle e^{-j\Omega\tau} d\tau \ , \quad (3.1)$$

where $\langle\,\rangle$ denotes ensemble average. Using this definition, the spectral density of operator A is $S_{A,A}(\Omega)$ which will be abbreviated $S_A(\Omega)$. We also define the normalized cross spectral density $C_{A,B}(\Omega)$ between two operators A and B

$$C_{A,B}(\Omega) \equiv \frac{S_{A,B}(\Omega)}{[S_A(\Omega)S_B(\Omega)]^{1/2}} \ . \quad (3.2)$$

The cross spectral density is in general a complex function which always satisfies $|\, C_{A,B}(\Omega) \,| \leq 1$.

To calculate the intensity-noise spectra and the correlation spectra between the lasers we use (2.5) and (2.8). We also need the correlation functions of F_N and F_p. They are [10]

$$\langle F_N(t)F_N(u)\rangle = 2\left(\frac{I_p}{e} + \frac{N_0}{\tau_{sp}} + \frac{N_0}{\tau_{nr}} + \frac{\beta(N_0 + N_{tr})p_0}{\tau_{sp}} \right)\delta(t-u) \ , \quad (3.3)$$

$$\langle F_p(t)F_p(u)\rangle = 2\left(\frac{\beta N_0}{\tau_{sp}} + \frac{\beta(N_0 + N_{tr})p_0}{\tau_{sp}} + \gamma_i p_0 \right)\delta(t-u) \ , \quad (3.4)$$

and

$$\langle F_m(t)F_m(u)\rangle = 2\gamma_m p_0 \delta(t-u) \ . \quad (3.5)$$

Due to the fact that the current I_{net} in the photon circuit stems from the current through the resistor R_{gen} and the negative conductance G_{abs} in the free-carrier circuit, there exists a cross correlation between $I_{abs} + I_{gen}$ and i_{net}, and hence between F_N and F_p. It can be expressed

$$\langle F_N(t)F_p(u)\rangle = \langle F_p(t)F_N(u)\rangle = -2\left(\frac{\beta N_0}{\tau_{sp}} + \frac{\beta(N_0 + N_{tr})p_0}{\tau_{sp}} \right)\delta(t-u) \ . \quad (3.6)$$

Using (2.5), (2.8), (3.1), (3.3)–(3.6), and Wiener-Khintchin's theorem, the final expression for the intensity-noise spectral density is

$$S_{\Delta I}(\Omega) = 2\frac{\gamma_m^2}{|\, D_2 \,|^2}\left(\frac{I_p}{e} + \frac{N_0(1-\beta)}{\tau_{sp}} + \frac{N_0}{\tau_{nr}} + |\, D_1 \,|^2 \gamma_i p_0 + |\, D_1 - D_2/\gamma_m \,|^2 \gamma_m p_0 + |\, 1 - D_1 \,|^2 \frac{\beta[N_0(p_0+1) + N_{tr}p_0]}{\tau_{sp}} \right) . \quad (3.7)$$

The six distinct noise contributions on the right-hand side (RHS) of this equation stem from (left to right) injection-current noise, spontaneous emission into nonlasing modes, nonradiative recombination, cavity internal losses, mirror losses, and finally, stimulated emission, absorption, and spontaneous emission into the lasing mode. In the low-frequency region, $\Omega \ll \gamma_m + \gamma_i$, and at pumprates several times the threshold pumping, (3.7) can be substantially simplified. First it can be noted that when these two conditions are satisfied, we have $D_1 \to 1$ and $D_2 \to \gamma_m + \gamma_i$. As $D_1 \to 1$, the contribution to the spectral density from stimulated emission and absorption fluctuations will become negligible. Furthermore, if in addition, an electrical and an optical quantum efficiency is defined, it is possible to express (3.7) in a particularly simple manner. Inspecting (2.3), the electrical quantum efficiency can be expressed

$$\eta_e = \frac{\beta[N_0(p_0+1) - N_{tr}p_0]/\tau_{sp}}{N_0/\tau_{sp} + N_0/\tau_{nr} + \beta(N_0 - N_{tr})p_0/\tau_{sp}} \ , \quad (3.8)$$

and the optical quantum efficiency is

$$\eta_p = \frac{\gamma_m}{\gamma_m + \gamma_p} \ . \quad (3.9)$$

Using these relations, the low-frequency intensity-noise spectral density at high pump levels reduces to

$$S_{\Delta I} \mid_{\Omega \ll \gamma_m} = \frac{2I_p}{e} \left[\eta_0^2 + \eta_0 \eta_e (1 - \eta_0) + 1 - \eta_e \right] \ . \tag{3.10}$$

