Schrödinger-cat states in the resonant Jaynes-Cummings model: Collapse and revival of oscillations of the photon-number distribution

V. Bužek,* H. Moya-Cessa,† and P. L. Knight
Optics Section, The Blackett Laboratory, Imperial College, London SW7 2BZ, England

S. J. D. Phoenix
BT Laboratories, Martlesham Heath, Ipswich IP5 7RE, England
(Received 16 December 1991)

The Jaynes-Cummings model of optical resonance describes the simplest fully quantized interaction between two quantum systems of different nature: a two-level atom (fermionic system) and a quantized field mode (bosonic system). This interaction leads to extreme quantum entanglement of the atom and field. However, the model also predicts that, at precisely half of the revival time, the atom and field become asymptotically disentangled. This disentanglement becomes more exact as the coherent-state amplitude increases. In this paper we investigate the nature of the pure-field-state superposition generated at such times. We show that this superposition is of distinguishable states of the field with the same amplitude but opposite phase. Interference between these components leads to nonclassical oscillations in photon-number distributions and squeezing in quadratures of the field. The Schrödinger-cat states of the field are highly transient, and depend very sensitively on the interaction time, the initial intensity of the field, and the atom-field detuning.

PACS number(s): 42.50.−p, 03.65.Bz, 42.52.+x

I. INTRODUCTION

In the last few years considerable attention has been devoted to the generation and detection of nonclassical states of light. In particular, squeezed [1] and sub-Poissonian [2] states of light have been produced in various laboratories. It has recently been shown that squeezing (that is, the reduction of quadrature fluctuations below the vacuum limit) has its origin in quantum interference between various components of quantum-superposition states [3,4]. For instance, squeezed-vacuum or squeezed coherent states can be expressed as one-dimensional continuous superpositions of coherent states with suitably chosen distribution functions [4]. It has also been shown that in addition to squeezing, higher-order squeezing [5], sub-Poissonian photon statistics [6], and oscillations of the photon-number distribution [3,7] emerge from a superposition of coherent states.

There have been several proposals recently for the generation of optical superposition states in various nonlinear processes [8]; for example, in quantum-nondemolition or back-action-evading measurements [9] and in micromaser experiments [10]. One of the simplest quantum nonlinear systems in quantum optics is that of the single-mode quantized electromagnetic field interacting with a two-level atom. This system is described by the Jaynes-Cummings model (JCM) [11] of optical resonance and exhibits many features that are purely quantum mechanical in origin. The most famous of these is the revival of the atomic inversion [12], which has recently been observed experimentally by Walther and coworkers [13].

If the atom and the field in the JCM are initially in a disentangled pure state, that is, the initial state of the atom-field (A-F) system can be described by the state vector

$$|\Psi_{A\text{-}F}(t=0)\rangle = |\Psi_A(t=0)\rangle \otimes |\Psi_F(t=0)\rangle , \qquad (1.1)$$

then the quantum dynamics leads to $t>0$ to a very strong, approximately maximal, entanglement between the atom and the field. In other words, if the entropy of the field subsystem is initially zero (the field is in a pure state) then at $t>0$ it is nonzero; that is, the field is in the statistical mixture [14] (obviously the total entropy of the atom-field system is equal to zero for any time $t>0$). In spite of this increase of the entropy because of the interaction of the field with the atom, there are some moments in the time evolution during which the entropy is dramatically reduced. Recently, Phoenix and Knight [14] and Gea-Banacloche [15] have shown that if the field is initially prepared in a coherent state then it evolves into an almost pure state again at half of the atomic-inversion revival time.

It is the main purpose of this paper to study in detail states of the cavity field at half the revival time. As we will demonstrate, these approximately pure states are in fact "macroscopic" superposition states composed of two states of light having the same amplitude but opposite phase. Gea-Banacloche [15] has recently derived an approximate analytical expression for the solutions of the JCM with the field initially prepared in a coherent state with a large number of photons (high-intensity field). In the course of the paper we will show to what extent the solutions proposed by Gea-Banacloche [15] are applicable for the description of the atom and the field in the JCM.

In Sec. II we briefly describe the Jaynes-Cummings model and give its exact solution. In Sec. III we summarize the approximate analytical solution by Gea-Banacloche [15]. Using the field entropy we estimate the time interval during which the approximate solution is applicable. In Sec. IV we discuss the statistical properties of the cavity field at one half of the revival time. Section V is devoted to conclusions.

II. QUANTUM DYNAMICS OF THE JCM

The Jaynes-Cummings Hamiltonian describing an interaction of a two-level atom with a single-mode cavity field in the dipole and the rotating-wave approximation is given by (we adopt $\hbar=1$) [12]

$$\hat{H}=\omega_F(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})+\tfrac{1}{2}\omega_A\hat{\sigma}_3+\lambda(\hat{a}^\dagger\hat{\sigma}_-+\hat{\sigma}_+\hat{a}) , \quad (2.1)$$

where ω_A is the atomic transition frequency; ω_F is the frequency of the cavity field; λ is the atom-field coupling constant; $\hat{a}$ and $\hat{a}^\dagger$ are the field annihilation and creation operators, respectively ($[\hat{a},\hat{a}^\dagger]=1$); $\hat{\sigma}_3$ is the atomic-inversion operator; and $\hat{\sigma}_\pm$ are the atomic "spin-flip" operators ($[\hat{\sigma}_+,\hat{\sigma}_-]=\hat{\sigma}_3$ and $[\hat{\sigma}_3,\hat{\sigma}_\pm]=\pm2\hat{\sigma}_\pm$). In the interaction picture the Hamiltonian (2.1) takes the form

$$\hat{H}_I=\tfrac{1}{2}\Delta\hat{\sigma}_3+\lambda(\hat{a}^\dagger\hat{\sigma}_-+\hat{\sigma}_+\hat{a}) , \quad (2.2)$$

where Δ is the detuning ($\Delta=\omega_A-\omega_F$). In the two-dimensional atomic basis the interaction Hamiltonian (2.2) is

$$\hat{H}_I=\begin{bmatrix}\Delta/2 & \lambda\hat{a}\\ \lambda\hat{a}^\dagger & -\Delta/2\end{bmatrix} , \quad (2.3)$$

and the corresponding evolution operator $\hat{U}(t)=\exp(-i\hat{H}_I t)$ can be written in the form [16]

$$\hat{U}(t)=\begin{bmatrix}\hat{U}_{11}(t) & \hat{U}_{12}(t)\\ \hat{U}_{21}(t) & \hat{U}_{22}(t)\end{bmatrix} , \quad (2.4)$$

where

$$\hat{U}_{11}(t)=\cos\hat{\Omega}_{n+1}t-i\frac{\Delta}{2}\frac{\sin\hat{\Omega}_{n+1}t}{\hat{\Omega}_{n+1}} , \quad (2.5a)$$

$$\hat{U}_{21}(t)=-i\lambda\hat{a}^\dagger\frac{\sin\hat{\Omega}_{n+1}t}{\hat{\Omega}_{n+1}} , \quad (2.5b)$$

$$\hat{U}_{12}(t)=-i\lambda\hat{a}\frac{\sin\hat{\Omega}_n t}{\hat{\Omega}_n} , \quad (2.5c)$$

$$\hat{U}_{22}(t)=\cos\hat{\Omega}_n t+i\frac{\Delta}{2}\frac{\sin\hat{\Omega}_n t}{\hat{\Omega}_n} , \quad (2.5d)$$

and

$$\hat{\Omega}_n=\left[\frac{\Delta^2}{4}+\lambda^2\hat{n}\right]^{1/2} , \quad \hat{\Omega}_{n+1}=\left[\frac{\Delta^2}{4}+\lambda^2(\hat{n}+1)\right]^{1/2} . \quad (2.6)$$

If we assume that at $t=0$ the atom-field system is in a pure state while the atom and the field are uncorrelated, then the initial state of the system can be described by the factorized density matrix $\hat{\rho}_{A\text{-}F}(t=0)\equiv\hat{\rho}_{A\text{-}F}(0)$,

$$\hat{\rho}_{A\text{-}F}(0)=\hat{\rho}_A(0)\otimes\hat{\rho}_F(0) , \quad (2.7)$$

where $\hat{\rho}_F$ and $\hat{\rho}_A$ are the density matrices of the field and atomic subsystems, respectively. In the case when the atom is initially in the excited state $|e\rangle$, and the field is also in the pure state $|\Psi_F(0)\rangle$, the $\hat{\rho}$ matrix (2.7) takes the form

$$\hat{\rho}_{A\text{-}F}(0)=|\Psi_F(0)\rangle\langle\Psi_F(0)|\otimes|e\rangle\langle e| . \quad (2.8)$$

The time evolution of this vector is governed by the evolution operator (2.4) and at $t>0$ we have

$$|\Psi_{A\text{-}F}(t)\rangle=\hat{U}_{11}(t)|\Psi_F(0)\rangle\otimes|e\rangle+\hat{U}_{21}(t)|\Psi_F(0)\rangle\otimes|g\rangle , \quad (2.9)$$

where $|g\rangle$ denotes the lower state of the atom. The operators $\hat{U}_{ij}(t)$ are given by Eqs. (2.5).

If the cavity field is initially prepared in the coherent state $|\alpha\rangle$,

$$|\alpha\rangle=\hat{D}(\alpha)|0\rangle=\sum_{n=0}^{\infty}e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}|n\rangle , \quad (2.10)$$

where $\hat{D}(\alpha)$ is the Glauber displacement operator $\hat{D}(\alpha)=\exp(\alpha\hat{a}^\dagger-\alpha^*\hat{a})$, and $|n\rangle$ is the Fock number state of the cavity field, then the atomic inversion $W(t)=\langle\hat{\sigma}_3\rangle$ exhibits collapses and revivals [12,13] [see Fig. 1(a)]. In the case of exact resonance the period t_R of the revivals (that is, the revival time) has been evaluated approximately by Eberly and co-workers [12] and is

$$t_R=\frac{2\pi}{\lambda}\sqrt{\bar{n}} , \quad (2.11)$$

where $\bar{n}=|\alpha|^2$ is the intensity of the coherent field.

