Nonstationary electron distribution functions in a laser field

Boris N. Chichkov, Sergey A. Shumsky, and Sergey A. Uryupin
P. N. Lebedev Physical Institute, Leninsky Prospect 53, Moscow, Russia
(Received 12 November 1991)

Time-dependent functions giving distributions over instantaneous values of electron momenta and energies in a laser field are derived. These distribution functions are found for arbitrary values of the ratio of the electron thermal velocity to the electron oscillation velocity. They are necessary in studies of laser-radiation interaction with plasma, laser-plasma x-ray lasers, etc.

PACS number(s): 52.40.Nk, 42.55.Vc, 52.50.Jm, 34.80.Qb

I. INTRODUCTION

The correct description of an electron distribution function in a laser-produced plasma remains a difficult problem, especially in the case when the plasma is created by a very short laser pulse. It is also clear now that in studies of processes having duration much shorter than the period of the laser field $\mathbf{E}(t)=\mathbf{E}\cos\omega t$, nonstationary electron distribution functions are needed [1–3]. As an example of such very short processes, electron-ion excitation and ionization can be considered.

The duration of electron-ion collision can be approximated by $\tau_c=\rho/v_i$, where ρ is the impact parameter and v_i is the velocity of the impact electron. Using the connection between the classical and quantum expressions for the angular momentum of the incident electron $mv_i\rho=\hbar l$ (where l is lower than about ten for cases of interest), the condition $\tau_c \ll 2\pi/\omega$ can be written as $\mathcal{E}_i \gg l\hbar\omega/4\pi$, or simply as $\mathcal{E}_i \gg \hbar\omega$, where $\mathcal{E}_i$ is the energy of the impact electron. For inelastic collisions, allowed when $\mathcal{E}_i \geq \mathcal{E}_{\rm th}$, where $\mathcal{E}_{\rm th}$ is the threshold excitation or ionization energy of ions, the condition $\mathcal{E}_i \gg \hbar\omega$ is fulfilled practically for all available laser frequencies. Therefore inelastic collisions in a laser-produced plasma can be assumed to be instantaneous. This means that for calculations of the rate coefficients of electron-ion-collision processes in the presence of a laser field the distribution functions over instantaneous values of electron momenta and energies are necessary. These functions should describe the plasma heating, the oscillations of electrons in a laser field, change of the rates of various elementary electron-ion processes in a laser-produced plasma [1–3], etc.

The aim of the present paper is to derive these distribution functions and to study their applicability.

II. OSCILLATING ELECTRON DISTRIBUTION FUNCTIONS

The equation of evolution of the electron distribution function $f=f(\mathbf{v},t)$ due to electron-ion and electron-electron scattering in a homogeneous laser-produced plasma is [4]

$$\left[\frac{\partial}{\partial t}+\frac{e}{m}\mathbf{E}\cos\omega t\frac{\partial}{\partial \mathbf{v}}\right]f=\frac{1}{2}\frac{\partial}{\partial v_k}\nu_{ei}(v)\left[(v^2\delta_{kj}-v_kv_j)\frac{\partial f}{\partial v_j}\right]+\frac{1}{2N}\frac{\partial}{\partial v_k}\int d\mathbf{v}'\nu_{ee}(|\mathbf{v}-\mathbf{v}'|)[(\mathbf{v}-\mathbf{v}')^2\delta_{kj}-(v_k-v_k')(v_j-v_j')]\left[\frac{\partial}{\partial v_j}-\frac{\partial}{\partial v_j'}\right]f(\mathbf{v},t)f(\mathbf{v}',t)\,, \tag{1}$$

where $\nu_{ee}(v)=4\pi e^4N\Lambda m^{-2}v^{-3}$ and $\nu_{ei}(v)=Z\nu_{ee}(v)$ are the electron-electron- and electron-ion-collision frequencies, $Z|e|$ is the ion charge, N is the electron density, and Λ is the Coulomb logarithm, which is assumed to be a constant. The laser is described by a single-mode linearly polarized electric field.

