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Momentum source of the plasma maser

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The plasma maser, an interesting nonlinear process in plasma, is an effective means of energy up-conversion in frequency from Langmuir turbulence to the electrostatic Bernstein mode. It is shown here that the primary momentum and energy source of the plasma maser is the external magnetic field, which is not the case for the standard mode-mode coupling processes. For strongly magnetized plasma ($\Omega_e \geq \omega_{pe}$, where Ω_e and ω_{pe} are the electron cyclotron frequency and the plasma frequency, respectively), the growth rate of the Bernstein mode arises from the polarization term. The growth rate is large in contrast to the previous ion-sound-turbulence case. These results have potential importance in the interpretation of anomalous radiation phenomena in astrophysical, laboratory, and fusion plasmas.

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I. INTRODUCTION

According to recent weak-turbulence theory [1], the lowest-order mode-mode coupling processes in plasmas are composed of three parts. The first one is the three-wave resonance [Fig. 1(a): with matching conditions $\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3$, $\omega_1+\omega_2=\omega_3$] and the second one is the nonlinear Landau resonance [Fig. 1(b): with the condition $\Omega-\omega=(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}$]. Both processes (a) and (b) are well known in plasma physics [2]. The third process, which we consider here, is the plasma maser [Fig. 1(c): with conditions $\omega=\mathbf{k}\cdot\mathbf{v}$, $\Omega\neq\mathbf{K}\cdot\mathbf{v}$]. It corresponds to the vertex correction [3]. Most authors in the past have considered energy up-conversion from ion-sound turbulence to Langmuir waves in an unmagnetized plasma. The growth rate in this case is too small to be observed in experiment. Accordingly, the process has not attracted much attention so far. Here, we consider plasma-maser interactions in a magnetized plasma and show that the growth rate is enhanced. Along these lines, quantum-electrodynamical methods are used to investigate some of the nature of the plasma maser [4]. The aim of the present paper is to clarify the reason that this process is effective only in a magnetized, and not in an unmagnetized, plasma.

According to the previous studies on the plasma maser, the slow time change of the plasmon number (N_K) in the presence of resonant mode (k,ω) is given by Eq. (47) in Chap. 14 of Ref. [5]

$$\frac{\partial}{\partial t}N_K=\left[2\gamma_0+2\hat{\gamma}_K+(\epsilon_K)^{-1}\frac{\partial}{\partial t}\epsilon_K\right]\times N_K+R_{\mathrm{TW}}+R_{\mathrm{NL}}+T_{\mathrm{PM}}\ ,$$

where R_{TW} is the three-wave resonance term ($\Omega\pm\Omega'=\omega$, $K\pm K'=k$), R_{NL} is the nonlinear Landau resonance term $(\Omega\pm\omega)=(K\pm k)v$, and T_{PM} is the plasma-maser term ($\omega=kv$, $\Omega\neq Kv$), and where γ_0 is the linear growth rate,

$$\hat{\gamma}_K=(\omega_{pe}/K)^2(\epsilon_K)^{-1}\times\int dv\ \mathrm{P}(\Omega-Kv)^{-1}\left[\frac{\partial}{\partial t}\right](\Omega-Kv)^{-1}K\left[\frac{\partial}{\partial v}\right]f_{0e}\ ,$$

$$\epsilon_K=-(\omega_{pe}/K)^2\int dv\ \mathrm{P}(\Omega-Kv)^{-2}K\left[\frac{\partial}{\partial v}\right]f_{0e}=\partial\epsilon_0/\partial\Omega\ ,$$

and ω_{pe} is the electron plasma frequency. ϵ_0 is the linear dielectric constant of the nonresonant mode (K,Ω), while P denotes the principal value. $\gamma_0=0$ because $\Omega\neq Kv$ by assumption. Both terms, $\hat{\gamma}_K$ and $(\epsilon_K)^{-1}(\partial/\partial t)\epsilon_K$, represent the slow time change of the medium due to quasilinear interactions between resonant electrons and the low-frequency mode (k,ω), and are combined into an absorption term $\partial^2\epsilon_0/2\partial\Omega\,\partial t$. As has been shown recently [6], only for an unmagnetized plasma, the plasma-maser contributions exactly cancel out with the reverse absorption effect if the above slow time change of medium due to quasilinear interaction is considered. For a magnetized plasma [7], the plasma-maser interaction is effective for the energy up-conversion in frequency from resonant low-frequency modes to nonresonant high-frequency modes.