The three terms on the RHS of this equation stem from injection-current noise, internal and external cavity losses, and finally, nonradiative recombination and spontaneous emission into nonlasing modes, processes which do not contribute energy to the lasing mode.

From (3.10), several known features from laser noise theory can be derived. First, in an ideal laser, stimulated emission will become the dominant recombination process at high pumping, so that $\eta_e \rightarrow 1$. Hence, $S_{\Delta I} \mid_{\Omega \ll \gamma_m}$ can be simplified to $2I_p\eta_0/e$ (single sided spectrum). At the same time the output photon flux is given by $I_p\eta_0/e$. This means that the intensity noise is at the standard quantum limit (Poissonian photon-counting statistics). This is to be expected since a shot-noise-limited injection current was assumed. If, on the other hand, the pump noise is negligible, that is, the first term on the RHS of (3.10) can be neglected, the output intensity-noise spectral density will be $2I_p\eta_0(1-\eta_0)/e$. That is, the output intensity noise will be a factor $1-\eta_0$ below the standard quantum limit. This is the pump-noise-suppressed laser. It is well known that the noise suppression is limited by the optical quantum efficiency of the device.

If the optical quantum efficiency can be increased to unity, and the pump noise still could be made negligible, then the term $1-\eta_e$ can no longer be neglected in (3.10). The intensity-noise spectral density in this case is given by $2I_p(1-\eta_e)/e$. Since $1-\eta_e \propto 1/p_0$, the intensity noise can be very low at high pumping.

The intensity-noise reduction in this case is limited only by nonradiative recombination and spontaneous emission into nonlasing modes. In an ideal microcavity laser with $\tau_{\rm nr} \rightarrow \infty$ and $\beta = 1$ [14, 15], these detrimental effects can be overcome, so there would be no intensity-noise floor, and at frequencies near zero the light intensity (photon flux) would be perfectly regular. Examples of intensity-noise spectra for these driving conditions are given in Fig. 3. All intensity-noise spectra in this paper are normalized so that a spectral density of onehalf corresponds to the standard quantum limit. The following parameters have been assumed for the laser: $\tau_{\rm sp} = 1$ ns, $\tau_{\rm nr} = 1\ \mu$s, $\beta = 10^{-5}$, $n_{\rm tr} = 10^{18}$ cm^{-3}, active volume $V = 60\ \mu$m^3, $\gamma_i = 10^{11}$ s^{-1}, $\gamma_m = 9 \times 10^{11}$ s^{-1}.

Before concluding this subsection, a few words should be added on the role of the electrical quantum efficiency. In the proposed laser model, the nonradiative recombination has been modeled simply by a recombination constant $\tau_{\rm nr}$. If, as has been implicitly assumed above, $\tau_{\rm nr}$ is independent of the pumping level, then η_e will be a monotonous function of I_p and as the injection current is increased, $\eta_e \rightarrow 1$. However, in real, nonideal lasers it is often found that due to leakage effects, it is often difficult to get the electrical quantum efficiency above, say, 90%. Therefore we will keep the electrical quantum efficiency as a parameter in our following equations instead of replacing it with a monotonous function of the injection current.