If the atom is initially prepared in a coherent superposition of the upper and lower states, that is,

$$|\Psi_A(0)\rangle=\cos(\Theta/2)|e\rangle+e^{i\phi}\sin(\Theta/2)|g\rangle , \quad (2.12)$$

then the state vector $|\Psi_{A\text{-}F}(t)\rangle$ at $t>0$ takes the form

$$\begin{aligned}|\Psi_{A\text{-}F}(t)\rangle=&[\cos(\Theta/2)\hat{U}_{11}(t)\\&+e^{i\phi}\sin(\Theta/2)\hat{U}_{12}(t)]|\alpha\rangle\otimes|e\rangle\\&+[\cos(\Theta/2)\hat{U}_{21}(t)\\&+e^{i\phi}\sin(\Theta/2)\hat{U}_{22}(t)]|\alpha\rangle\otimes|g\rangle . \quad (2.13)\end{aligned}$$

It has recently been shown by Zaheer and Zubairy [17] that the revival effect is almost completely suppressed when at $t=0$ the atom is prepared in the atomic coherent state

$$|\Psi_A(t)\rangle=\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle) , \quad (2.14)$$

that is, $\Theta=\pi/2$ and $\phi=0$ (the amplitude of the coherent state is supposed to be real). Moreover, these authors have shown that "coherent trapping" occurs in the two-level atom, i.e., the atom and the field are almost decoupled and evolve independently, which means that one can write

$$|\Psi_{A\text{-}F}(t)\rangle \simeq |\Psi_F(t)\rangle \otimes |\Psi_A(t)\rangle \ . \tag{2.15}$$

We will turn to this point in detail when analyzing approximate solutions of the Jaynes-Cummings model proposed by Gea-Banacloche [15].

Entropy of the cavity field in the JCM

If we assume that initially the atom and the field are decoupled and both are in a pure state, then at $t>0$ because of the quantum dynamics described by the Hamiltonian (2.1), the atom-field system evolves into an entangled state. In this entangled state the atom and the field subsystems separately are in mixed states. Nevertheless, the evolution in the JCM is such that the dynamics eventually force the atomic and the field subsystems into the almost pure state at one half of the revival time (see below). Recently Phoenix and Knight [14] have shown that entropy is a very useful operational measure of the purity of the quantum state, which automatically includes all moments of the density operator. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher the entropy, the greater the entanglement.

The entropies of the atom and the field, when treated as separate system, are defined through the corresponding reduced density operators by [18]

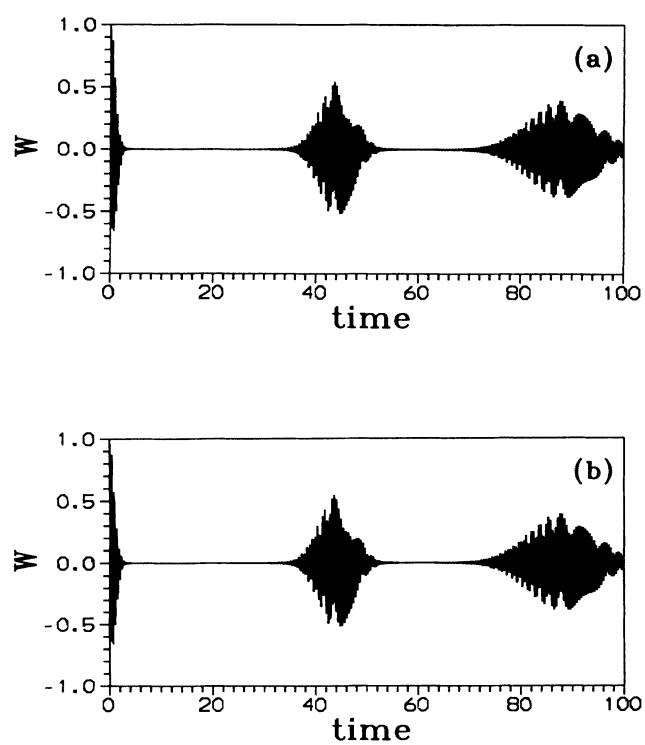

FIG. 1. Time evolution of the atomic inversion $W(t)=\langle\hat{\sigma}_3\rangle$ of the atom initially prepared in the excited state and the field in the coherent state ($\bar{n}=49$). The revival of the atomic inversion at the revival time $t_R=2\pi\sqrt{\bar{n}}/\lambda$ is clearly seen. (a) corresponds to the exact solution, while (b) is derived from the approximate solution given by Gea-Banacloche.

$$S_{A(F)}=-\mathrm{Tr}_{A(F)}\{\hat{\rho}_{A(F)}\ln(\hat{\rho}_{A(F)})\} \ , \tag{2.16}$$

where the reduced density operators $\hat{\rho}_{A(F)}$ are

$$\hat{\rho}_{A(F)}=\mathrm{Tr}_{F(A)}\{\hat{\rho}\} \ , \tag{2.17}$$

and we have used the subscript A (F) to denote the atom (field). We should note here that from the theorem of Araki and Lieb [19] it follows that if the atom-field system is initially in a pure state (that is, the total entropy of the system is equal to zero), then at $t>0$ the entropies of the field and the atomic subsystems are precisely equal (for details see [14]).

Following the work by Phoenix and Knight [14], we can express the field (atomic) entropy S_F (S_A) in terms of the eigenvalues $\pi_{1,2}$ of the reduced field (atomic) density operator

$$S_F=-\pi_1\ln\pi_1-\pi_2\ln\pi_2 \ . \tag{2.18}$$

The explicit expressions for π_i's are given in Appendix A. In Fig. 2 we plot the field entropy for various values of detuning Δ. From these figures we can conclude the following: (i) the maximum entropy of the field subsystem that is achieved during the time evolution is inversely proportional to the detuning, that is, the bigger the detuning, the smaller the maximum entropy; (ii) the first maximum of the field entropy at $t>0$ is achieved at the collapse time, that is, when $W(t)$ reaches its steady value, which is equal to zero; (iii) at one-half of the revival time, that is, at $t_0=t_R/2$, where the revival time of the atomic inversion in the case when the atom and the field are not in resonance can be estimated as

$$t_R \simeq \frac{\pi}{[\Delta^2/4+\lambda^2(\bar{n}+1)]^{1/2}-[\Delta^2/4+\lambda^2\bar{n}]^{1/2}} \ , \tag{2.19}$$

the entropy reaches its local minimum; (iv) at exact resonance ($\Delta=0$), this minimum is very pronounced and the entropy is almost equal to zero, which means that the field is almost in a pure state (see also Sec. IV); and (v) with the increase of the detuning, the difference between the maximum entropy and the entropy at one-half of the revival time is less pronounced.

From the above we can conclude that the resonant quantum dynamics leads to a strong entanglement between the atom and the field during the first stages of the time evolution. Subsequently, it also leads to a very strong disentanglement at one half of the revival time.

This effect can be explained as a consequence of quantum interference between two parts of the Q function defined as $Q(\alpha)=\langle\alpha|\hat{\rho}_F|\alpha\rangle$ that evolve from the Q function of the initial coherent state (for details on bifurcation of the Q function, see the recent papers by Risken and co-workers [20]). Namely, in the case of exact resonance, the Q function splits because of the interaction of the field with the atom into two identical components that interfere with each other and eventually lead to an increase in the field entropy at $t>0$. This quantum interference leads to suppression of the entropy at $t=t_R/2$. On the other hand, in the off-resonant case, the atom and the field are not as strongly coupled as in the resonant case, so the entanglement between the atomic and the field subsystems is not as strong, which means that the maximum entropy is smaller than in the resonant case. This effect is also seen in the time evolution of the Q function, which for $\Delta \neq 0$ splits at $t > 0$ into two unequal pieces that are able to interfere much less effectively than in the case of exact resonance. From here it follows that the maximum entropy is smaller, but simultaneously the suppression of the entropy at half the revival time is not as pronounced as in the case when $\Delta = 0$. It should be noted at this point that the disentanglement at one half of the revival time is less and less effective as the detuning increases. We should also mention that the entropy at the *revival* time significantly decreases with an increase of the detuning and for large detunings the system is obviously purest not at half the revival time but at the full revival time. This is due to the fact that in the far-off-resonance limit the revival of the initial state of the system at $t = t_R$ is more complete, as can be seen from the evolution of the Q function in Fig. 3(c), which shows a nearly perfect reconstruction of the initial coherent-state contours of the Q function at the revival time. This should be contrasted with the rather more elliptical shape of the Q function for the resonant case at t_R [see Fig. 3(a)]. Obviously, in the limit $\Delta \rightarrow \infty$ there will be effectively no interaction between the atom and the field, which will be in the pure state for any time $t > 0$. The Q function will simply rotate in the phase space without splitting into components.

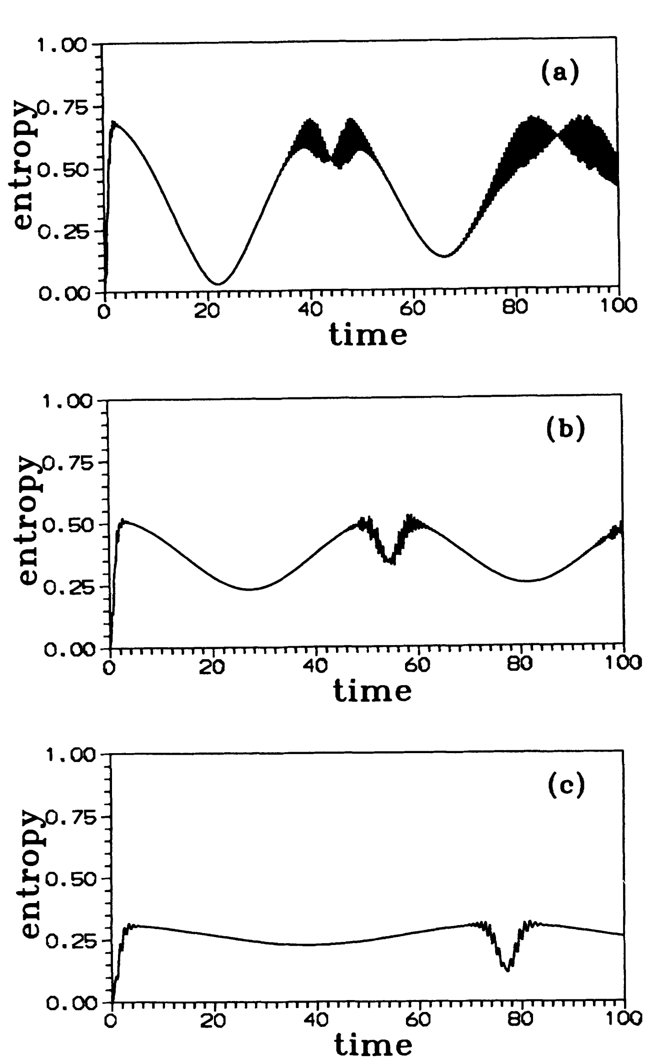

FIG. 2. Time evolution of the field entropy $S_F(t)$ computed from the exact solution (the initial condition of the atom-field system is the same as in Fig. 1). Each figure corresponds to a different value of detuning: (a) $\Delta = 0$, (b) $\Delta/\lambda = 10$, and (c) $\Delta/\lambda = 20$.