In spite of many studies on plasma maser, the problems of momentum and energy sources in the plasma maser have not been cleared up yet. To elucidate the momentum sources of the instability, we consider the case $\mathbf{K}\perp\mathbf{k}$ and $\mathbf{E}_l\perp\delta\mathbf{E}_h$, where $\mathbf{E}_l$ and $\delta\mathbf{E}_h$ are the electric fields of the low-frequency (Langmuir wave) and high-frequency (Bernstein) modes, respectively, and $\mathbf{k}$ and $\mathbf{K}$ are their respective wave vectors. In contrast to the previous cases, it is found that only the polarization term contributes to plasma-maser instability. The contribution from the direct-coupling term vanishes identically. The present study clearly shows that the plasma-maser interaction strongly depends on the wave modes, in addition to the condition of the system being open or closed.

In Sec. II, the nonlinear dielectric function of the Bernstein mode in the presence of stationary Langmuir turbulence is obtained under a random-phase approximation. The plasma-maser interaction is considered and the growth rate of the Bernstein mode is estimated in Sec. III. The momentum sources of plasma-maser instability are clarified and conclusions are summarized in Sec. IV.

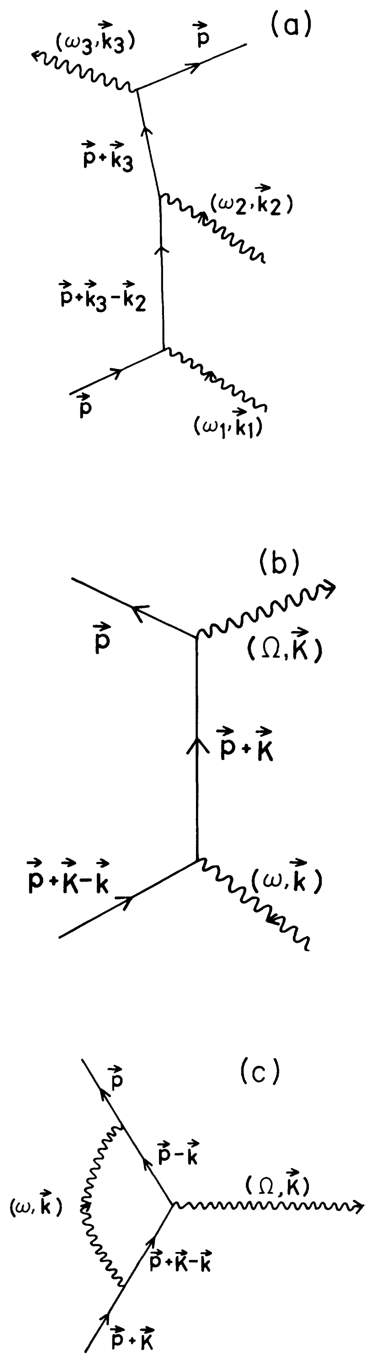

FIG. 1. (a)–(c) show three-wave resonance, nonlinear scattering, and plasma maser, respectively.

II. NONLINEAR DIELECTRIC FUNCTION OF THE BERNSTEIN MODE IN THE PRESENCE OF STATIONARY LANGMUIR TURBULENCE

We consider a homogeneous magnetized plasma in the presence of an enhanced Langmuir wave turbulence driven by a weak electron beam drifting with velocity v_0 through a background plasma of Maxwellian electrons and ions with the following distribution functions [8]:

$$\begin{aligned} f_{0e}(\mathbf{v}) &= (1-\delta)(m/2\pi T_b)^{3/2} e^{-mv_\perp^2/2T_b} e^{-mv_\parallel^2/2T_b} \\ &\quad + \delta(m/2\pi T)^{3/2} e^{-mv_\perp^2/2T} e^{-m(v_\parallel - v_0)^2/2T} , \\ f_{0i}(\mathbf{v}) &= (M/2\pi T_i)^{3/2} e^{-Mv^2/2T_i} , \end{aligned} \tag{1}$$

where $f_{0e}(\mathbf{v})$ is the electron distribution function, T_b and $T(>T_b)$ are temperatures for the background and beam plasma, and $\delta = n_b/n_0 \ll 1$ is the ratio of the beam electron density to the background plasma density. Here, $v_0 \gg (2T/m)^{1/2}$ is assumed, and $\perp$ and $\parallel$ mean perpendicular and parallel to the external magnetic field. $f_{0i}(\mathbf{v})$ is the ion distribution function.