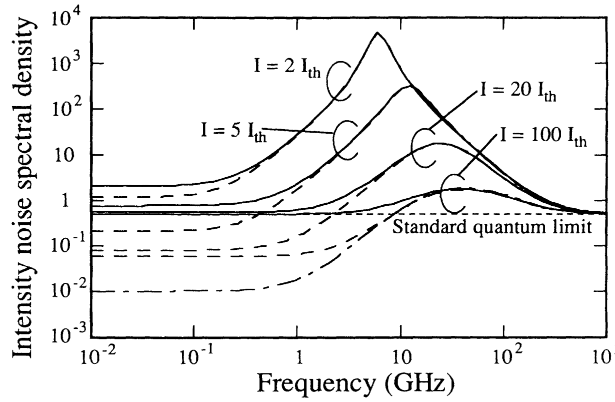

FIG. 3. The normalized intensity-noise spectral density as a function of frequency. The curves are drawn at an injection current 2, 5, 20, and 100 times the threshold current. The solid lines represent a pump current with full shot noise, the dashed lines represent a noiseless pump current. The dash-dotted line represents a laser with no pump noise and with $\gamma_i = 0$, and $\gamma_m = 10^{12}$ s^{-1} yielding $\eta_0 = 1$. The internal-photon number at this injection-current level is ≈ 15.5 million and the overall quantum efficiency is slightly below 99%.

Finally, in a recent paper Richardson and Yamamoto have claimed that splitting a current between two parallel resistors need not introduce any additional noise [16]. (In contrast, splitting a light beam into two will introduce additional noise.) If Richardson and Yamamoto's claim is correct, then the intensity-noise correlation between two lasers will be even better than is predicted below. However, this possibility will not be considered in this paper.

B. Correlation spectra

The intensity-noise correlation from two series-coupled lasers (denoted 1 and 2) can readily be obtained from (3.7) by noting that only the first noise component, the injection-current noise, is correlated in the two lasers. Assuming that the two lasers are identical, the normalized correlation spectrum becomes

$$C_{\Delta I_1, \Delta I_2}(\Omega) = \frac{I_p/e}{\dfrac{I_p}{e} + \dfrac{N_0}{\tau_{\rm sp}} + \dfrac{N_0}{\tau_{\rm nr}} + \mid D_1 \mid^2 \gamma_i p_0 + \mid D_1 - D_2/\gamma_m \mid^2 \gamma_m p_0 + \mid 1 - D_1 \mid^2 \left(\dfrac{\beta[N_0(p_0+1) + N_{\rm tr} p_0]}{\tau_{\rm sp}} \right)} \ . \tag{3.11}$$

Although it cannot be seen explicitly from this equation, the correlation is lost for frequencies higher than the inverse of the longer of the carrier lifetime and the cavity lifetime. The former goes to zero at sufficiently high pumping, so the fundamental correlation-bandwidth limit is the cavity bandwidth, which for a typical semiconductor laser is on the order of 50 GHz. In practice, however, at moderate pump levels the carrier lifetime in a laser is often on the order of 100 ps, yielding a correlation bandwidth of perhaps 10 GHz. Examples of the normalized correlation spectrum given by (3.11) are shown in Fig. 4.

It is straightforward to generalize (3.11) to the case where the lasers have different characteristics. Using η_{ei} and η_{0i}, where $i = 1, 2$, and looking only in the low-frequency limit, (3.11) can be reexpressed

$$C_{\Delta I_1,\Delta I_2}\,|_{\Omega \ll \gamma_m} = \frac{\eta_{01}\eta_{02}}{\{[\eta_{01}^2 + \eta_{01}\eta_{e1}(1-\eta_{01}) + 1 - \eta_{e1}][\eta_{02}^2 + \eta_{02}\eta_{e2}(1-\eta_{02}) + 1 - \eta_{e2}]\}^{1/2}} \,. \tag{3.12}$$

High above threshold, when η_{e1} and $\eta_{e2} \to 1$, the normalized correlation is simplified to

$$C_{\Delta I_1,\Delta I_2}\,|_{\Omega \ll \gamma_m} = \sqrt{\eta_{01}\eta_{02}} \,. \tag{3.13}$$

This result is of course expected, since under this driving condition, the only significant uncorrelated noise processes between the two lasers are the respective cavity internal losses. If the cavity internal losses are low, and consequently the optical quantum efficiencies are high, the two lasers will exhibit near-unity intensity-fluctuation correlation.

If the optical quantum efficiencies of both lasers are unity, then the correlation will be given by

$$C_{\Delta I_1,\Delta I_2}\,|_{\Omega \ll \gamma_m} = [(2-\eta_{e1})(2-\eta_{e2})]^{-1/2} \approx (\eta_{e1}+\eta_{e2})/2 \,. \tag{3.14}$$

As both η_{e1} and η_{e2} approach unity as $p_0/(1+p_0)$, the correlation will also approach unity just as rapidly.