It is useful to remove the rapid oscillations from Eq. (1) by the transformation to the new variables $\mathbf{u}=\mathbf{v}-\mathbf{v}_E\sin\omega t$, $\tau=t$ (where $\mathbf{v}_E=e\mathbf{E}/m\omega$). This gives the equation for the function $F(\mathbf{u},\tau)=f(\mathbf{u}+\mathbf{v}_E\sin\omega\tau,\tau)$. Assuming weak anisotropy of function $F(\mathbf{u},\tau)\approx F(u,\tau)$ and averaging the equation for $F(\mathbf{u},\tau)$ over spherical angles of $\mathbf{u}$ we get

$$\frac{\partial F}{\partial \tau}=\frac{1}{3u^2}\frac{\partial}{\partial u}\left[v_E^2u^2\nu_{ei}(u)R(u,v_E,\tau)\frac{\partial F}{\partial u}\right]+\frac{4\pi}{3Nu^2}\frac{\partial}{\partial u}\left[u^3\nu_{ee}(u)\int_0^\infty u'du'[u^3\eta(u'-u)+u'^3\eta(u-u')]\left[\frac{1}{u}\frac{\partial}{\partial u}-\frac{1}{u'}\frac{\partial}{\partial u'}\right]F(u,\tau)F(u',\tau)\right]\,, \tag{2}$$

$$R(u,v_E,\tau)=\sin^2\omega\tau\,\eta(u-v_E|\sin\omega\tau|)+\frac{u^3}{v_E^3|\sin\omega\tau|}\eta(v_E|\sin\omega\tau|-u)\ ,$$

where

$$\eta(x)=\begin{cases}1 & (x>0)\\ \frac{1}{2} & (x=0)\\ 0 & (x<0)\ .\end{cases}$$

In a weak laser field the assumption of a weak anisotropy of function $F(\mathbf{u},\tau)$ is clear and is justified by the weak anisotropy of the electron-ion-collision frequency ν_{ei}. In a strong laser field the anisotropy of the frequency ν_{ei} can also be strong. However, with the growth of the laser field the frequency ν_{ei} decreases and the electron-electron-collision frequency ν_{ee} remains the same. It means that the anisotropic part connected with the electron-ion-collision term decreases and the isotropic electron-electron part dominates. This gives another reason for weak anisotropy of the function $F(\mathbf{u},\tau)$ in a sufficiently strong laser field. For the time $\tau\gg 2\pi/\omega$ we can replace in Eq. (2) $R(u,v_E,\tau)$ by its average value $R(u,v_E)$,

$$R(u,v_E)=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}d\tau\,R(u,v_E,\tau)$$
$$=\tfrac{1}{2}\eta(u-v_E)+\frac{1}{\pi}\eta(v_E-u)\left\{\arcsin\frac{u}{v_E}-\frac{u}{v_E}\left[1-\left[\frac{u}{v_E}\right]^2\right]^{1/2}+2\left[\frac{u}{v_E}\right]^3\ln\frac{u}{v_E-(v_E^2-u^2)^{1/2}}\right\}\ . \qquad (3)$$

In the case of a weak laser field for the majority of electrons the condition $v_E\le u$ is valid and Eq. (2) with $R(u,v_E)=\frac{1}{2}$ is identical to the equations studied by other authors [6,7]. We study Eq. (2) with $R(u,v_E)$ from (3), which is applicable also for a strong laser field.

A. Maxwellian distribution function

For the electrons with $u\sim v_T$ the electron-electron-collision term in Eq. (2) is greater than the electron-ion-collision term, when the condition

$$Zv_E^2R(v_T,v_E)\ll v_T^2 \qquad (4)$$

is fulfilled. Here $v_T=(T/m)^{1/2}$ is the electron thermal velocity. For weak fields, (4) is equivalent to the condition $v_T^2\gg Zv_E^2$ and for strong fields to the condition $v_E\gg Zv_T$. When the electron-electron-collision term dominates we can try to find the solution of Eq. (2) in the form

$$F=F_M(x,\tau)[1+Z\Psi(x)]\ ,$$
$$x=(\mathbf{v}-\mathbf{v}_E\sin\omega\tau)^2/2v_T^2(\tau)=u^2/2v_T^2(\tau)\ , \qquad (5)$$

where $F_M(x,\tau)=N(2\pi)^{-3/2}v_T^{-3}(\tau)\exp(-x)$ is the Maxwellian distribution function, which takes into account time dependence of the temperature $T(\tau)=mv_T^2(\tau)$ and electron oscillations in the laser field. To emphasize this fact we call $F_M(x,\tau)$ the oscillating (or laser-assisted) Maxwellian distribution function. $Z|\Psi(x)|\ll 1$ is a small correction.