III. APPROXIMATE SOLUTION OF THE JCM FOR LARGE INITIAL FIELD

Recently, Gea-Banacloche [15] proposed a very elegant approximate analytical solution of the JCM with the cavity field initially in a coherent state with a large average photon number. In particular, he has shown that, if the atom is initially in the superposition state $|\pm\rangle$,

$$|\pm\rangle = \frac{1}{\sqrt{2}}(e^{i\phi}|e\rangle \pm |g\rangle) , \tag{3.1}$$

and the field is in the coherent state $|\alpha\rangle$ ($\alpha = |\alpha| e^{i\phi}$), then the state vector of the atom-field system at $t > 0$, in the high-intensity limit (i.e., $\bar{n} = |\alpha|^2 \rightarrow \infty$), can be expressed as

$$|\Psi_{A\text{-}F}^{(\pm)}(t)\rangle = |\Psi_F^{(\pm)}(t)\rangle \otimes |\Psi_A^{(\pm)}(t)\rangle , \tag{3.2}$$

where

$$|\Psi_A^{(\pm)}(t)\rangle = \frac{1}{\sqrt{2}}\left[\exp\left[i\phi \mp i\frac{\lambda t}{2\sqrt{\bar{n}}}\right]|e\rangle \pm |g\rangle\right] , \tag{3.3a}$$

and

$$\begin{aligned} |\Psi_F^{(\pm)}(t)\rangle &= \exp(\mp i\lambda t\sqrt{\hat{n}}\,)|\alpha\rangle \\ &= e^{-\bar{n}/2}\sum_{n=0}^{\infty}\frac{\bar{n}^{\,n/2}}{\sqrt{n!}}\exp[it(n\phi \mp \lambda\sqrt{n}\,)]|n\rangle . \end{aligned} \tag{3.3b}$$

From Eq. (3.2) it follows that if the atom is initially prepared in the superposition state (3.1), then at $t > 0$ the atom and the field are completely disentangled in this limit; that is, the field entropy is equal to zero.

It has also been shown by Gea-Banacloche [15] that because of the fact that the states $|+\rangle$ and $|-\rangle$ given by Eq. (3.1) form a basis set for the atom, the evolution of any other initial state can be expressed as a simple linear combination of Eq. (3.2). In particular, if the atom is initially in the upper state $|e\rangle = e^{-i\phi}(|+\rangle + |-\rangle)/\sqrt{2}$, then at $t > 0$ the atom-field state vector evolves into

$$|\Psi_{A\text{-}F}(t)\rangle = \frac{e^{-i\phi}}{\sqrt{2}}(|\Psi_{A\text{-}F}^{(+)}(t)\rangle + |\Psi_{A\text{-}F}^{(-)}(t)\rangle) , \tag{3.4a}$$

which can be written as

$$\begin{aligned} |\Psi_{A\text{-}F}(t)\rangle &= \cos\left[\lambda t\left[\sqrt{\hat{n}} + \frac{1}{2\sqrt{\bar{n}}}\right]\right]|e\rangle \otimes |\alpha\rangle \\ &\quad - ie^{-i\phi}\sin(\lambda t\sqrt{\hat{n}}\,)|g\rangle \otimes |\alpha\rangle . \end{aligned} \tag{3.4b}$$

No matter how the initial atomic state is chosen (e.g.,

$|+\rangle$ or $|-\rangle$) the evolution forces the atom at $t_0=t_R/2$ into the unique (apart from a global phase factor) attractor state $|\Psi_A(t=t_R/2)\rangle$ [14,15]

$$|\Psi_A(t=t_R/2)\rangle=\frac{1}{\sqrt{2}}(ie^{i\phi}|e\rangle-|g\rangle) .$$

If the atom is prepared either in $|+\rangle$ or $|-\rangle$, it remains in a pure state approximately throughout its evolution. On the other hand, if the atom is initially prepared in a superposition of $|+\rangle$ and $|-\rangle$, that is,

$$|\Psi_A(0)\rangle=[\cos(\theta/2)|+\rangle+\sin(\theta/2)|-\rangle] , \quad (3.5)$$

then at $t>0$ the atomic state will *not* be pure, but nevertheless at one half of the revival time it evolves into the "attractor" state $|\Psi_A(t=t_R/2)\rangle$, which is almost pure. Simultaneously the cavity field at $t=t_R/2$ also evolves into a pure state (as we saw in our previous discussion in Sec. II and Fig. 1). This pure state is described by the state vector

$$|\Psi_F(t=t_R/2)\rangle\simeq\cos(\theta/2)|\Psi_F^{(+)}(t=t_R/2)\rangle + \sin(\theta/2)|\Psi_F^{(-)}(t=t_R/2)\rangle . \quad (3.6)$$

It will be the purpose of Sec. IV to study in detail the statistical properties of the superposition state of the cavity field (3.6). But before doing so, we will investigate the extent to which the asymptotic solution of Gea-Banacloche [15] is valid; that is, we will compare the exact results from the solution for the JCM with the results obtained from the approximate solutions (3.6).

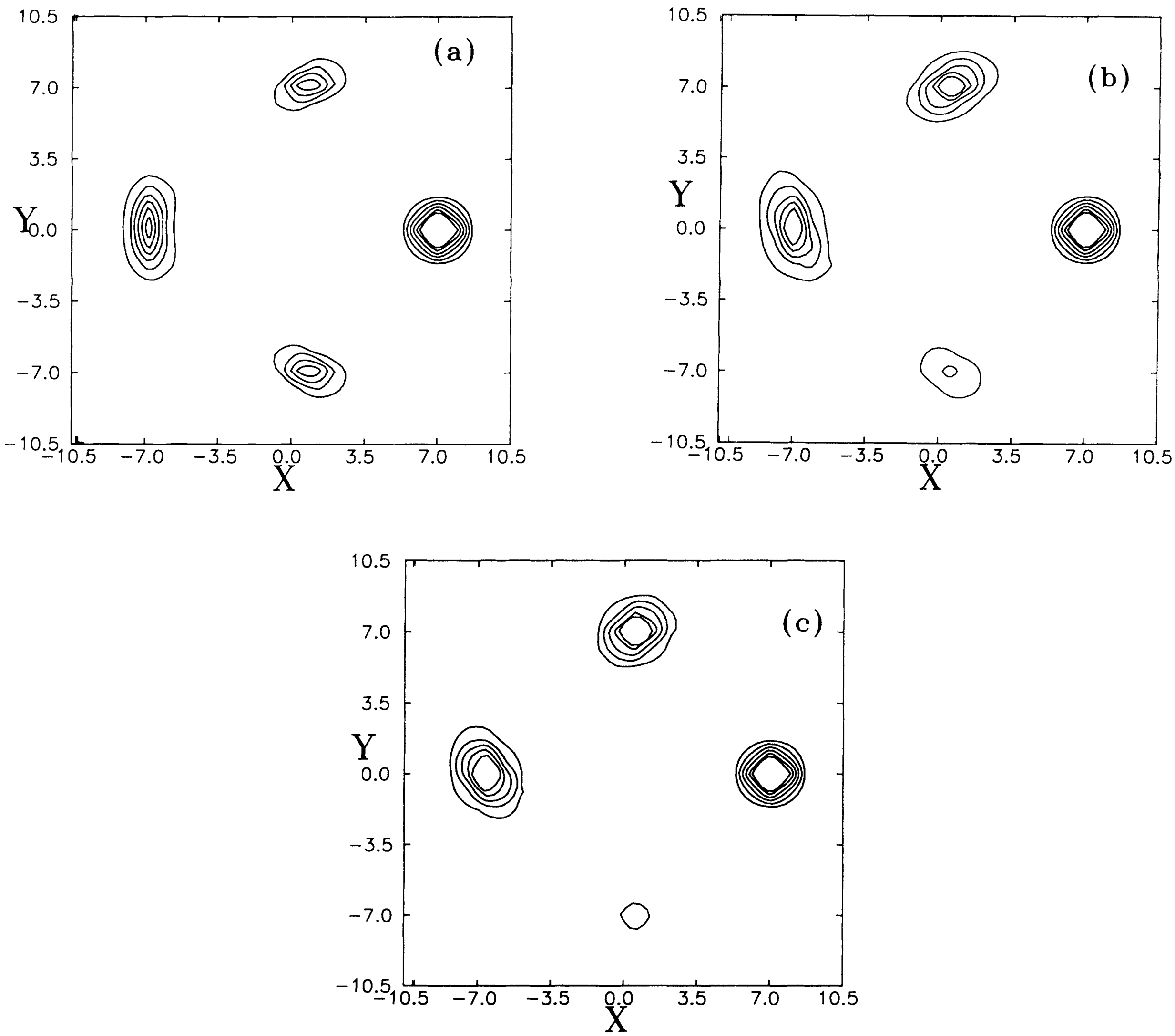

FIG. 3. Phase contours of the Q function of the cavity field at $t=0$, $t=t_R/2$, and $t=t_R$ (the initial condition of the atom-field system is the same as in Fig. 1). The Q function of the coherent state with $\alpha=7.0$ is represented by phase contours [circles centered at the point (7.0;0.0)]. At $t=t_R/2$ the Q function is composed of two parts located approximately at the points (0.0;7.0) and (0.0;−7.0). In the case of exact resonance, these two parts of the Q function are "locally" equal, but if $\Delta\neq 0$ then the Q function at $t=t_R$ is asymmetric [see (b) and (c), with $\Delta/\lambda=10$ and 20, respectively]. For nonzero detuning, the revival time t_R is given by Eq. (2.19).

A. Atomic inversion

Let us suppose that the atom is initially excited. The time evolution of the atomic inversion $W(t)=\langle\hat{\sigma}_3\rangle$ obtained from the exact solution for the JCM is plotted in Fig. 1(a), while in Fig. 1(b) we display the atomic inversion obtained from the asymptotic solution. Comparing Figs. 1(a) and 1(b), we can conclude that they are almost identical, that is, the asymptotic solution (3.4) is extremely well suited for the description of the time evolution of the atomic inversion and precisely describes both the collapse and the revival of this observable. In Appendix B we show how by using the asymptotic solution (3.4) one can easily derive an approximate analytical expression describing the collapse (decay) of the atomic-level population of the upper level.

B. Mean photon number

It is well known that the so-called excitation number $\hat{E}=\hat{\sigma}_3+\hat{n}$ is an integral of motion in the JCM, that is, $[\hat{E},\hat{H}_I]=0$, which reflects excitation-number conservation in this model within the rotating-wave approximation. From this it follows that if the atom is prepared initially in the upper level and with the field in the coherent state with intensity $\bar{n}$, then at $t>0$ the mean photon number should be

$$\bar{n}(t)=\bar{n}+1-W(t)\ . \tag{3.7}$$

On the other hand, we can easily evaluate the mean photon number using the asymptotic solution (3.4)

$$\bar{n}(t)=\sum_{n=0}^{\infty}P_n n[\cos^2\{\lambda t[\sqrt{n}+1/(2\sqrt{\bar{n}})]\}+\sin^2(\lambda t\sqrt{n})]\ . \tag{3.8}$$

In Fig. 2 we plot the parameter N, defined as

$$N=\bar{n}(t)-\bar{n}-1+W(t)\ , \tag{3.9}$$

where $\bar{n}(t)$ and $W(t)$ are defined by Eqs. (3.8) and (3.6), respectively. The parameter N oscillates around -1, which is in contradiction with the exact solution (3.7), from which it follows that in the exact JCM the parameter N is identically equal to zero. In other words, the asymptotic solution due to Gea-Banacloche [15] describes quite well the collapse-revival pattern of the mean photon number that is typical for the JCM, but it leads to a violation of the conservation of the mean excitation number. As we see from Fig. 4, one photon is missing. This can easily be seen when we evaluate the time-averaged value of the mean value of the operator $\hat{E}$, that is,

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T dt[\bar{n}(t)+W(t)]\ , \tag{3.10}$$

which is equal to $\bar{n}$ using Eqs. (3.6) and (3.8) instead of $\bar{n}+1$. On the other hand, one can say that the asymptotic solution of Gea-Banacloche [15] is valid in the limit $\bar{n}\to\infty$, so that $N/\bar{n}$ goes to zero as $\bar{n}\to\infty$ and the portion of the "missing" energy is negligible.