We start with the set of Vlasov-Poisson equations for the electrons,

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left[\mathbf{E}(\mathbf{r},t) + \frac{\mathbf{v}\times\mathbf{B}(\mathbf{r},t)}{c} \right]\cdot\frac{\partial}{\partial \mathbf{v}} \right] f_e(\mathbf{r},\mathbf{v},\mathrm{t}) = 0 , \tag{2}$$

$$\nabla\cdot\mathbf{E}(\mathbf{r},t) = -4\pi e \int f_e(\mathbf{r},\mathbf{v},t)d\mathbf{v} , \tag{3}$$

where the notations are standard. According to the linear-response theory of a turbulent plasma [9], the unperturbed electron distribution function and fields are

$$\begin{aligned} F_{0e} &= f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e} , \\ E_{0e} &= \epsilon \mathbf{E}_l, \quad B_{0e} = \mathbf{B}_0 , \end{aligned} \tag{4}$$

where ϵ is a small parameter associated with the Langmuir turbulence field ($\mathbf{E}_l$) with the wave vector $\mathbf{k} = (0,0,k_\parallel)$ propagating along to an ambient magnetic field $\mathbf{B}_0 = \mathbf{Z}B_0$. In Eq. (4) the second-order electric field ($\epsilon^2 E_2$) is omitted, which can be justified under the random-phase approximation. The Fourier components of f_{1e} are

$$f_{1e}(\mathbf{k},\omega) = \frac{(e/m)E_l(\mathbf{k},\omega)\dfrac{\partial}{\partial v_\parallel} f_{0e}}{-i(\omega - k_\parallel v_\parallel + i0)} . \tag{5}$$

We now perturb the quasi-steady-state by a high-frequency test electrostatic (ES) Bernstein-mode wave field $\mu\delta\mathbf{E}_h(\mathbf{K},\Omega)$ with a propagation vector $\mathbf{K} = (K_\perp,0,0)$ and a frequency Ω, here $\mu \ll \epsilon$. We note that both the beam electrons and Langmuir turbulences carry momentum along the Z direction. Thus they carry no momentum in the ES Bernstein-mode propagation direction. Accordingly, if the ES Bernstein mode grows, a question arises as to where the wave momentum comes from. This is the main motivation of the present study. The total perturbed electric fields and the electron distribution function can be written as

$$\begin{aligned}\delta\mathbf{E}&=\mu\delta\mathbf{E}_h+\mu\epsilon\delta\mathbf{E}_{lh}\ ,\\ \delta f&=\mu\delta f_h+\mu\epsilon\delta f_{lh}+\mu\epsilon^2\Delta f\ .\end{aligned}\tag{6}$$

In the above, we have omitted the modulation electric field $\mu\epsilon^2\Delta\mathbf{E}$ because it gives a small nonlinear frequency shift in the final result. To the orders μ, $\mu\epsilon$, and $\mu\epsilon^2$ we obtain from the Vlasov equation,

$$\begin{aligned}&P\delta f_h-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}=0\ ,\\ &P\delta f_{lh}-\frac{e}{m}\mathbf{E}_l\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_h-\frac{e}{m}\delta\mathbf{E}_{lh}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{1e}=0\ ,\\ &P\Delta f-\frac{e}{m}\mathbf{E}_l\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_{lh}-\frac{e}{m}\delta\mathbf{E}_{lh}\cdot\frac{\partial}{\partial\mathbf{v}}f_{1e}-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{2e}=0\ ,\end{aligned}\tag{7}$$

where

$$P\equiv\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}-\frac{e}{m}\frac{\mathbf{v}\times\mathbf{B}_0}{c}\cdot\frac{\partial}{\partial\mathbf{v}}\right]\ .\tag{8}$$