Calculating with the distribution (5) and Eq. (2) the average electron energy, we get the equation for the thermal electron velocity $v_T=v_T(\tau)$,

$$v_T^4(\tau)\frac{dv_T(\tau)}{d\tau}=\tfrac{1}{9}\sqrt{2/\pi}\,\nu_{ei}(v_E)v_E^5\int_0^\infty R(x)\exp(-x)dx\ , \qquad (6)$$

where $R(x)=R(\sqrt{2x}\,v_T(\tau),v_E)$. In the case of strong laser fields, when $v_E\gg Zv_T$ from (6) one obtains the linear time dependence of the temperature

$$T(\tau)=T(0)+\frac{4}{3\pi}\tau mv_E^2\nu_{ei}(v_E)\ln\frac{v_E}{v_T(0)}\ , \qquad (7)$$

where $T(0)$ is the initial temperature. This coincides with the results of the previous papers [4,5]. In weak laser fields, when $v_T^2\gg Zv_E^2$ the time dependence becomes slower,

$$T^{5/2}(\tau)=T^{5/2}(0)+\frac{5}{9\sqrt{2\pi}}\tau\nu_{ei}(v_E)(mv_E^2)^{5/2}\ . \qquad (8)$$

The direct substitution of the distribution (5) in (2) gives the equation for the function Ψ

$$\frac{1}{\sqrt{x}}\frac{d}{dx}\left[e^{-x}\int_0^\infty dx'e^{-x'}[x^{3/2}\eta(x'-x)+x'^{3/2}\eta(x-x')]\times\left[\frac{d\Psi(x)}{dx}-\frac{d\Psi(x')}{dx'}\right]\right]=\frac{v_E^2}{v_T^2}e^{-x}\rho(x)\ , \qquad (9)$$

$$\rho(x)=\tfrac{1}{3}(x-\tfrac{3}{2})\int_0^\infty dx'e^{-x'}R(x')+\tfrac{1}{4}\sqrt{\pi/x}\,e^x\frac{d}{dx}[e^{-x}R(x)]\ ,$$

having the solution

$$\Psi(x)=\left[\frac{v_E}{v_T}\right]^2\left[C_1-C_2x-\frac{2}{3}\int_1^x dy\frac{1}{\gamma(\frac{3}{2},y)}(x-y-1)e^y \times\int_y^\infty dz\sqrt{z}\,\rho(z)e^{-z} \times(z-y-1)\right], \tag{10}$$

where $\gamma(\frac{3}{2},y)=\int_0^y dz\sqrt{z}e^{-z}$ and C_1, C_2 are found from the conditions $\int_0^\infty dx\sqrt{x}e^{-x}\Psi(x)=0$ and $\int_0^\infty dx\,x^{3/2}e^{-x}\Psi(x)=0$, which mean that there are no contributions from function Ψ to the electron density and temperature. In weak laser fields $C_1=0.15$, $C_2=-0.11$. In strong laser fields $C_1=-(v_T/v_E)^3 0.20$ and $C_2=-(v_T/v_E)^3 0.22$. In the weak-field limit the formula (10) coincides with the expression derived in Ref. [8].

In Fig. 1 the dependence of the function Ψ on $x=u^2/2v_T^2$ and on $0.5\ln(v_E^2/2v_T^2)$ is shown. Note that in the derivation of Eq. (9) it was assumed $Z|\Psi(x)|\ll 1$. As can be seen, $\Psi(x)$ is very close to zero for $Zv_E^2\ll v_T^2$ and $v_E\gg Zv_T$ and the oscillating Maxwellian distribution is a good solution of Eq. (2).