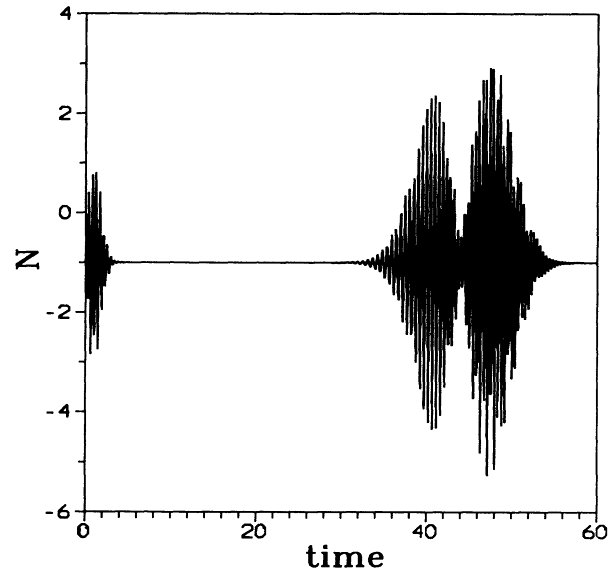

FIG. 4. Time evolution of the excitation-number deviation parameter $N(t)$ given by Eq. (3.9) ($\bar{n}=100$). The fact that $N(t)$ is not exactly equal to zero reflects the "violation" of the energy-conservation law. Nevertheless, in the high-intensity limit ($\bar{n}\gg 1$) the parameter $N(t)/\bar{n}$ is negligible.

C. Field entropy

Using the asymptotic solution (3.4) we can easily evaluate the field entropy. To do so, we first find the eigenvalues and eigenstates of the reduced field density matrix corresponding to the state (3.4) (see Appendix C), and then using Eq. (2.7) we can study the time evolution of the entropy of the field subsystem, which is plotted in Fig. 5(b). Comparing Figs. 5(a) and 5(b) describing the evolution of the entropy using the exact and asymptotic solutions of the JCM, respectively, we can conclude that qualitatively the time evolutions of the entropy, obtained from the approximate solution and the exact solution, are very similar. Nevertheless, there are some differences. Namely, at half the revival time the entropy emerging from the exact solution only approaches zero, and the degree of purity of the field at subsequent minima progressively deteriorates. On the other hand, the entropy evaluated from the approximate solution is *periodically equal to zero* at half-integer values of the revival period. That is, according to the approximate solution, the field periodically completely disentangles from the atom. From this it follows that one can use the approximate solution (3.4) only for times smaller than the revival time t_R.

D. Photon-number distribution

The photon-number distribution P_n is defined as

$$P_n=|\langle n|\Psi_F\rangle|^2\ . \tag{3.11}$$

Using the exact solution of the JCM (2.9), we find the following expression for the photon-number distribution:

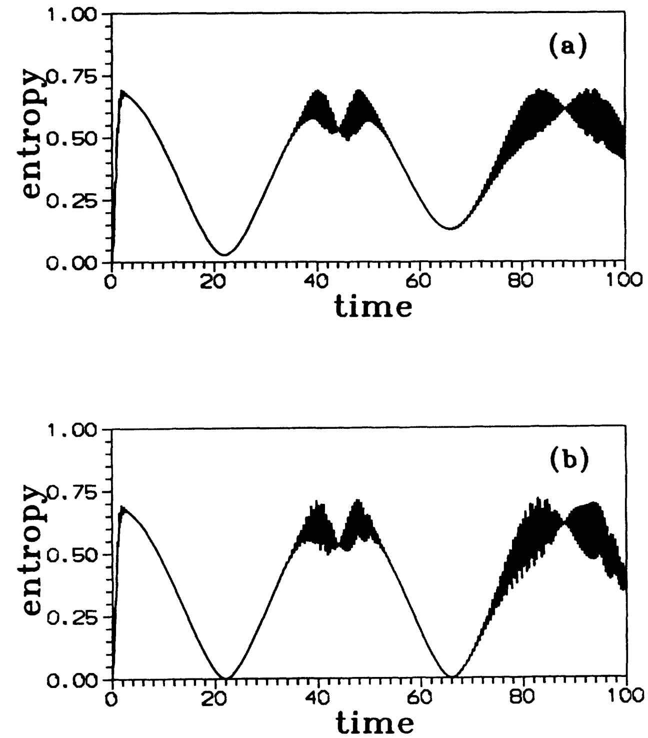

FIG. 5. Time evolution of the field entropy (the initial condition of the atom-field system is the same as in Fig. 1 and the detuning $\Delta=0$). (a) is derived from the exact solution of the JCM, while (b) is derived from the approximate solution given by Gea-Banacloche.

$$P_n(t)=P_n\left[\cos^2\Omega_{n+1}t+\frac{\Delta^2}{4}\frac{\sin^2\Omega_{n+1}t}{\Omega_{n+1}^2}\right]+P_{n-1}\lambda^2 n\frac{\sin^2\Omega_n}{\Omega_n^2}\,, \tag{3.12a}$$

while from the approximate solution (3.4b) we find that

$$P_n(t)=P_n\left\{1+\tfrac{1}{2}\cos\left[2\lambda\left[\sqrt{n}+\frac{1}{2\sqrt{\bar{n}}}\right]\right]-\frac{1}{2}\cos(2\lambda t\sqrt{n}\,)\right\}, \tag{3.12b}$$

where P_n is the initial Poissonian photon-number distribution. From Eq. (3.12b) it follows that at the revival time the approximate $P_n(t_R)$ is simply a Poissonian distribution, which periodically recovers at times $kt_R/2$. On the other hand, from the exact solution (3.12a) it follows that $P_n(t_R)$ is changed because of the atom-field interaction, i.e., it deviates from the Poissonian distribution. In fact, at the first revival $P_n(t_R)$ can be approximated by the Poissonian distribution, but at the later revival times it deviates from the Poissonian shape significantly [see Fig. 6(a)].

E. Quadrature squeezing

In his paper, Gea-Banacloche [15] mentioned that the approximate solution (3.4) is not very suitable for evaluation of correlation functions of the field operators when for some reason the leading order in $\bar{n}$ vanishes. That is, $\langle \hat{a}^2\rangle$ and $\langle \hat{a}\rangle$ will be correct to leading order in $\bar{n}$, but the difference $\langle \hat{a}^2\rangle-\langle \hat{a}\rangle^2$ may not be given correctly. To check this point in more detail, we evaluate here the time evolution of the variances of the quadrature operators, which are known to be squeezed [21] in the JCM.

We define quadrature operators of the cavity field $\hat{X}_i$ as

$$\hat{X}_1=\hat{a}+\hat{a}^\dagger,\quad \hat{X}_2=-i(\hat{a}-\hat{a}^\dagger)\,, \tag{3.13}$$

and evaluate their variances $\langle(\Delta X_i)^2\rangle=\langle X_i^2\rangle-\langle X_i\rangle^2$ at $t>0$ when at $t=0$ the atom is in the upper level and the field is in the coherent state. In Fig. 7(a) we plot the time evolution of the variance of the quadrature operator $\hat{X}_i$

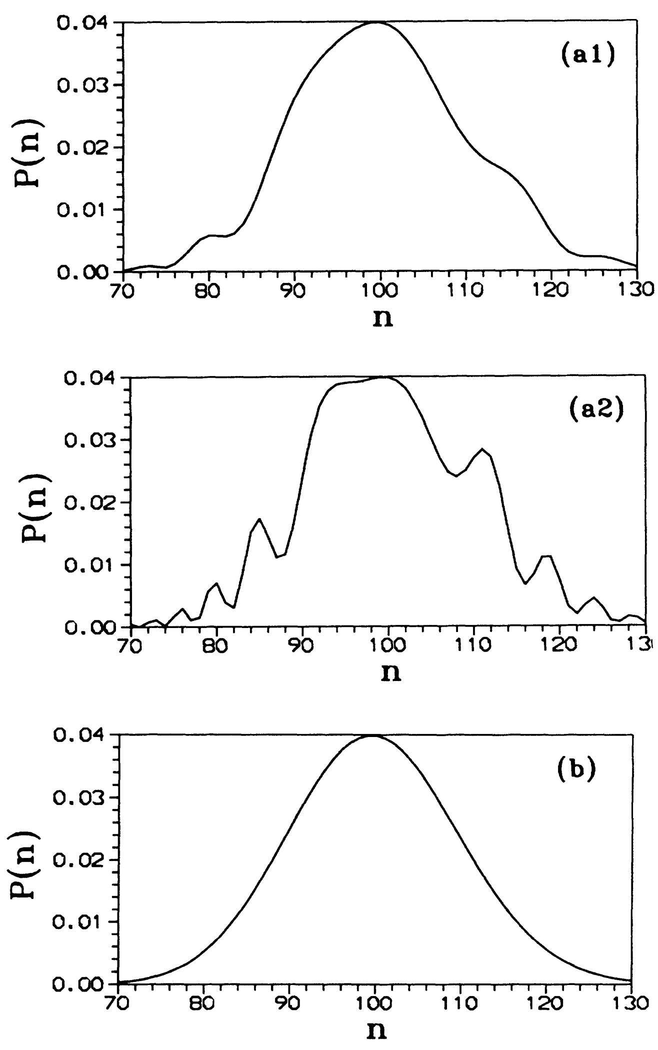

FIG. 6. Photon-number distribution of the cavity field at $t=t_R$ [see (a1)] and $t=2t_R$ [see (a2)] (the initial condition of the atom-field system is the same as in Fig. 1, but $\bar{n}=100$). (a) corresponds to the P_n of the cavity field evaluated from the exact solution of the JCM at $t=t_R$ and $t=2t_R$, respectively, while (b) corresponds to P_n obtained from the approximate solution at $t=kt_R$ ($k=0,1,2,\dots$). Using the approximate solution we find that at the revival time P_n is always Poissonian, which obviously is not the case for the exact solution [see (a1) and (a2)].