C. Intensity-noise reduction

Measuring the intensity fluctuations of a single semiconductor laser, and feeding back the obtained signal to regulate the current generator, Fig. 5, both the current through the laser, and the intensity fluctuations from the laser, can be stabilized to fluctuation levels below the standard quantum limit. However, if one tries to extract the sub-Poissonian light from the circuit by splitting off a portion with a semitransparent beam splitter, the sub-Poissonian characteristics of the light are quickly lost, and, in fact, the light-beam split off will be super-Poissonian due to the anticorrelation imposed by the beam splitter between the intensity fluctuations of the transmitted (measured by the feedback loop) and the reflected light [17]. Since, as shown above, the low-frequency intensity noise of a high-quantum-efficiency laser is essentially a replica of the current noise in the driving circuit, a better strategy for feedback noise suppression now emerges. Using two lasers in series, the first can be used to measure the current noise in the circuit. This information can be fed back to stabilize the current fluctuations to a level below the standard quantum limit. The output intensity of the second laser will subsequently have fluctuations at a level below the standard quantum limit. In Fig. 6 an alternative strategy for noise reduction is shown. Here, the correlation information is used to reduce the signal intensity noise directly.

In practice it may not always be desirable to use the correlation information to produce a nonclassical photon state. When coding information onto a light beam by using intensity modulation, the signal-to-noise ratio is inversely proportional to the intensity noise. To increase the sensitivity of the detector readout for a given signal power, the noise in the signal beam should be minimized. This could be achieved with feedforward or feedback manipulation described above. To use the correlation infor-

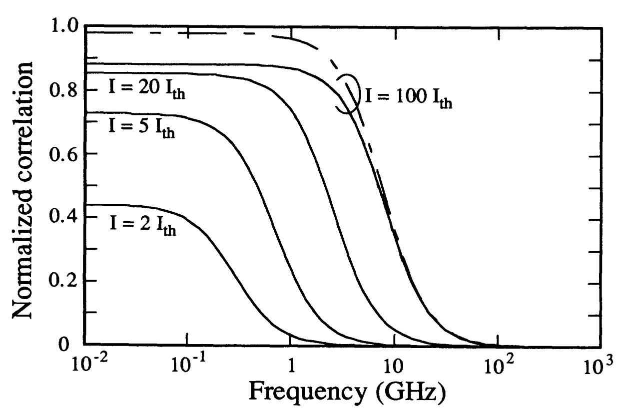

FIG. 4. The normalized intensity-noise-correlation spectral density as a function of frequency. Since the lasers have been assumed to be identical, the spectra are real. The parameters for the simulated lasers are the same as for the preceding figure. The injection current has been assumed to have full shot noise. The dash-dotted line represents a laser with $\gamma_i = 0$, and $\gamma_m = 10^{12}\ \mathrm{s}^{-1}$ so that $\eta_0 = 1$.

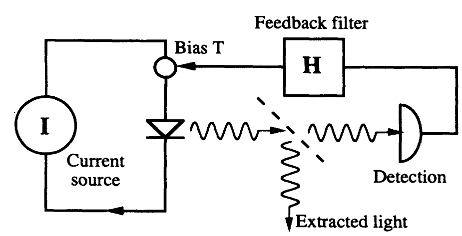

FIG. 5. An intensity-noise-suppressing feedback loop. As long as the loop is closed (no beamsplitter) the photon-counting statistics of the laser light is sub-Poissonian. However, opening the closed loop by inserting a beam splitter (dashed) while still operating the feedback loop with the transmitted light, results in the extracted light being super-Poissonian.

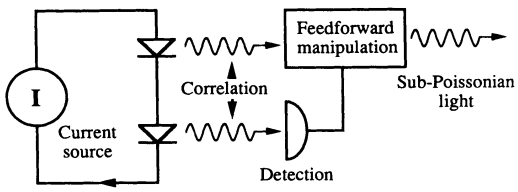

FIG. 6. An intensity-noise suppression scheme. Instead of regulating the signal beam by feedforward as shown, the information could be fed back to regulate the injection current. Thus, effectively, the second laser performs a quantum nondemolition measurement of the current in the circuit.