As was pointed out in the Introduction the duration of inelastic electron-ion collisions in a laser-produced plasma is usually much shorter than the laser-field period. For calculations of rate coefficients of such processes the distribution function over instantaneous values of electron energy $\mathcal{E}=m\mathbf{v}^2/2$ is necessary. We get this distribution function by integrating (5) over the solid angle of $\mathbf{v}$ (z axis is parallel to $\mathbf{E}$),

$$F(\mathcal{E},\tau)d\mathcal{E}=F_M(\mathcal{E})\Phi_E(\tau)d\mathcal{E}\ ,$$
$$\Phi_E(\tau)=\frac{\sinh(a\sin\omega\tau)}{a\sin\omega\tau}\exp\left[-\frac{\mathcal{E}_E}{T}\sin^2\omega\tau\right], \tag{11}$$

where $a=(vv_E)/v_T^2$, $\mathcal{E}_E=mv_E^2/2$, and $F_M(\mathcal{E})=2N\pi^{-1/2}\mathcal{E}^{1/2}T^{-3/2}\exp(-\mathcal{E}/T)$ is the usual Maxwellian distribution function over electron energy. The time-dependent rate coefficient for any inelastic process with the threshold energy $\mathcal{E}_{\rm th}$ is given by

$$\chi=\int_{\mathcal{E}_{\rm th}}^\infty vF(\mathcal{E},\tau)\sigma(\mathcal{E})d\mathcal{E}\ , \tag{12}$$

where $\sigma(\mathcal{E})$ is the cross section for this process. Note that the above expression is valid only when the collision is short enough and the velocity $v=\sqrt{2\mathcal{E}/m}$ of the impact electron is not changed by the laser field during the collision.

For applications it is useful to have values of the rate coefficients averaged over the laser-field period. We get these by replacing $F(\mathcal{E},\tau)$ in (12) by its averaged value over the laser-field period, $F(\mathcal{E})=F_M(\mathcal{E})\Phi_E$, where $\Phi_E=\overline{\Phi_E(\tau)}$. When $v_E\ll v_T/\sqrt{Z}$

$$\Phi_E=\frac{1}{a}\int_0^a I_0(x)dx\rightarrow\begin{cases}1 & \text{if } a\ll 1\\ (2\pi a^3)^{-1/2}\exp(a) & \text{if } a\gg 1\ .\end{cases} \tag{13}$$

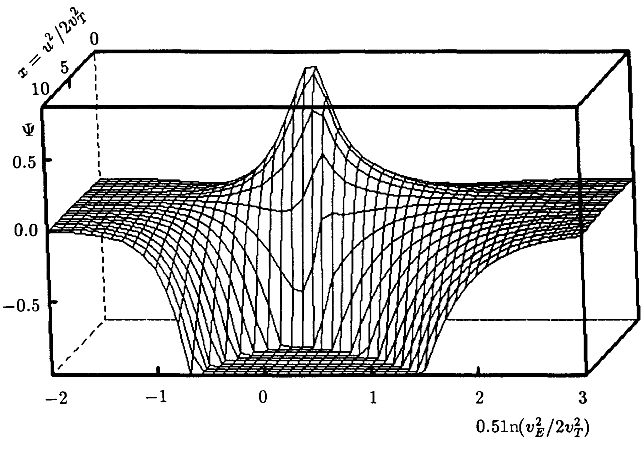

FIG. 1. Dependence of function Ψ on electron energy and laser intensity.

For $v_E\gg Zv_T$ using that $(\pi\Delta)^{-1/2}\exp(-x^2/\Delta)\rightarrow\delta(x)$, when $\Delta\rightarrow 0$, it is easy to show that $F(\mathcal{E})$ gives the well-known distribution over kinetic energy for oscillators

$$F(\mathcal{E})=N\{\pi[\mathcal{E}(\mathcal{E}_E-\mathcal{E})]^{1/2}\}^{-1}\ . \tag{14}$$

Now we turn to the case $v_E/Z\le v_T\le\sqrt{Z}v_E$ and the distribution is non-Maxwellian (see Fig. 2, regions II and III). In these regions, where inequality (4) has the opposite sign, for the majority of electrons the electron-ion-collision term is greater than the electron-electron-collision term.