The Poisson equation for the ES Bernstein mode is

$$\nabla\cdot\delta\mathbf{E}_h(\mathbf{K},\Omega)=-4\pi e\int[\delta f_h(\mathbf{K},\Omega)+\Delta f(\mathbf{K},\Omega)]d\mathbf{v}\ .\tag{9}$$

Taking a transform of the form

$$A(\mathbf{r},\mathbf{v},t)=\sum_{\mathbf{K}}A(\mathbf{K},\mathbf{v},\Omega)\exp(i\mathbf{K}\cdot\mathbf{r}-i\Omega t)\ ,$$

we obtain the nonlinear dielectric constant of the ES Bernstein mode $[\epsilon_h(\mathbf{K},\Omega)]$ in the presence of Langmuir turbulences as

$$\epsilon_h(\mathbf{K},\Omega)=\epsilon_0(\mathbf{K},\Omega)+\epsilon_d(\mathbf{K},\Omega)+\epsilon_p(\mathbf{K},\Omega)\ ,\tag{10}$$

where $\epsilon_0(\mathbf{K},\Omega)$ is the linear part given by

$$\epsilon_0(\mathbf{K},\Omega)=1+\left[\frac{\omega_{pe}}{K_\perp}\right]^2\sum_{n=-\infty}^{\infty}\int\frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega-n\Omega_e}\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}f_{0e}d\mathbf{v}\ ,\tag{11}$$

$\epsilon_d(\mathbf{K},\Omega)$ is the direct-mode-coupling term given by

$$\begin{aligned}\epsilon_d(\mathbf{K},\Omega)=\left[\frac{\omega_{pe}}{K_\perp}\right]^2\left[\frac{e}{m}\right]^2\sum_{n,a,s=-\infty}^{\infty}\sum_{\mathbf{k},\omega}|E_l(\mathbf{k},\omega)|^2\int\frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega-a\Omega_e}\frac{\partial}{\partial v_\parallel}\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{\Omega-\omega+k_\parallel v_\parallel-s\Omega_e}\\ \times\left[\frac{\partial}{\partial v_\parallel}\frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega-s\Omega_e}\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\frac{1}{-\omega+k_\parallel v_\parallel+i0}\frac{\partial}{\partial v_\parallel}\right]f_{0e}d\mathbf{v}\ ,\end{aligned}\tag{12}$$

and $\epsilon_p(\mathbf{K},\Omega)$ is the polarization-mode-coupling term given by

$$\epsilon_p(\mathbf{K},\Omega)=\left[\frac{\omega_{pe}}{K_\perp}\right]^2\left[\frac{e}{m}\right]^2\sum_{\mathbf{k},\omega}|E_l(\mathbf{k},\omega)|^2\frac{\omega_{pe}^2}{\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2}[(A+B)(C+D)]\ ,\tag{13}$$

with

$$A=\sum_{a,s=-\infty}^{\infty}\int\frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega-a\Omega_e}\frac{\partial}{\partial v_\parallel}\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{\Omega-\omega+k_\parallel v_\parallel-s\Omega_e}\left[\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}-k_\parallel\frac{\partial}{\partial v_\parallel}\right]f_{0e}d\mathbf{v}\ ,\tag{14}$$

$$B=\sum_{a=-\infty}^{\infty}\int\frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega-a\Omega_e}\left[\frac{a\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}-k_\parallel\frac{\partial}{\partial v_\parallel}\right]\frac{1}{\omega-k_\parallel v_\parallel+i0}\frac{\partial}{\partial v_\parallel}f_{0e}d\mathbf{v}\ ,\tag{15}$$

$$C=\sum_{n,s=-\infty}^{\infty}\int\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{\Omega-\omega+k_\parallel v_\parallel-s\Omega_e}\frac{\partial}{\partial v_\parallel}\frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega-n\Omega_e}\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}f_{0e}d\mathbf{v}\ ,\tag{16}$$

$$D=\sum_{s=-\infty}^{\infty}\int\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{\Omega-\omega+k_\parallel v_\parallel-s\Omega_e}\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\frac{1}{-\omega+k_\parallel v_\parallel+i0}\frac{\partial}{\partial v_\parallel}f_{0e}d\mathbf{v}\ .\tag{17}$$