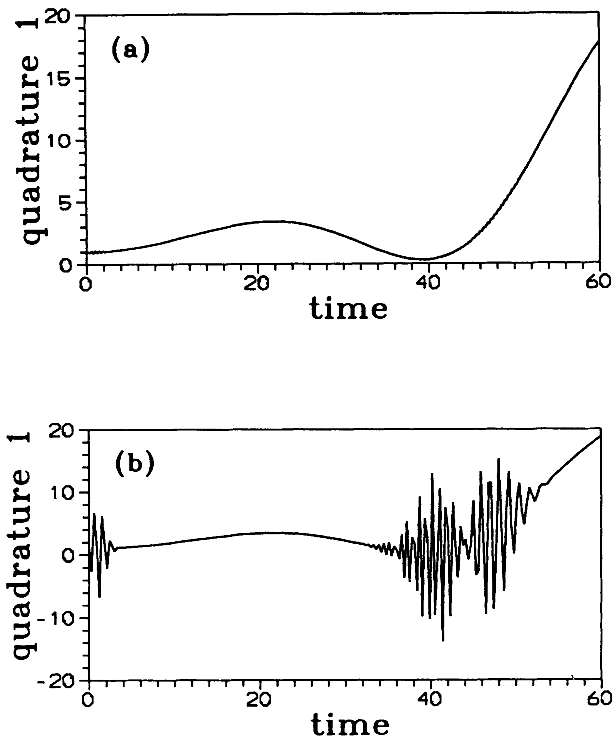

FIG. 7. Time evolution of the variance of the quadrature operator $\hat{X}_1$ (initial condition of the atom-field system is the same as in Fig. 1). From (a), which is evaluated from the exact solution, we see that the cavity field becomes squeezed because of the interaction with the atom. On the other hand, the picture obtained from the approximate solution [see (b)] is completely wrong because it leads to negative values of the variance of the quadrature operator.

using the exact solution for the JCM. From this figure it follows that fluctuations in this quadrature become squeezed [that is, $\langle(\Delta X_i)^2\rangle<1$] because of the interaction of the cavity field with a two-level atom (for details on this subject see Refs. [21]). On the other hand, in Fig. 7(b) we plot the time evolution of the same quadrature operator using the asymptotic solution (3.4). From this picture we learn that one *cannot* use the asymptotic solution to evaluate the fluctuations of quadrature operators; in particular, this solution leads to the completely incorrect result that fluctuations can have negative values.

From the above discussion it follows that the asymptotic (approximate) solution (3.4) of the JCM due to Gea-Banacloche [15] can be adopted for description of the time evolution of the atomic inversion and for mean values of correlation functions for field operators for which the leading term of order $\bar{n}$ does not vanish. Simultaneously we should stress that the approximate solution in general is valid only in the time interval $1/\lambda<t<t_R$.

In the next section we will study the statistical properties of the cavity field at one-half of the revival time $t_0=t_R/2$, when the field is almost in a pure state. For our purposes the approximate solution (3.4) will be adequate to use and will give us a very simple insight into the nature of the state of the cavity field at this time.

IV. SUPERPOSITION STATES OF THE CAVITY FIELD

As we stated earlier in Sec. II, the entropy of the field subsystem reaches its local minima at one-half of the revival time (see Fig. 2). Moreover, the higher the intensity, the smaller the value of the entropy, which means that in the high-intensity limit one can assume the field to be in a pure state. On the other hand, we know that the Q function of the cavity field at one-half of the revival time is composed of two independent components that are out of phase by π (see Fig. 3 and for further details, the work of Eiselt and Risken [20]). From this we can conclude that the state of the cavity field at one-half of the revival time should be close to a pure macroscopic superposition state (see also Ref. [10]).

First of all we will study the photon-number distribution P_n of the field state at $t_0=t_R/2$, which in the case of the exact solution is given by Eq. (3.12a) (the atom is initially in the upper state). We plot this photon-number distribution for the case of exact resonance at one-half of the revival time in Fig. 8. As can be seen from this figure, the photon-number distribution at t_0 exhibits very significant oscillations. The origin of these oscillations is in the quantum interference [3] between two components in phase space of the Q function (see Fig. 3). In the vicinity of $\bar{n}$, P_n looks very much like the photon-number distribution of the *even* or *odd* coherent state [22] (for more details on the statistical properties of these states see Ref. [7]). These states are defined as follows:

$$|\alpha\rangle_{\text{even}}=A_{\text{even}}(|\alpha\rangle+|-\alpha\rangle)\,,\qquad A_{\text{even}}^{-2}=2[1+\exp(-2|\alpha|^2)]\,, \tag{4.1a}$$

$$|\alpha\rangle_{\text{odd}}=A_{\text{odd}}(|\alpha\rangle-|-\alpha\rangle)\,,\qquad A_{\text{odd}}^{-2}=2[1-\exp(-2|\alpha|^2)]\,. \tag{4.1b}$$

To be more precise, if the initial photon number $\bar{n}$ is even (odd), then $P_n(t_0)$ around $\bar{n}$ is similar to the photon-number distribution of the odd (even) coherent state. Nevertheless, the superposition state $|\Psi_F(t_0)\rangle$ of the cavity field is *not* an even coherent state or an odd coherent state; that is, it is not a superposition of two coherent states. It is rather a superposition of two states, which, as indicated by Gea-Banacloche [15], can in the high-intensity limit be expressed as

$$|\Psi_F(t_0)\rangle\simeq|\Psi_F^{(+)}(t_0)\rangle+|\Psi_F^{(-)}(t_0)\rangle\,, \tag{4.2}$$

where

$$|\Psi_F^{(+)}(t_0)\rangle=e^{-\bar{n}/2}\sum_{n=0}^{\infty}\frac{\bar{n}^{n/2}}{\sqrt{n!}}\exp[it_0(n\phi-\lambda\sqrt{n})]|n\rangle\,, \tag{4.3a}$$

$$|\Psi_F^{(-)}(t_0)\rangle=e^{-\bar{n}/2}\sum_{n=0}^{\infty}\frac{\bar{n}^{n/2}}{\sqrt{n!}}\exp[it_0(n\phi+\lambda\sqrt{n})]|n\rangle\,, \tag{4.3b}$$

so that the photon-number distribution can approximately be described as

$$P_n(t_0) \simeq 2P_n \cos^2 \pi \sqrt{n\bar{n}} \ . \qquad (4.4)$$

From the last expression we easily find that if the initial photon number of the cavity field $\bar{n}$ is an integer, then $P_{\bar{n}}(t_0)=0$, which explains why the cavity field with an even (odd) initial number of photons evolves at one-half of the revival time to the state with similar photon-number distribution as the odd (even) coherent state.

One can qualitatively explain the oscillations of the photon-number distribution at t_0 as a consequence of quantum interference in phase space. Let us suppose the atom is initially prepared in the superposition state $|+\rangle$ and the field is in a coherent state. In this case the Q function of the cavity field does not split into two components, as is the case when the atom is initially prepared in the upper or lower level (see Fig. 3 and Ref. [15] for further discussion), but rotates in phase space without changing its shape. This is due to the fact that the atomic and the field subsystems are essentially decoupled and the field remains in a pure state. The same is true in the case when the atom is initially prepared in the state $|-\rangle$, except that the Q function rotates in the opposite direction. In both cases the photon-number distribution is Poissonian throughout the time evolution. If the atom is initially prepared in the upper (lower) state; then, following the discussion in Sec. III, the atom-field state vector at $t>0$ can be expressed as a properly normalized sum of solutions $|\Psi^{(\pm)}_{A\text{-}F}(t)\rangle$ with the atom prepared in the superposition state $|\pm\rangle$. Similarly, the Q function of the field can be expressed as a "sum" of two Q functions emerging from the state vectors $|\Psi^{(\pm)}_F(t_0)\rangle$. In fact, the quantum interference between these two component Q functions leads to a complete change of the initial photon-number distribution and the appearance of oscillations in $P_n(t_0)$ as a function of n. This is very similar to the case of even or odd coherent states, except for the fact that in this case oscillations in n are not affected by the mean photon number $\bar{n}$. For instance, for the even coherent state we find that

$$P_n^{(\text{even})} = \frac{\exp(-|\alpha|^2)}{1+\exp(-2|\alpha|^2)} \frac{2|\alpha|^{2n}}{n!} \quad \text{if } n=2m \ , \qquad (4.5)$$

irrespective of the value of $|\alpha|^2$ and $P_n^{(\text{even})}=0$ for $n=2m+1$. In the case of the superposition state (4.2), at one-half of the revival time we find that the oscillations in P_n are extremely sensitive to the initial photon number, which is clearly seen from Fig. 8 where we plot $P_n(t)$ at $t=t_R/2$ for $\bar{n}=100$, 101, and 100.5. In the first two cases the oscillations of $P_n(t_0)$ are similar to the oscillations for the odd and even coherent states, while in the case $\bar{n}=100.5$ the quantum interference leads to a totally different pattern, which, as we will see later, is very similar to that seen when the atom and the field are out of resonance or when the field is studied at a time not exactly equal to half the revival time.

In Fig. 8 we plot the photon-number distribution of the cavity field in the case of nonzero detuning. We have to remind ourselves here that in this case the purity of the field is less than it is in the case of exact resonance (see Fig. 2, where the entropy of the field is plotted for various values of detuning). At one-half of the revival time t_R, which in the case of nonzero detuning is given by Eq. (2.19), the photon-number distribution resembles a Poissonian with some kind of periodical modulation. We should stress here that the approximate solution of Gea-Banacloche [15] cannot be used for large detunings (that is, in the case $\Delta^2 \gg 4\lambda^2\bar{n}$). In the large detuning limit, the state of the field is again almost pure, and the photon statistics is Poissonian. In this case the Q function simply rotates in the phase space (there is no bifurcation of this function for $t>0$, see Fig. 9). When the field is substantially detuned from resonance, the only effect of the interaction is to cause a frequency shift in the atomic transition, or equivalently a Kerr modification of the field Hamiltonian. But the shift derived from a two-level Hamiltonian is of near-negligible magnitude. Were it to be substantially larger, we would in fact see a bifurcation of the quasiprobability and the generation of a superposition of distinct field states. This has been investigated by