B. Self-similar distribution function

In the weak laser field, when $\sqrt{Z}v_E\gg v_T\gg v_E$ (region II in Fig. 2) and $Z\gg 1$, the anisotropy of the electron-ion-collision term is small and a self-similar solution of Eq. (2) with $R(u,v_E)$ from (3) can be found [6,7]. In this case the electron distribution function is

$$F=\frac{5N}{4\pi\Gamma(\frac{3}{5})u_T^3(\tau)}\exp\left[-\frac{u^5}{u_T^5(\tau)}\right]\times\left[1+\frac{u_T^2(\tau)}{Zv_E^2}g(u/u_T(\tau))\right], \tag{15}$$

where $u_T^2 g/Zv_E^2$ is a small correction defined by

$$xg''-(1+5x^5)g'-10x^4g=\frac{50}{\Gamma(\frac{3}{5})}x^2\exp(x^5)\frac{d}{dx}\left[\int_0^\infty d\tilde{x}\,\tilde{x}[x^3\eta(\tilde{x}-x)+\tilde{x}^3\eta(x-\tilde{x})](x^3-\tilde{x}^3)\exp(-x^5-\tilde{x}^5)\right] \tag{16}$$

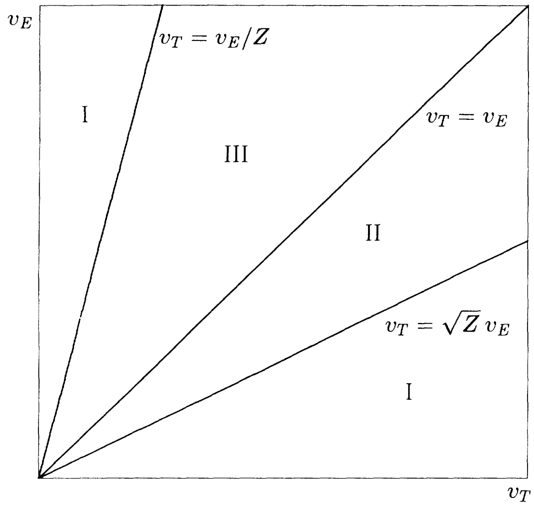

FIG. 2. Regions of validity of various electron distributions: I, Maxwellian distribution regions; II, region of self-similar distribution; III, region of anisotropic distribution.

and is shown in Fig. 3. In (15) the behavior of the $u_T(\tau)$ in time is given by the equation [6,9]

$$u_T^4(\tau)\frac{du_T(\tau)}{d\tau}=\tfrac{5}{6}v_E^5\nu_{ei}(v_E)\ , \tag{17}$$

describing the electron heating in the case of the non-Maxwellian distribution function. The velocity u_T is connected with the effective thermal velocity of electrons by the equation $v_T^2(\tau)=u_T^2(\tau)/3\Gamma(\frac{3}{5})$. The new point to note is that the distribution (15) is also oscillating, since u is connected with the real electron velocity by $u=|\mathbf{v}-\mathbf{v}_E\sin\omega\tau|$.

The self-similar electron distribution function (15) is formed due to very rapid inverse bremsstrahlung heating of electrons. The energy is mainly absorbed by the slow electrons, therefore only the distribution of slow electrons has the form (15) and g is close to zero. The part of the energy absorbed from the laser field by the fast electrons is much smaller and their distribution remains close to Maxwellian due to collisions between electrons. This leads to anomalous growth of $|g|$ function in Fig. 3.

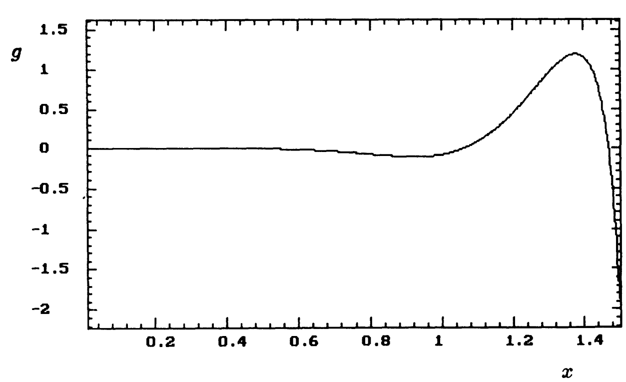

FIG. 3. Dependence of function g on electron energy.

C. Anisotropic distribution function

In the strong laser field, when $v_E/Z \ll v_T \ll v_E$ (region III in Fig. 2) and $Z \gg 1$, we must take into account anisotropy of the electron-ion-collision term in directions parallel (z) and perpendicular ($\perp$) to the laser field. This is the most interesting region, where the analytical solution was unknown.