Here,

$$\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)=1+\frac{4\pi e^2}{m|\mathbf{K}-\mathbf{k}|^2}\sum_s\int\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{\Omega-\omega+k_\parallel v_\parallel-s\Omega_e}\left[\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}-k_\parallel\frac{\partial}{\partial v_\parallel}\right]f_{0e}d\mathbf{v}\ , \tag{18}$$

where ω_{pe}, Ω_e, and $J_s(K_\perp v_\perp/\Omega_e)$ are the electron plasma frequency, the cyclotron frequency, and Bessel functions, respectively. In deriving Eq. (10), integration is performed over the unperturbed orbit, and a random-phase approximation on Langmuir turbulences is adopted. It is easy to show, after a little algebra, that the result obtained above [Eq. (10)] agrees with the standard expressions for the nonlinear dielectric constant [10].

III. PLASMA-MASER INTERACTION

We consider the plasma-maser interactions between electrons and Langmuir turbulences driven by an electron beam, with the electron distribution function as given by Eq. (1). Inserting Eq. (1) into the linear dielectric constant of an ES Bernstein mode [Eq. (11)], we get

$$\epsilon_0(\mathbf{K},\Omega)=1-\left[\frac{\omega_{pe}}{K_\perp}\right]^2\sum_{n=1}^{\infty}\frac{2n^2 I_n(K_\perp^2 a_e^2)e^{-K_\perp^2 a_e^2}}{(\Omega^2-n^2\Omega_e^2)a_e^2}\ , \tag{19}$$

where a_e is the Larmor radius for the background plasma electrons defined by $a_e=(T_b/m\Omega_e^2)^{1/2}$, and I_n are the modified Bessel functions. We now put $\epsilon_0(\mathbf{K},\Omega)=0$ to obtain the linear dispersion relation of the ES Bernstein mode. Then we get

$$\Omega^2=n^2\Omega_e^2(1+\beta_n)\ , \tag{20}$$

where

$$\beta_n=\frac{2\omega_{pe}^2 I_n(K_\perp^2 a_e^2)e^{-K_\perp^2 a_e^2}}{K_\perp^2 a_e^2\Omega_e^2}\ . \tag{21}$$

From Eqs. (19) and (21), we obtain

$$\frac{\partial\epsilon_0(\mathbf{K},\Omega)}{\partial\Omega}=\sum_{n=1}^{\infty}\frac{2n^2\Omega_e^2\Omega\beta_n}{(\Omega^2-n^2\Omega_e^2)^2}\ . \tag{22}$$

The growth rate of the Bernstein mode is given by

$$\gamma(\mathbf{K},\Omega)=-\left.\frac{\mathrm{Im}\epsilon_h(\mathbf{K},\Omega)}{\dfrac{\partial\epsilon_0(\mathbf{K},\Omega)}{\partial\Omega}}\right|_{\Omega=\Omega_r}\ , \tag{23}$$

where Im shows the imaginary part of the dielectric constant, and Ω_r is the real frequency of the ES Bernstein mode.

A. Growth rate from the direct-coupling term

It is easy to show, by partial integration of Eq. (12), that $\epsilon_d=0$. Accordingly, the growth rate of the Bernstein mode from the direct-coupling term vanishes. This result is markedly different from the previous studies on plasma maser, where the direct coupling offers contributions. The reason is that wave electric fields of the Langmuir wave and the Bernstein mode are perpendicular to each other with the wave vectors $\mathbf{K}\perp\mathbf{k}$.

B. Growth rate from the polarization term

The growth rate of the Bernstein mode from the polarization term is

$$\gamma_p(\mathbf{K},\Omega)=-\left.\frac{\mathrm{Im}\epsilon_p(\mathbf{K},\Omega)}{\dfrac{\partial\epsilon_0(\mathbf{K},\Omega)}{\partial\Omega}}\right|_{\Omega=\Omega_r}\ . \tag{24}$$