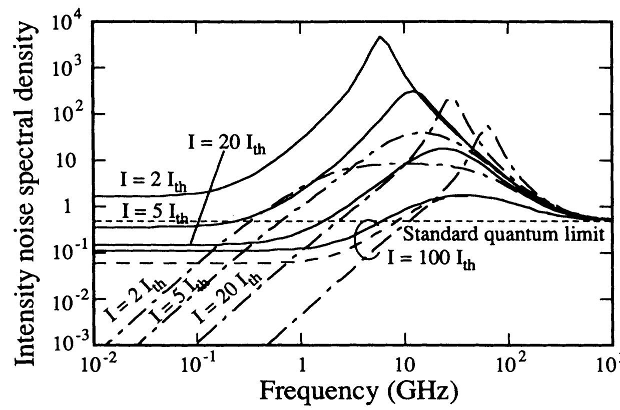

FIG. 7. Normalized intensity-noise spectral density vs frequency, for a laser whose intensity is regulated by using correlation information from a second, identical laser coupled in series. The dashed line represents the intensity noise of an identical solitary laser driven by a noiseless current source at 100 times the threshold current. The dash-dotted lines represent the intensity noise of an ideal microcavity laser with $\beta = 1$ and negligible nonradiative recombination after feedforward noise reduction using an identical laser.

mation fully to reduce the optical noise, either a high-gain feedback loop is needed, or some nonlinear feedforward manipulation of the photon beam. However, the correlation information may equally well be used to subtract the noise after detection of the (modulated but noisy) signal in some systems. This would probably be simpler in practice, since it involves only linear electrical subtraction. The requirement that the overall generation-to-detection conversion efficiency is high remains the same in both cases.

To show how much can be gained by using the correlation information, we will calculate the resulting intensity-noise spectrum assuming that ideal, wideband, noiseless feedforward control of the laser light is employed. (The "noiseless" condition requires nonlinear manipulation of the beam intensity. However, it can be shown [18] that the noise added by a linear device, e.g., a variable attenuator, is negligible if the optical power is high.) It can rather easily be demonstrated that the resulting intensity-noise spectral density from such an ideal circuit is given by [3, 19]

$$S_{\Delta I_1,\mathrm{min}}(\Omega) = S_{\Delta I_1}(\Omega)[1- \mid C_{\Delta I_1,\Delta I_2}(\Omega) \mid^2] \, , \qquad (3.15)$$

when the overall transfer function $H(\Omega)$ (the Fourier transform of the impulse response) is given by

$$H(\Omega) = \frac{S^*_{\Delta I_1,\Delta I_2}(\Omega)}{S_{\Delta I_2}(\Omega)} \, , \qquad (3.16)$$

where the asterisk denotes complex conjugation. An example of noise-reduction spectra are shown in Fig. 7.

It is easy to compute the limit for noise reduction at low frequencies using (3.10), (3.12), and (3.15). The result is

$$S_{\Delta I_1,\mathrm{min}}\,|_{\Omega \ll \gamma_m} = \frac{2I_p}{e}\left(\frac{[\eta_{01}^2 + \eta_{01}\eta_{e1}(1-\eta_{01}) + 1 - \eta_{e1}][\eta_{02}^2 + \eta_{02}\eta_{e2}(1-\eta_{02}) + 1 - \eta_{e2}] - \eta_{01}^2\eta_{02}^2}{\eta_{02}^2 + \eta_{02}\eta_{e2}(1-\eta_{02}) + 1 - \eta_{e2}}\right) \, . \qquad (3.17)$$

Assuming that the lasers are identical, so that we can suppress indices 1 and 2, and assuming that the lasers are driven several times above threshold so that $\eta_e \approx 1$, (3.17) is reduced to

$$S_{\Delta I_1,\mathrm{min}}\,|_{\Omega \ll \gamma_m} = \frac{2I_p\eta_0}{e}(1-\eta_0)(1+\eta_0) \, . \qquad (3.18)$$

This should be compared with the standard quantum-limit spectral density at this power, $2I_p\eta_0/e$, and the spectral density for a pump-noise-suppressed laser derived in Sec. III A, $2I_p\eta_0(1-\eta_0)/e$. When $\eta_0 \to 0$ the three spectral densities tend to the same value. In this case the intensity noise is dominated by the cavity internal-loss fluctuations and not by injection-current fluctuations. Thus the intensity fluctuations of the two lasers are uncorrelated and reduction of the pump noise has no effect.