Turning back to Eq. (1) and removing the rapid oscillations, we get

$$\frac{\partial F}{\partial \tau}=\frac{1}{u_\perp}\frac{\partial}{\partial u_\perp}\left[u_\perp\left[D_{\perp\perp}\frac{\partial F}{\partial u_\perp}+D_{\perp z}\frac{\partial F}{\partial u_z}\right]\right] +\frac{\partial}{\partial u_z}\left[D_{\perp z}\frac{\partial F}{\partial u_\perp}+D_{zz}\frac{\partial F}{\partial u_z}\right]\ , \tag{18}$$

$$\begin{bmatrix} D_{\perp\perp}\\ D_{\perp z}\\ D_{zz}\end{bmatrix}=\frac{v_E^3\nu_{ei}(v_E)}{2|\mathbf{u}+\mathbf{v}_E\sin\omega\tau|^3}\begin{bmatrix}(u_z+v_E\sin\omega\tau)^2\\ -u_\perp(u_z+v_E\sin\omega\tau)\\ u_\perp^2\end{bmatrix}\ ,$$

where $\mathbf{u}=u_\perp\mathbf{e}_\perp+u_z\mathbf{e}_z$. For the time interval $\tau \gg 2\pi/\omega$, taking into account that $u \ll v_E$, we can replace in the above equation the diffusion tensor D by its average value over the laser-field period,

$$\begin{bmatrix} D_{\perp\perp}\\ D_{\perp z}\\ D_{zz}\end{bmatrix}=\frac{1}{\pi}v_E^2\nu_{ei}(v_E)\begin{bmatrix}\ln(4v_E/eu_\perp)\\ 0\\ 1\end{bmatrix}\ . \tag{19}$$

Equation (18) with coefficients (19) can be solved, by approximating $\ln(4v_E/eu_\perp)$ to $\ln[v_E/v_T(0)]$. The solution, meeting the conditions

$$F(u,0)=\frac{N}{(2\pi)^{3/2}v_T^3(0)}\exp[-u^2/2v_T^2(0)]\ , \qquad F(\infty,\tau)=0\ , \tag{20}$$

has the following form:

$$F(u,\tau)=\frac{Nm^{3/2}}{(2\pi)^{3/2}T_\perp(\tau)[T_z(\tau)]^{1/2}} \times\exp\left[-\frac{mu_\perp^2}{2T_\perp(\tau)}-\frac{mu_z^2}{2T_z(\tau)}\right]\ ,$$

$$T_\perp(\tau)=mv_T^2(0)+\frac{2}{\pi}mv_E^2\nu_{ei}(v_E)\tau\ln[v_E/v_T(0)]\ , \tag{21}$$

$$T_z(\tau)=mv_T^2(0)+\frac{2}{\pi}mv_E^2\nu_{ei}(v_E)\tau\ .$$

This is the anisotropic two-temperature electron distribution function which is formed in the strong laser field (region III in Fig. 2). In (21) $u_\perp$ and u_z are connected with the real velocities $\mathbf{v}$ of electrons by $u_\perp=v_\perp$; $u_z=v_z-v_E\sin\omega\tau$. The distribution (21) exists for the rather short period of time $\tau\sim\{\nu_{ei}(v_E)\ln[v_E/v_T(0)]\}^{-1}$, but it has a practical application in the rapidly developing field of very-short-laser-pulse–matter interactions. When the thermal velocity becomes comparable with the velocity of electron oscillations in the laser field, the distribution function is isotropic.

III. CONCLUSION

We derived the oscillating (or laser-assisted) Maxwellian, self-similar, and anisotropic distributions and studied the regions of their applicability. These distributions should be used in calculations of rates of various atomic processes in laser-produced plasmas instead of those in conventional use.

Drawing mentally a horizontal line in Fig. 2, the evolution of electron distribution function in a laser-produced plasma becomes clear: from Maxwellian to anisotropic, then to self-similar and then back to Maxwellian distribution. The analytical formulas derived in this paper give understanding of how much time the electron distribution function spends in each region.