By inspection, it is clear from Eqs. (14) and (16) that $A=0$ and $\mathrm{Im}C=0$. Thus,

$$\mathrm{Im}[(A+B)(C+D)]=\mathrm{Im}(B)\mathrm{Re}(C+D)+\mathrm{Re}B\times\mathrm{Im}D\ , \tag{25}$$

where Re shows the real part. For the electron distribution function as given by Eq. (1), we obtain from Eq. (15), after a lengthy but straightforward calculation,

$$B=\sum_{a=1}^{\infty}\left[\frac{2a^2k_\parallel I_a(K_\perp^2a_e^2)e^{-K_\perp^2a_e^2}}{\omega^2(\Omega^2-a^2\Omega_e^2)a_e^2}-i\frac{2a^2\pi^{1/2}\delta}{(\Omega^2-a^2\Omega_e^2)\rho_e^2}\frac{m}{|k_\parallel|T}\frac{(\omega-k_\parallel v_0)}{(k_\parallel v_e)}I_a(K_\perp^2\rho_e^2)e^{-K_\perp^2\rho_e^2}e^{-[m(\omega/k_\parallel-v_0)^2]/2T}\right]\ , \tag{26}$$

where I_a is the modified Bessel function, and $v_e=(2T/m)^{1/2}$ and $\rho_e=(T/m\Omega_e^2)^{1/2}$ are the electron thermal velocity and the Larmor radius for the beam electrons, respectively. The real part of B, which is an odd term in (k,ω), comes from the background plasma. The imaginary part contribution of B even in (k,ω) comes from the plasma-maser interactions between Langmuir wave and beam electrons, for which the condition is $\omega=k_\parallel v_\parallel$. From Eq. (16), we obtain

$$C=-\sum_{s,n=1}^{\infty}\frac{2n^2k_\parallel b}{\Omega^2(\Omega^2-n^2\Omega_e^2)a_e^2}\times\left[1+\frac{2\Omega^2(\Omega^2+s^2\Omega_e^2)}{(\Omega^2-s^2\Omega_e^2)^2}\left[\frac{c}{b}\right]\right]\ , \tag{27}$$

where

$$b=\int_0^\infty J_0^2(K_\perp v_\perp/\Omega_e)J_n^2(K_\perp v_\perp/\Omega_e)f_{0e}(v_\perp)2\pi v_\perp dv_\perp\,,$$
$$c=\int_0^\infty J_s^2(K_\perp v_\perp/\Omega_e)J_n^2(K_\perp v_\perp/\Omega_e)f_{0e}(v_\perp)2\pi v_\perp dv_\perp\,. \tag{28}$$

In deriving Eq. (27), we expand the denominator in powers of $(k_\parallel v_\parallel-\omega)^2/(\Omega^2-s^2\Omega_e^2)$, $(k_\parallel v_\parallel-\omega)/\Omega$ for $\Omega\simeq\Omega_e\geq\omega\simeq\omega_{pe}$ and keep the dominant odd terms in (k,ω) with $f_{0e}(\mathbf{v})\equiv f_{0e}(v_\parallel)f_{0e}(v_\perp)$. We obtain from Eq. (17)

$$D=\sum_{s=1}^{\infty}\left[-\frac{2s^2k_\parallel I_s(K_\perp^2a_e^2)e^{-K_\perp^2a_e^2}}{\omega^2(\Omega^2-s^2\Omega_e^2)a_e^2}\left[1+\frac{\omega^2}{\Omega^2-s^2\Omega_e^2}-\frac{4\Omega^2\omega^2}{(\Omega^2-s^2\Omega_e^2)^2}\right]\right.$$
$$\left.-i\frac{2s^2\pi^{1/2}\delta}{(\Omega^2-s^2\Omega_e^2)\rho_e^2}\frac{m}{|k_\parallel|T}\frac{(\omega-k_\parallel v_0)}{(k_\parallel v_e)}I_s(K_\perp^2\rho_e^2)e^{-K_\perp^2\rho_e^2}e^{-m(\omega/k_\parallel-v_0)^2/2T}\right]. \tag{29}$$

In deriving Eq. (29), we have retained only dominant terms to make ReD an odd function of ω and k. On the other hand, the dominant imaginary-part contribution (ImD) from the plasma maser ($\omega=k_\parallel v_\parallel$) is even in (k,ω).