On the other hand, if $\eta_0 \to 1$, the intensity noise of both the series-coupled laser scheme and the pump-noise-suppressed laser will go below the standard quantum limit. The residual noise of the series-coupled laser scheme will essentially be twice as large as that of an identical pump-noise-suppressed laser. This is easily explained by the fact that in subtracting the correlated noise of the measured laser, one simultaneously is adding the uncorrelated noise of the laser. Thus the final noise level after pump-noise subtraction is twice the residual noise of a solitary, pump-noise-suppressed laser. In Fig. 7 one can see that high above threshold pumping, the noise of the pump-noise-suppressed laser (dashed) is at a level one tenth of the standard quantum limit since the optical quantum efficiency of the modeled laser is 90%. The ultimate residual noise level of the feedforward scheme (solid) is twice as large.

If the optical quantum efficiency of both the lasers is unity, then

$$S_{\Delta I_1,\min}\,|_{\Omega\ll\gamma_m}=\frac{2I_p}{e}\left(\frac{(2-\eta_{e1})(2-\eta_{e2})-1}{2-\eta_{e2}}\right) . \tag{3.19}$$

At high pumping when $\eta_e \to 1$, and assuming that the lasers are identical, the residual noise spectral density can be written $4I_p(1-\eta_e)/e$. This is once again twice the noise level achievable with a pump-noise-suppressed laser, for the same reason as stated above. We can note that using two ideal microcavity lasers, with unity quantum efficiency, the intensity-noise correlation is perfect at frequencies near zero, irrespective of the pumping level (the residual noise after correction is proportional to Ω^2 in Fig. 7).

Before leaving the noise-reduction issue, one may ask if the residual intensity noise of a series-coupled feedback scheme cannot be smaller that roughly twice the noise level of an identical, solitary, noise-suppressed laser. The answer is that it can. The residual intensity noise after, e.g., feedforward correction can be arbitrarily close to the noise level of the pump-noise-suppressed laser if one is willing to pay the price of increasing circuit complexity. Assume that $M+1$ identical lasers are coupled in series with a current source, and the lasers are driven high above threshold. Then, the light intensities of M of the lasers are detected, and the information from each laser is fed forward to regulate the intensity of the last laser beam. Using the nonideal transfer function $H(\Omega)=-1/M$ for each laser (it can be shown that when $M\to\infty$ this is the ideal transfer function in the low-frequency limit), the residual intensity noise will be $2I_p\eta_0(1-\eta_0)(M+1)/(eM)$. This noise level is a factor $1+1/M$ above the residual noise of an identical, solitary, pump-noise-suppressed laser. The reason the noise level approaches that of a pump-noise-suppressed laser is that when measuring the uncorrelated noise processes of M lasers and taking the ensemble average, the average will be proportional to $1/M$. As M gets large, the residual noise of the laser subject to feedforward manipulation is simply its own residual intensity noise which is uncorrelated to all noise processes in the other M lasers.

IV. LIGHT-EMITTING DIODES

Most of the reasoning and the arguments for intensity noise correlation and reduction pertaining to laser diodes can also be applied to light-emitting diodes. There are only two significant differences between the two devices. First of all, the emission from an ordinary LED is emitted into many modes. This makes the exact analysis difficult, but here we will not deal with the individual modes, but only with the total detected photon flux. Second, there is very little optical feedback (ideally none) in a LED, so stimulated emission and absorption processes can be neglected. The fact that stimulated emission is negligible in a LED unfortunately means that contrary to an ideal laser, where $\eta_e \to 1$ as the injection current is increased above the threshold, η_e in a LED is almost independent of the injection current. It is simply given by the ratio

$$\eta_e=\frac{1/\tau_{\rm sp}}{1/\tau_{\rm sp}+1/\tau_{\rm nr}} , \tag{4.1}$$

between the spontaneous-emission lifetime and the nonradiative lifetime, both of which remain relatively constant with increasing injection current. Of course, the intensity correlation is degraded by a nonunity η_e.