APPENDIX: CIRCULARLY POLARIZED LASER FIELD

In this appendix results for the circularly polarized laser field $\mathbf{E}_c=(1/\sqrt{2})E(\mathbf{e}_x\cos\omega t-\mathbf{e}_y\sin\omega t)$ having the same intensity as in the linear polarization case are presented. The oscillations from Eq. (1) are removed by the transformation

$$\tau=t,\qquad \mathbf{u}=\mathbf{v}-\frac{1}{\sqrt{2}}v_E(\mathbf{e}_x\sin\omega t+\mathbf{e}_y\cos\omega t)\ . \tag{A1}$$

(I) The Maxwellian distribution function $F_M=N(2\pi)^{-3/2}v_T^{-3}\exp(-u^2/2v_T^2)$ is valid in the same regions as in Fig. 2. The time dependence of the temperature in the strong laser field $v_E \gg Zv_T$ is given, instead of (7), by

$$T(\tau)=T(0)+\frac{4}{3\sqrt{2}}\tau m v_E^2\nu_{ei}(v_E)\ . \tag{A2}$$

In the weak laser field, $v_E \ll v_T/\sqrt{Z}$, the time dependence of the temperature remains the same (8).

The distribution functions over electron energy $\mathcal{E}=m\mathbf{v}^2/2$ we get by integrating the Maxwellian distribution over the solid angle of $\mathbf{v}$,

$$F(\mathcal{E})d\mathcal{E}=F_M(\mathcal{E})\Phi_E d\mathcal{E}\ ,\qquad \Phi_E=\frac{\sinh(a)}{a}\exp\left[-\frac{\mathcal{E}_E}{2T}\right]\ , \tag{A3}$$

where $a=(vv_E/\sqrt{2})/v_T^2$, $\mathcal{E}_E$, and $F_M(\mathcal{E})$ are the same as in (11). Differently from (11), the distribution (A3) is time independent.

(II) For the self-similar distribution function (15), (17), and also the region of validity remain the same with u defined by (A1).

(III) The equation for the electron distribution function, taking into account anisotropy of the electron-ion-collision term in directions parallel (z) and perpendicular ($\perp$) to the direction of propagation of the laser beam, has the same form as (18), except the expressions for the diffusion tensor D. The averaged values over the laser-field period for $u \ll v_E$ are

$$\begin{bmatrix} D_{\perp\perp} \\ D_{\perp z} \\ D_{zz} \end{bmatrix}=\frac{1}{\sqrt{2}}v_E^2\nu_{ei}(v_E)\begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}\ . \tag{A4}$$

The solution of Eq. (18) $F(u,\tau)$ is given by the same expression (21), but the time dependence of the temperatures is different,

$$T_\perp(\tau)=mv_T^2(0)+\frac{1}{\sqrt{2}}mv_E^2\nu_{ei}(v_E)\tau\ ,\qquad T_z(\tau)=mv_T^2(0)+\sqrt{2}mv_E^2\nu_{ei}(v_E)\tau\ . \tag{A5}$$

[1] B. N. Chichkov, J. Phys. B. **23**, 103 (1990).
[2] B. N. Chichkov and E. E. Fill, Opt. Commun. **78**, 35 (1990).
[3] B. N. Chichkov and D. B. Uskov, in *X-Ray Lasers 1990,* edited by G. J. Tallents, Inst. Phys. Conf. Ser. N116 (IOP, Bristol, 1991), p. 127.
[4] V. P. Silin, Zh. Eksp. Teor. Fiz. **47**, 2254 (1964) [Sov. Phys.—JETP **20**, 1510 (1965)].
[5] V. P. Silin and S. A. Uryupin, Zh. Eksp. Teor. Fiz. **81**, 910 (1981) [Sov. Phys.-JETP **54**, 485 (1981)].
[6] A. B. Langdon, Phys. Rev. Lett. **44**, 575 (1980).
[7] R. Balescu, J. Plasma Phys. **27**, 553 (1982).
[8] M. V. Chegotov, V. P. Silin, and A. V. Maximov, Fiz. Plazmy **16**, 575 (1990) [Sov. J. Plasma Phys. **16**, 331 (1990)].
[9] V. P. Silin and S. A. Uryupin, Zh. Eksp. Teor. Fiz. **98**, 117 (1990) [Sov. Phys.-JETP **71**, 64 (1990)].