For strongly magnetized plasma ($\Omega_e\geq\omega_{pe}$), expanding the denominator of Eq. (18) in powers of $(k_\parallel v_\parallel-\omega)^2/(\Omega^2-s^2\Omega_e^2)$ etc., we obtain the dominant contribution as follows:

$$\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2\simeq k_\parallel^2+2\omega_{pe}^2\Omega\omega k_\parallel^2\left[\sum_{s=1}^{\infty}\frac{6I_s(K_\perp^2a_e^2)}{(\Omega^2-s^2\Omega_e^2)^2}\left[1-\frac{s^2}{3k_\parallel^2a_e^2}\right]-\frac{I_0(K_\perp^2a_e^2)}{\Omega^4}\right]e^{-K_\perp^2a_e^2}. \tag{30}$$

In obtaining Eq. (30), we use $\epsilon_0(\mathbf{K},\Omega)=0$ [Eq. (19)]. We note that the first and the second terms in the right-hand side of Eq. (30) are even and odd in (k,ω), respectively. Inserting Eqs. (25), (26), (27), and (29) into Eqs. (13), we obtain

$$\mathrm{Im}\epsilon_p(\mathbf{K},\Omega)=\sum_{a,s,n=1}^{\infty}\sum_{\mathbf{k},\omega}\frac{|E_l(\mathbf{k},\omega)|^2}{4\pi NT}\frac{\omega_{pe}^6 4a^2\pi^{1/2}\delta}{\Omega^2(\Omega^2-a^2\Omega_e^2)}\frac{bk_\parallel}{\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2K_\perp^2\rho_e^2a_e^2|k_\parallel|}\frac{(\omega-k_\parallel v_0)}{k_\parallel v_e}I_a(K_\perp^2\rho_e^2)$$
$$\times e^{-K_\perp^2\rho_e^2}e^{-m(\omega/k_\parallel-v_0)^2/2T}\left\{\frac{n^2}{\Omega^2-n^2\Omega_e^2}\left[1+\frac{2\Omega^2(\Omega^2+s^2\Omega_e^2)}{(\Omega^2-s^2\Omega_e^2)^2}\left[\frac{c}{b}\right]\right]\right.$$
$$\left.+\frac{s^2\Omega^2I_s(K_\perp^2a_e^2)e^{-K_\perp^2a_e^2}}{(\Omega^2-s^2\Omega_e^2)^2}\left[1-\frac{4\Omega^2}{\Omega^2-s^2\Omega_e^2}\right]\right\}. \tag{31}$$

Inserting Eqs. (22) and (31) into Eq. (24), we get

$$\frac{\gamma_p(\mathbf{K},\Omega)}{\Omega}=\sum_{a=1}^{\infty}\sum_{\mathbf{k},\omega}\frac{|E_l(\mathbf{k},\omega)|^2}{4\pi NT}\frac{\omega_{pe}^6 2a^2\pi^{1/2}\delta}{\Omega^4\beta_a\Omega_e^2}\frac{bk_\parallel}{\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2K_\perp^2\rho_e^2a_e^2|k_\parallel|}\frac{(k_\parallel v_0-\omega)}{k_\parallel v_e}I_a(K_\perp^2\rho_e^2)$$
$$\times e^{-K_\perp^2\rho_e^2}e^{-m(\omega/k_\parallel-v_0)^2/2T}\left[1+\frac{2\Omega^2(\Omega^2+a^2\Omega_e^2)}{(\Omega^2-a^2\Omega_e^2)^2}\frac{c}{b}+\frac{\Omega^2I_a(K_\perp^2a_e^2)e^{-K_\perp^2a_e^2}}{(\Omega^2-a^2\Omega_e^2)}\left[1-\frac{4\Omega^2}{\Omega^2-a^2\Omega_e^2}\right]\right], \tag{32}$$

where β_a and $\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2$ are given by Eqs. (21) and (30), respectively. In deriving Eq. (32), we put $a=s=n$. The final growth rate is even in ω and k, because we take odd functions in ω and k for $[\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2]^{-1}$ [Eq. (30)]. Equation (32) is the main result of this paper.