We will start the analysis with the rate equation for the free carriers. Neglecting the stimulated emission and absorption terms, Eq. (2.3) reads

$$\frac{dN}{dt}=\frac{I_p}{e}-\frac{N}{\tau_{\rm sp}}-\frac{N}{\tau_{\rm nr}}+F_N . \tag{4.2}$$

The steady-state solution to this equation is trivially found to be $N_0=I_p/[e(1/\tau_{\rm sp}+1/\tau_{\rm nr})]$. The correlation function for F_N in this case (when the stimulated processes and their corresponding noise sources are absent) can be written

$$\langle F_N(t)F_N(u)\rangle=2\left(\frac{I_p}{e}+\frac{N_0}{\tau_{\rm sp}}+\frac{N_0}{\tau_{\rm nr}}\right)\delta(t-u) . \tag{4.3}$$

We will not write any corresponding rate equations for the photons. The reason is that in order to have a good intensity-noise-correlation between two LED's, the overall detection efficiency of the emitted photons must be high. This effectively means that the photodetector must measure the added power of many modes, each one weakly coupled to every other via (4.2). Such a multimode analysis goes beyond the scope of this paper. However, it is well known that the photon-counting statistics of each of the individual modes satisfies a thermal (exponential) distribution. It is also known that the photon-counting statistics of a superposition of these modes approaches the statistics of the injection-current noise when the overall generation-to-detection efficiency approaches unity, and when the photon-counting time is much longer than the free-carrier decay time [13]. This is a consequence of energy conservation. Thus we will assume that the low-frequency intensity noise of a LED driven by a shot-noise limited current source is

$$S_{\Delta I}\,|_{\Omega\ll 1/\tau_{\rm sp}}=2I_p\eta_e\eta_0/e , \tag{4.4}$$

where η_0 is the overall detection quantum efficiency of the generated photons. The detected intensity is $I_p\eta_e\eta_0/e$.

Inspecting (4.2) and (4.3), and taking the optical quantum efficiency into account, it can be deduced that the pump noise contributes a fraction $2I_p\eta_e^2\eta_0^2/e$ to the total output noise. The low-frequency intensity-noise-correlation spectrum of two LED's, 1 and 2, can therefore be written

$$S_{\Delta I_1,\Delta I_2}\,|_{\Omega\ll\tau_{\rm sp}}=2I_p\eta_{e1}\eta_{01}\eta_{e2}\eta_{02}/e . \tag{4.5}$$

It is clear from (4.2) that the correlation is rapidly degraded for frequencies higher than $\Omega > 1/\tau_{\rm sp}+1/\tau_{\rm nr} =$

$1/\tau_{sp}\eta_e$. For a typical LED, this bandwidth is in the order of 1 GHz.

The normalized intensity-noise-correlation spectral density is found from (4.4) and (4.5)

$$C_{\Delta I_1,\Delta I_2}\,|_{\Omega\ll\tau_{sp}} = \sqrt{\eta_{e1}\eta_{01}\eta_{e2}\eta_{02}}\ . \qquad (4.6)$$

This somewhat heuristic result seems plausible, since for a LED, which is a linear device, both the electrical and the optical quantum efficiencies should enter the correlation equations on equal footing. It should be noted that since (4.6) is partly derived from energy conservation arguments it holds only when $\eta_{e1}\eta_{01} \approx \eta_{e2}\eta_{02} \approx 1$. This is not a serious limitation since only when this condition is satisfied can significant correlation be observed.