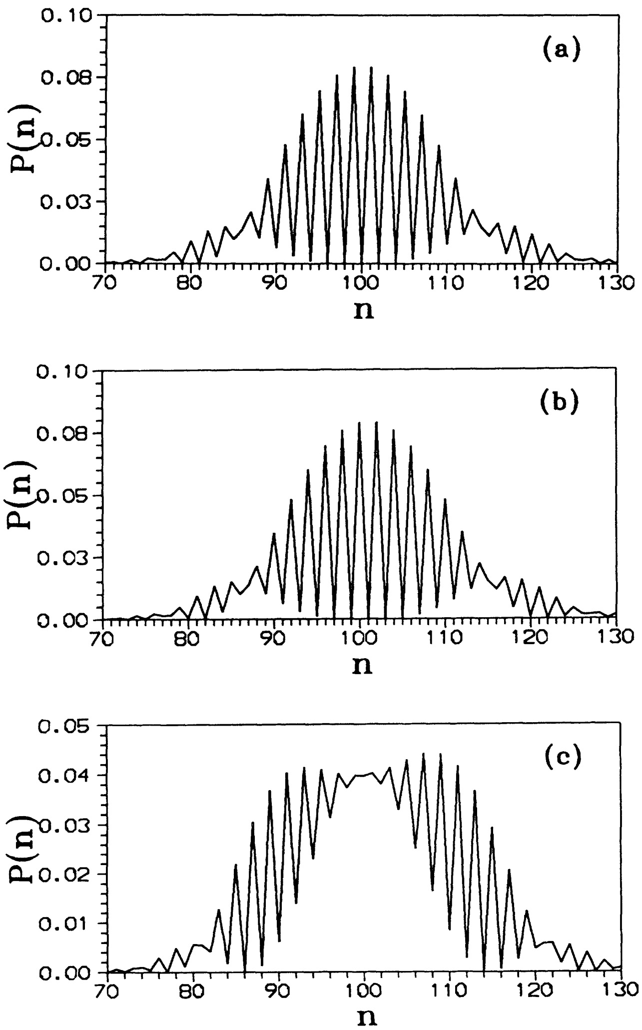

FIG. 8. Photon-number distribution of the cavity field at one-half of the revival time. The initial condition of the atom-field system is the same as in Fig. 1, but with $\bar{n}=100.0$ (a), $\bar{n}=101.0$ (b), and $\bar{n}=100.5$ (c).

Brune *et al.* [10] for atom-field interactions and by many authors for the Kerr effect [8].

V. CONCLUSIONS

We have studied the cavity field interacting with a two-level atom at one-half of the revival time. We have shown that this state is first a pure state and second a superposition state of two distinct components. We have shown that the quantum interference between the two components of this superposition leads to oscillations in the photon-number distribution (Figs. 10–12). To understand the nature of these oscillations we have utilized the approximate solution of the Jaynes-Cummings model of Gea-Banacloche [15]. Using this approximation one can express the field state at a time close to one-half of the revival time as a sum of two vectors,

$$|\Psi_F(t)\rangle \simeq |\Psi_F^{(+)}(t)\rangle + |\Psi_F^{(-)}(t)\rangle, \quad t = t_0 + \delta \tag{5.1}$$

where $|\Psi_F^{(\pm)}(t)\rangle$ can generally be expressed as

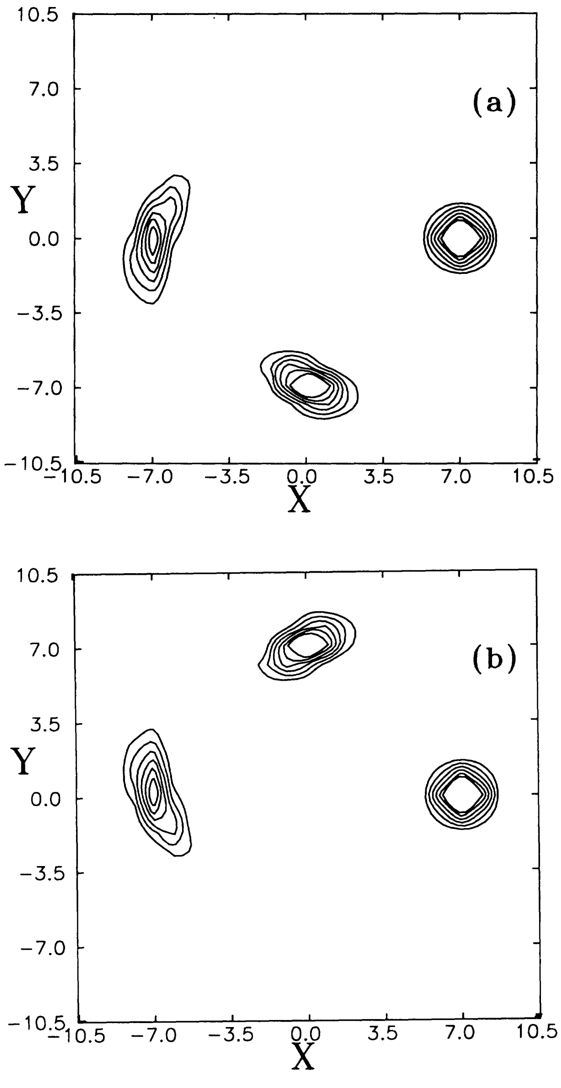

FIG. 9. Q function of the cavity field initially prepared (a) in the coherent state ($\bar{n}=49$) and the atom prepared in the superposition state $|+\rangle$, and (b) in the state $|-\rangle$. The atom and the field are in resonance. The Q function is plotted at $t=0$, $t=t_R/2$, and $t=t_R$. We see that there is no splitting of the Q function when the atom is prepared in the states $|\pm\rangle$. The Q function simply rotates in the phase space around the origin (0.0;0.0) and its shape is modified by the interaction with the atom. The direction of rotation depends on the initial state of the atom.

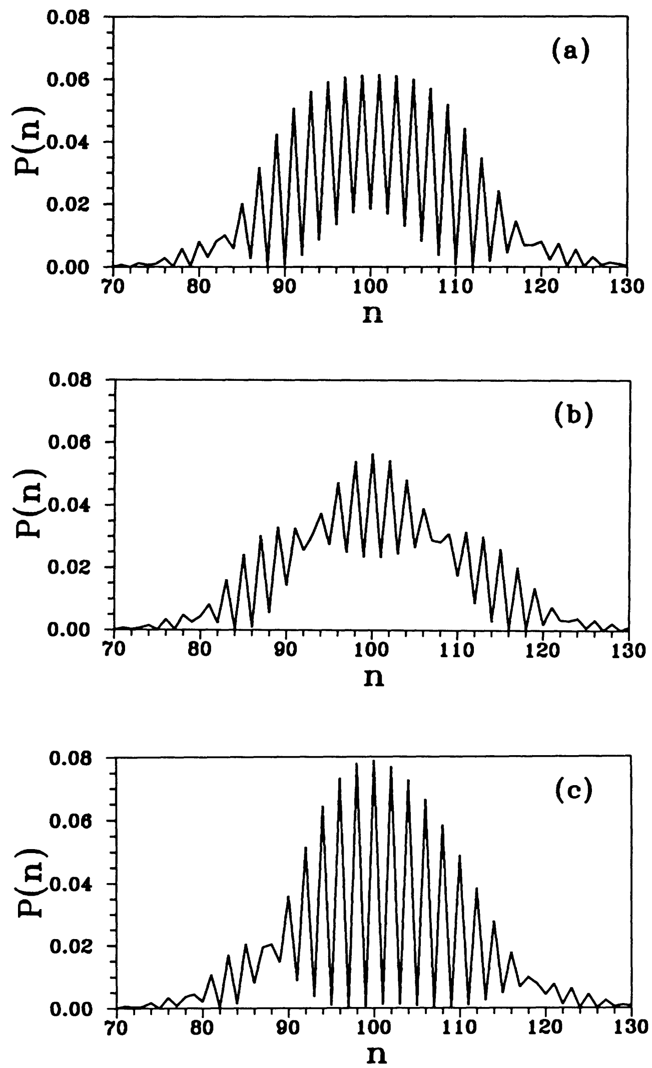

FIG. 10. Photon-number distribution of the cavity field resonantly interacting with the atom (the initial condition of the atom-field system is the same as in Fig. 1, but with $\bar{n}=100.0$). The moments at which the P_n is plotted are (a) $t_1=t_R/2+0.05/\lambda$, (b) $t_2=t_R/2+0.1/\lambda$, and (c) $t_3=t_R/2+0.48/\lambda$. Comparing these figures, we can conclude that the oscillations in P_n are strongly affected by the duration of the interaction between the atom and field. This is because the oscillations in P_n are due to the quantum interference between the two parts of the Q function and the quantum interference is very sensitive to the relative phase between these two parts. Finally, the relative phase depends on the interaction time, so the shape of the photon-number distribution P_n depends on very small variations of the interaction time.

$$|\Psi_F^{(\pm)}(t)\rangle = e^{-\bar{n}/2}\sum_{n=0}^{\infty}\frac{\bar{n}^{n/2}}{\sqrt{n!}}\exp[i\phi^{(\pm)}(\bar{n},\Delta,\delta)]|n\rangle\ , \tag{5.2}$$

and the phases $\phi^{(\pm)}$ depend on the initial photon number $\bar{n}$, on the detuning Δ, and finally on the exact interaction time; that is, on the value of δ ($|\delta| \ll t_0$). The character of the quantum interference between state vectors $|\Psi_F^{(+)}(t)\rangle$ and $|\Psi_F^{(-)}(t)\rangle$ depends on the values of these parameters. As we have shown, oscillations in the photon-number distribution are very sensitive to the particular values of $\bar{n}$, Δ, and δ; that is, they are sensitive to whether the quantum interference is constructive or destructive [3].

In this paper we have discussed the lossless Jaynes-Cummings model. Obviously, losses affect the way in which the cavity field evolves in the JCM [23]. Nevertheless, we can assume that the time of interaction between the atom and the field is short enough to treat the dynamics of the atom-field system in the framework of a lossless Jaynes-Cummings model. Then, when the superposition state of the field is created in the cavity, one can study the decay of this state due to the finite Q of the cavity. As follows from Refs. [6,7], one should expect very rapid destruction of the oscillations in the photon-number distribution of the cavity field. The question is whether these oscillations can be recovered when another atom is injected into the cavity; that is, whether in the micromaser model with losses as proposed by Meystre *et al.* [24] the oscillations in the photon-number distribution can be observed. We will discuss this elsewhere, including an analysis of how the repeated injection of atoms, each having a specific interaction time (controlled through a careful choice of atomic velocities) equal to one-half of the revival time, can lead to the creation of *stable* superpositions.

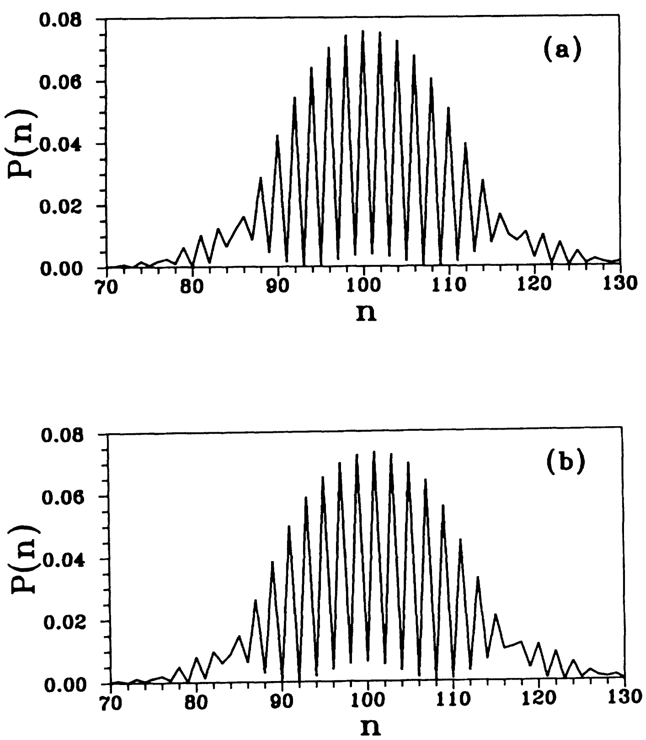

FIG. 11. Photon-number distribution of the cavity field (the initial condition of the atom-field system is the same as in Fig. 1, but with $\bar{n}=100.5$). The interaction time t is (a) $t_1=t_R/2+0.1/\lambda$ and (b) $t_2=t_R/2+0.05/\lambda$. From this figure it follows that for times slightly differing from exactly one-half of the revival time, the photon-number distribution of the field with a noninteger initial mean photon number [see Fig. 8(c)] can exhibit oscillations similar to those of the even coherent state (a) or the odd coherent state (b). This obviously is the consequence of very subtle quantum interference in phase space.

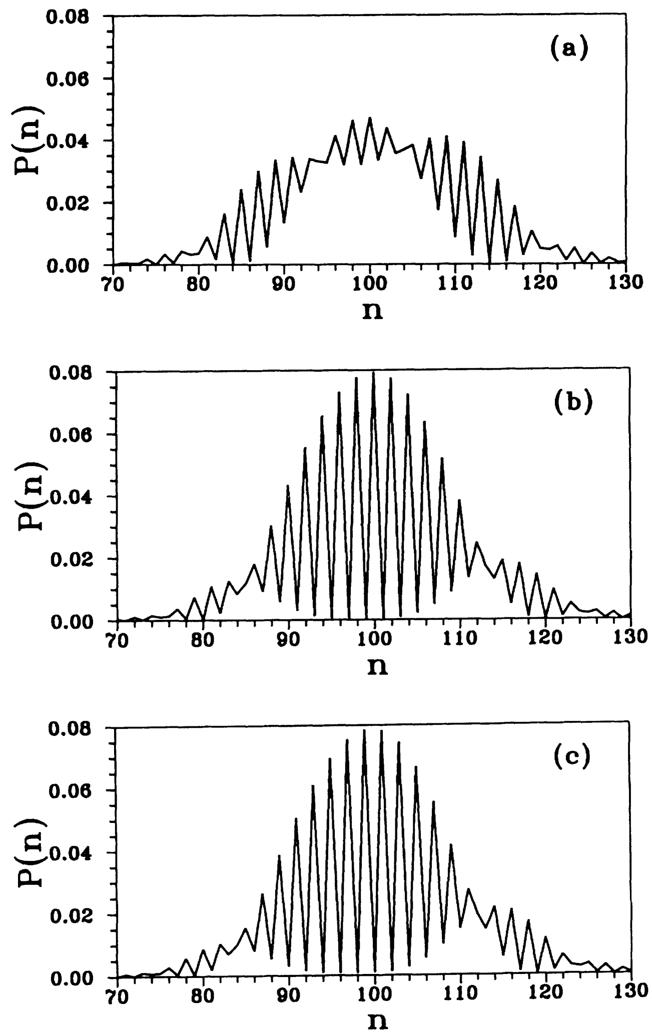

FIG. 12. The relative phase between the two components of the Q function can be changed not only by varying the interaction time, but also by changing the value of the detuning Δ. In this figure we plot P_n at $t=t_R/2=\pi\sqrt{\bar{n}}/\lambda$ for various values of the detuning. (The initial condition of the atom-field system is the same as in Fig. 1, but $\bar{n}=100$.) From (a) ($\Delta/\lambda=1.5$) we see that the oscillations of P_n [compare with Fig. 8(a)] can be destroyed by varying the detuning. As seen from (b) ($\Delta/\lambda=2.0$) and (c) ($\Delta/\lambda=\sqrt{8}$), there exist nonzero values of Δ for which the oscillations can be observed.

ACKNOWLEDGMENTS

This work was supported by the United Kingdom Science and Engineering Research Council, the Mexican Consejo Nacional de Ciencia y Tecnología (CONACyT), and the British Overseas Research Student Awards Scheme.

APPENDIX A: EIGENVALUES AND EIGENSTATES OF THE FIELD DENSITY OPERATOR

If the atom is initially prepared in the upper state $|e\rangle$ and the field in a coherent state $|\alpha\rangle$; that is, the initial density matrix $\hat{\rho}_{A\text{-}F}(0)$ is

$$\hat{\rho}_{A\text{-}F}(0)=|\alpha\rangle\langle\alpha|\otimes|e\rangle\langle e|\equiv\hat{\rho}_F(0)\otimes\hat{\rho}_A(0)\ , \tag{A1}$$

then at $t>0$ the reduced field density operator $\hat{\rho}_F(t)=\mathrm{Tr}_A\{\hat{\rho}_{A\text{-}F}(t)\}$ evolves according to

$$\hat{\rho}_F(t)=(\hat{C}-i\hat{R})\hat{\rho}_F(0)(\hat{C}+i\hat{R})+\hat{S}\hat{\rho}_F(0)\hat{S}^\dagger\ , \tag{A2}$$

where [see Eq. (2.5)]

$$\hat{C}=\cos\hat{\Omega}_{n+1}t=\mathrm{Re}[\hat{U}_{11}(t)]\ , \tag{A3a}$$

$$\hat{R}=\frac{\Delta}{2}\frac{\sin\hat{\Omega}_{n+1}t}{\hat{\Omega}_{n+1}}=-\mathrm{Im}[\hat{U}_{11}(t)]\ , \tag{A3b}$$

$$\hat{S}=\lambda\hat{a}\frac{\sin\hat{\Omega}_{n+1}t}{\hat{\Omega}_{n+1}}=i\hat{U}_{21}(t)\ . \tag{A3c}$$

The field density operator (A2) can be rewritten as

$$\hat{\rho}_F(t)=|C\rangle\langle C|+|S\rangle\langle S|\ , \tag{A4}$$

where the field states $|C\rangle$ and $|S\rangle$ are

$$|C\rangle=(\hat{C}-i\hat{R})|\alpha\rangle,\quad |S\rangle=\hat{S}|\alpha\rangle\ . \tag{A5}$$

Following Phoenix and Knight [14], we write the eigenstate of the reduced field density operator in the form

$$|\Psi\rangle=\mu|C\rangle+\nu|S\rangle\ . \tag{A6}$$

If we apply the density matrix given by Eq. (A4) to the state (A6) we find

$$\hat{\rho}_F(t)|\Psi\rangle=\left[\langle C|C\rangle+\langle C|S\rangle\frac{\nu}{\mu}\right]\mu|C\rangle+\left[\langle S|S\rangle+\langle S|C\rangle\frac{\mu}{\nu}\right]\nu|S\rangle\ , \tag{A7}$$

where

$$\langle C|C\rangle=\sum_{n=0}^{\infty}P_n\left[\cos\Omega_{n+1}t+\frac{\Delta^2}{4}\frac{\sin^2\Omega_{n+1}t}{\Omega_{n+1}^2}\right]\ , \tag{A8}$$

$$\langle S|S\rangle=\lambda^2\sum_{n=0}^{\infty}P_n\left[(n+1)\frac{\sin^2\Omega_{n+1}t}{\Omega_{n+1}^2}\right]\ , \tag{A9}$$

$$\langle C|S\rangle=\lambda\alpha^*\sum_{n=0}^{\infty}P_n\left[\cos\Omega_{n+2}t+i\frac{\Delta}{2}\frac{\sin\Omega_{n+2}t}{\Omega_{n+2}}\right]\times\frac{\sin\Omega_{n+1}t}{\Omega_{n+1}}\equiv|\langle C|S\rangle|e^{i\phi}\ , \tag{A10}$$

P_n is the Poissonian photon-number distribution $P_n=\exp(-|\alpha|^2)|\alpha|^{2n}/n!$ and the generalized Rabi frequency Ω_n is

$$\Omega_n=\left[\frac{\Delta^2}{4}+\lambda^2 n\right]^{1/2}\ . \tag{A11}$$

From Eq. (A7) we find that the relation

$$\pi=\langle C|C\rangle+\langle C|S\rangle\frac{\nu}{\mu}=\langle S|S\rangle+\langle S|C\rangle\frac{\mu}{\nu} \tag{A12}$$

should be satisfied where π is an eigenvalue of the density operator $\hat{\rho}_F(t)$. Solutions for μ and ν can be written in the form

$$\mu=\exp(\pm\Theta)e^{i\phi/2},\quad \nu=\pm\exp(\mp\Theta)e^{-i\phi/2}\ , \tag{A13}$$

so that $\mu\nu=\pm1$, where

$$\sinh\Theta=\frac{\langle C|C\rangle-\langle S|S\rangle}{2|\langle C|S\rangle|}\ . \tag{A14}$$

The eigenvalues $\pi_{1,2}$ of the field density operator $\hat{\rho}_F(t)$ are

$$\pi_{1,2}=\langle C|C\rangle\pm e^{\mp\Theta}\langle C|S\rangle\ . \tag{A15}$$

The corresponding eigenstates can be written in the form

$$|\Psi_F^{(1,2)}(t)\rangle=\frac{1}{(2\pi_{1,2}\cosh\Theta)^{1/2}}\times(e^{-(i\phi\pm\Theta)/2}|C\rangle\pm e^{-(i\phi\mp\Theta)/2}|S\rangle)\ . \tag{A16}$$

APPENDIX B: COLLAPSE OF THE ATOMIC POPULATION

The asymptotic solution (3.4) for the state vector of the atom-field system is well suited for the description of the collapse (decay) of the atomic population of the upper level during the first moments of the time evolution of the system. To do so, we notice that for high intensities $\bar{n}$ of the coherent field, the Poissonian photon-number distribution P_n can be approximated by the continuous distribution $P(n)$

$$P(n)=\frac{1}{\sqrt{2\pi\bar{n}}}\exp\left[-\frac{(n-\bar{n})^2}{2\bar{n}}\right] \tag{B1}$$

and the summation over n by integration over the corresponding continuous parameter. Using this method, we can easily evaluate the overlap between the field states $|\Psi_F^{(+)}(t)\rangle$ and $|\Psi_F^{(-)}(t)\rangle$, for which we find

$$\langle \Psi_F^{(+)}(t)|\Psi_F^{(-)}(t)\rangle = \sum_{n=0}^{\infty} P_n \exp(2i\lambda\sqrt{n}\,t) \simeq \frac{1}{\sqrt{2\pi\bar{n}}}\int_0^{\infty} dn \exp\left[-\frac{(n-\bar{n})^2}{2\bar{n}} + 2i\lambda\sqrt{n}\,t\right] \simeq \exp\left[-\frac{\lambda^2t^2}{2} + 2i\lambda\sqrt{\bar{n}}t\right], \tag{B2}$$

from which it follows that state vectors $|\Psi_F^{(\pm)}(t)\rangle$ are not, strictly speaking, orthogonal at the first instants of the time evolution, but for times $t > 1/\lambda$ the relation (B2) tends approximately to zero, from which it follows that for $t > 1/\lambda$ these states are orthogonal and the asymptotic solution (3.4) is properly normalized.