When using the information about the injection-current noise obtained from one LED and feeding the information forward to regulate the second LED's intensity, the minimum residual intensity noise of a shot-noise-driven LED can be found from (3.15), (4.4), and (4.6). The result is

$$S_{\Delta I_1,\min}\,|_{\Omega\ll\gamma_m} = \frac{2I_p}{e}\eta_{e1}\eta_{01}(1-\eta_{e1}\eta_{01}\eta_{e2}\eta_{02})\ . \qquad (4.7)$$

This is to be compared with the minimum residual noise from an injection-current-noise-suppressed LED which can be obtained simply by subtracting $2I_p\eta_e^2\eta_0^2/e$ from (4.4), due to the fact that the pump noise is uncorrelated with every other noise process in the LED. The minimum noise spectral density in a noise-suppressed LED is

$$S_{\Delta I_1,\min}\,|_{\Omega\ll\gamma_m} = 2I_p\eta_e\eta_0(1-\eta_e\eta_0)/e\ . \qquad (4.8)$$

If the LED's are identical and have a high overall quantum efficiency, then (4.7) gives a residual noise level about twice that of (4.8). In analogy with the semiconductor laser, by coupling a large number of LED's in series, the residual noise of the manipulated LED can be made to come arbitrarily close to that of an equivalent, solitary, injection-current-noise-suppressed LED.

V. DISCUSSION

In the previous two sections it has been shown that the intensity-noise correlation between two semiconductor lasers, or LED's, coupled in series is sufficiently strong to produce nonclassical intensity-noise reduction. It has also been demonstrated that the residual noise after feedback or feedforward manipulation of one of the beams will always be roughly twice as large as if the laser injection-current noise was suppressed. Thus it is clear that injection-current noise suppression, easily achieved in practice by adding a large source resistance to the circuit, will always be more effective than using the correlation information for noise suppression. What should be noticed, however, is that a laser effectively monitors the current in a circuit in a nondestructive manner. This could potentially find applications.

It has also been demonstrated that the twin beams from two semiconductor lasers can have near-unity intensity correlation over a bandwidth of about 10 GHz. The fundamental bandwidth limit is almost an order of magnitude higher. Even LED's can have high correlation over a bandwidth of roughly 1 GHz. This is in sharp contrast to optical parametric oscillators which up to date have been the most commonly used generators of correlated twin beams. Due to weak optical nonlinearities in crystals, oscillators used for correlated twin beam generation have bandwidths only of the order of 20 MHz. With monolithic ring cavities, the bandwidth may increase to a few GHz [20], still not quite as wideband as semiconductor-lasers cold-cavity bandwidths. If one is willing to use pulsed light, then the bandwidth of optical parametric amplifiers is comparable with that of lasers, but the available mean power is rather small.

The laser twin beam generation scheme has another advantage over frequency down-conversion generation. In the latter, the sum of the signal and idler optical frequencies must always equal the pump frequency. With lasers, one is free from such constraints. One can use any two wavelengths where high-quantum-efficiency lasers and photodetectors are available. In principle, most of the visible spectrum and some of the near-infrared spectrum can be used. In addition, the semiconductor laser is monolithic and mechanically stable. It does not require a powerful and expensive pump laser and it is easy to handle. Thus, in many respects it looks more attractive to use for correlated twin beam generation then any frequency down-conversion device.

VI. CONCLUSION

It has been demonstrated that two lasers coupled in series with a shot-noise-limited current source have strong intensity-noise correlation. The correlation is limited only by the overall generation-to-detection quantum efficiency of the devices. At injection currents that are a few times above threshold, most of the injected free carriers are converted to photons due to the rapid stimulated-emission process. In this regime, the correlation is limited by the laser internal losses and the detection quantum efficiency only. If the latter approaches unity, then a nonclassical state with sub-Poissonian photon-counting statistics can be generated by measuring the intensity fluctuations of one of the lasers and feeding back, or forward, the information to manipulate the intensity of the other laser. The residual noise after feedback or feedforward manipulation is roughly two times larger than the noise of the laser if it were fed by an injection-current-noise-suppressed source. The noise-reduction bandwidth is the same in the two cases, fundamentally it is of the order of 50 GHz for a typical semiconductor laser. In practice it is from a few to 10 GHz. It has also been shown that the intensity noise of two LED's coupled in series is also correlated. Unfortunately it will be more difficult to realize a near-unity quantum efficiency and hence a high correlation with such devices. In addition, the correlation bandwidth (and thus potential noise-reduction bandwidth) is more limited, it is of the order of 1 GHz.

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