The time evolution of the population of the upper level of the atom under consideration is given by the relation

$$P_e(t) = |\langle e|\Psi_{F\text{-}A}(t)\rangle|^2 , \tag{B3}$$

from which we easily find that

$$P_e(t) = \tfrac{1}{4}|\exp[-i\lambda t/(2\sqrt{\bar{n}})]|\Psi_F^{(+)}(t)\rangle + \exp[i\lambda t/(2\sqrt{\bar{n}})]|\Psi_F^{(-)}(t)\rangle|^2 \simeq \tfrac{1}{2} + \tfrac{1}{2}\exp\left[-\frac{\lambda^2t^2}{2}\right]\cos 2\lambda\sqrt{(\bar{n}+1)}t , \tag{B4}$$

which is equivalent to the *semiclassical* solution obtained by Cummings [11] and later by Meystre and co-workers [11]. The function (B4) describes the collapse (decay) of the atomic level population and is valid for times $1/\lambda \ll t \ll t_R$, that is, the semiclassical approach is not acceptable to describe the revival of the initial level population.

APPENDIX C: EIGENVALUES AND EIGENSTATES OF THE FIELD DENSITY OPERATOR—ASYMPTOTIC SOLUTIONS

Using the asymptotic solution (3.4) for the state vector of the atom-field system (the atom is supposed to be in the upper state at $t=0$), we can represent the field density operator in the form

$$\hat{\rho}'_F(t) = |C'\rangle\langle C'| + |S'\rangle\langle S'| , \tag{C1}$$

where the states $|C'\rangle$ and $|S'\rangle$ are defined as

$$|C'\rangle = \cos\{\lambda t[\sqrt{\hat{n}} + 1/(2\sqrt{\bar{n}})]\}|\alpha\rangle , \tag{C2a}$$

$$|S'\rangle = \sin(\lambda t\sqrt{\hat{n}})|\alpha\rangle . \tag{C2b}$$

Following the procedure described in Appendix A we will look for the eigenstates of the density operator (C1) in the form

$$|\Psi'\rangle = \mu'|C'\rangle + \nu'|\Psi'\rangle , \tag{C3}$$

from which we find the following requirement for the eigenvalues $\pi'_{1,2}$:

$$\pi' = \langle C'|C'\rangle + \langle C'|S'\rangle\frac{\nu'}{\mu'} = \langle S'|S'\rangle + \langle S'|C'\rangle\frac{\mu'}{\nu'} , \tag{C4}$$

where

$$\langle C'|C'\rangle = \sum_{n=0}^{\infty} P_n \cos^2\{\lambda t[\sqrt{n} + 1/(2\sqrt{\bar{n}})]\} , \tag{C5a}$$

$$\langle S'|S'\rangle = \sum_{n=0}^{\infty} P_n \sin^2(\lambda t\sqrt{n}) , \tag{C5b}$$

$$\langle C'|S'\rangle = \sum_{n=0}^{\infty} P_n \cos\{\lambda t[\sqrt{n} + 1/(2\sqrt{\bar{n}})]\}\sin(\lambda t\sqrt{n}) . \tag{C5c}$$

Finally, the eigenvalues $\pi'_{1,2}$ can be expressed in the form (A15) with the parameter Θ defined by Eq. (A14).

---

*Permanent address: Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Czechoslovakia.

†Permanent address: Instituto Nacional de Astrofísica, Optica y Electrónica, Apartado Postal 51 y 216, Puebla, Puebla, Mexico.

[1] R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987); K. Zaheer and M. S. Zubairy, in *Advances in Atomic, Molecular, and Optical Physics,* edited by D. Bates and B. Bederson (Academic, New York, 1990), Vol. 28, p. 143; see also topical issues of J. Mod. Opt. **34**, No. 7/8 (1987) and J. Opt. Soc. Am. B **10**, No. 10 (1987), and references therein.

[2] L. Mandel, Opt. Lett. **4**, 205 (1979); Phys. Scr. **T12**, 34 (1986).

[3] W. P. Schleich and J. A. Wheeler, J. Opt. Soc. Am. B **4**, 1715 (1987); W. P. Schleich, D. F. Walls, and J. A. Wheeler, Phys. Rev. A **38**, 1177 (1988), W. P. Schleich, R. J. Horowicz, and S. Varro, in *Quantum Optics V,* edited by J. D. Harvey and D. F. Walls (Springer-Verlag, Berlin, 1989), p. 134; J. P. Dowling, W. P. Schleich, and J. A. Wheeler, Ann. Phys. (Leipzig) (to be published).

[4] J. Janszky and A. V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990); V. Bužek, and P. L. Knight, Opt. Commun. **81**, 331 (1991); P. Adam and J. Janszky, Phys. Lett. A **149**, 67 (1990).

[5] V. Bužek, I. Jex, and Tran Quang, J. Mod. Opt. **37**, 159 (1990).

[6] V. Bužek, P. L. Knight, and A. Vidiella-Barranco, in *Squeezed States and Uncertainty Relations,* NASA Conference Publication 3135, edited by D. Han, Y. S. Kim, and W. W. Zachary (NASA, Washington, DC, 1991), p. 181; A. Vidiella-Barranco, V. Bužek, P. L. Knight, and W. K. Lai, in *Quamtum Measurements in Optics,* edited by P. Tombesi and D. F. Walls (Plenum, New York, 1991).

[7] V. Bužek, A. Vidiella-Barranco, and P. L. Knight, Phys. Rev. A **45**, 6570 (1992).

[8] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986); G.

J. Milburn and C. A. Holmes, *ibid.* **56**, 2237 (1986); G. J. Milburn, Phys. Rev. A **33**, 674 (1986); A. Mecozzi and P. Tombesi, Phys. Rev. Lett. **58**, 1055 (1987); P. Tombesi and A. Mecozzi, J. Opt. Soc. Am. B **4**, 1700 (1987); M. Wolinsky and H. J. Carmichael, Phys. Rev. Lett. **60**, 1836 (1988); C. M. Savage and W. A. Cheng, Opt. Commun. **70**, 439 (1989); B. C. Sanders, Phys. Rev. A **39**, 4284 (1989); A. Miranowicz, R. Tanaś, and S. Kielich, Quantum Opt. **2**, 253 (1990); A. D. Wilson-Gordon, V. Bužek, and P. L. Knight, Phys. Rev. A **44**, 7647 (1991).

[9] A. La Porta, R. E. Slusher, and B. Yurke, Phys. Rev. Lett. **62**, 26 (1989); S. Song, C. M. Caves, and B. Yurke, Phys. Rev. A **41**, 5261 (1990); B. Yurke, J. Opt. Soc. Am. B **3**, 732 (1986); R. M. Shelby and M. D. Levenson, Opt. Commun. **64**, 553 (1987); B. Yurke, W. Schleich, and D. F. Walls, Phys. Rev. A **42**, 1703 (1990); T. Ogawa, M. Ueda, and N. Imoto, *ibid.* **43**, 6458 (1991).

[10] J. J. Slosser, P. Meystre, and E. M. Wright, Opt. Lett. **15**, 233 (1990); J. J. Slosser and P. Meystre, Phys. Rev. A **41**, 3867 (1990); M. Wilkens and P. Meystre, *ibid.* **43**, 3832 (1991); P. Meystre, J. J. Slosser, and M. Wilkens, *ibid.* **43**, 4959 (1991); S. Haroche, in *Fundamental Systems in Quantum Optics,* edited by J. Dalibard, J. M. Raimond, and J. Zinn-Justin (Elsevier, New York, 1992); M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45**, 5193 (1992).

[11] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 81 (1963); F. W. Cummings, Phys. Rev. **140**, A1051 (1965); P. Meystre, A. Quattropani, and H. P. Bates, Phys. Lett. **49A**, 85 (1974). For reviews, see L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975); S. Haroche and J. M. Raimond, in *Advances in Atomic and Molecular Physics,* edited by D. Bates and B. Bederson (Academic, New York, 1985), Vol. 20, p. 347; H.-I. Yoo and J. H. Eberly, Phys. Rep. **118**, 239 (1985); S. M. Barnett, P. Filipowicz, J. Javanainen, P. L. Knight, and P. Meystre, in *Quantum Optics,* edited by E. R. Pike and S. Sarkar (Hilger, Bristol, 1986), p. 485; E. A. Hinds, in *Advances in Atomic, Molecular, and Optical Physics,* edited by D. Bates and B. Bederson (Academic, New York, 1990), Vol. 28, p. 237.

[12] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **58**, 1323 (1980).

[13] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).

[14] S. J. D. Phoenix and P. L. Knight, Ann. Phys. (N.Y.) **186**, 381 (1988); Phys. Rev. Lett. **66**, 2833 (1991); Phys. Rev. A **44**, 6023 (1991).

[15] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990); Phys. Rev. A **44**, 5913 (1991).

[16] S. Stenholm, Phys. Rep. **6C**, 1 (1973).

[17] K. Zaheer and M. S. Zubairy, Phys. Rev. A **39**, 2000 (1989).

[18] See, for instance, the review by A. Wehrl, Rev. Mod. Phys. **50**, 221 (1978).

[19] M. Araki and E. Lieb, Commun. Math. Phys. **18**, 160 (1970).

[20] J. Eiselt and H. Risken, Opt. Commun. **72**, 351 (1989); K. Vogel and H. Risken, Phys. Rev. A **39**, 4675 (1989); J. Eiselt and H. Risken, *ibid.* **43**, 346 (1991); M. J. Werner and H. Risken, Quantum Opt. **3**, 185 (1991); M. J. Werner and H. Risken, Phys. Rev. A **44**, 4623 (1991).

[21] P. Meystre and M. S. Zubairy, Phys. Lett. **89A**, 390 (1982); J. R. Kuklinski and J. L. Madajczyk, Phys. Rev. A **37**, 317 (1988); P. K. Aravind and Hu Guanhui, Physica C **150**, 427 (1988).

[22] J. Peřina, *Quantum Statistics of Linear and Nonlinear Optical Phenomena* (Reidel, Dordrecht, 1984).

[23] Tran Quang, P. L. Knight, and V. Bužek, Phys. Rev. A **44**, 6092 (1991), and references therein.

[24] See, for instance, P. Meystre, Prog. Opt. (to be published).