Next, we estimate the growth rate for $\Omega\simeq\Omega_e$ and $\omega\simeq\omega_{pe}$ with $\Omega_e\geq\omega_{pe}$. For $K_\perp a_e=0.5$, $K_\perp\rho_e=1.5$, $k_\parallel=0.1(T_b/T)^{1/2}k_e$, and $v_0=11v_e$, where k_e is the Debye wave number for the background electrons, we get $(k_\parallel v_0-\omega)=k_\parallel v_e$ and $k_\parallel a_e=0.1(\omega_{pe}/\Omega_e)(T_b/T)^{1/2}$. First, we obtain from Eq. (21),

$$\beta_1\approx(\omega_{pe}/\Omega_e)^2\,. \tag{33}$$

In obtaining Eq. (33), we put $I_1(K_\perp^2a_e^2)\simeq(K_\perp^2a_e^2)/2$ for $K_\perp a_e<1$. For the above parameters, the dominant contribution of Eq. (30), which is an odd function in k and ω, comes from the last term in square brackets on the right-hand side and reduces to

$$\frac{1}{\epsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)|\mathbf{K}-\mathbf{k}|^2} \simeq \frac{1}{k_\parallel^2}\left[1+\frac{2\omega_{pe}^2\omega}{\Omega^3}I_0(K_\perp^2 a_e^2)e^{-K_\perp^2 a_e^2}\right] . \quad (34)$$

We note that the second term on the right-hand side of Eq. (34) is odd in k and ω.

Next, we estimate Eq. (28). For a small argument of the Bessel functions, we put $J_0^2(K_\perp v_\perp/\Omega_e)\simeq 1$ and $J_1^2(K_\perp v_\perp/\Omega_e)\simeq(K_\perp v_\perp)^2/(4\Omega_e^2)$. Then we obtain

$$b(n=1)\simeq(K_\perp^2/4\Omega_e^2)\int_0^\infty f_{0e}(v_\perp)2\pi v_\perp^3 dv_\perp=(K_\perp a_e)^2/4 ,$$
$$c(s=n=1)\simeq(K_\perp^4/16\Omega_e^4)\int_0^\infty f_{0e}(v_\perp)2\pi v_\perp^5 dv_\perp =(K_\perp a_e)^4/8 . \quad (28')$$

Inserting Eqs. (33), (28′), and the second term in Eq. (34) into the right-hand side of Eq. (32) with $a=1$, we obtain

$$\frac{\gamma_p(\mathbf{K},\Omega)}{\Omega}=\sum_{\mathbf{k},\omega}\frac{|E_l(\mathbf{k},\omega)|^2}{4\pi NT}\left[\frac{\omega_{pe}}{\Omega_e}\right]^7\frac{\pi^{1/2}(K_\perp a_e)^2\delta}{2(K_\perp\rho_e)^2(k_\parallel a_e)^2}I_1(K_\perp^2\rho_e^2)e^{-K_\perp^2\rho_e^2}I_0(K_\perp^2 a_e^2)e^{-K_\perp^2 a_e^2}\left[1-\frac{(K_\perp a_e)^2}{2}\left[\frac{\Omega_e}{\omega_{pe}}\right]^2\right] . \quad (35)$$

Figure 2 shows the normalized growth rate [Eq. (35)] versus plasma parameter (ω_{pe}/Ω_e). Here, $W[=\sum_{\mathbf{k},\omega}|E_l(\mathbf{k},\omega)|^2/4\pi NT]$ is the normalized Langmuir turbulence energy density, and we take $\delta=0.1$, $T/T_b=9$, $K_\perp a_e=0.5$, $K_\perp\rho_e=1.5$. It should be stressed that the growth rate obtained [Eq. (35)] is large in contrast to the previous ion-sound turbulence case [11], where $\gamma_d/\Omega\approx(m/M)^{1/2}(kK/k_e^2)W_T \ll W_T$ and $\gamma_p/\Omega=0$, here $W_T[=\sum_{\mathbf{k},\omega}|E_l(\mathbf{k},\omega)|^2(k_e/k)^2/4\pi NT]$ represents the normalized turbulence energy of ion-sound turbulences. Here, m and M are masses for an electron and an ion, and k and K are wave numbers for ion-sound and Langmuir waves, respectively. The maximum growth rate is comparable to those of the other two mode-mode couplings (three-wave resonance [Fig. 1(a)] and nonlinear scattering [Fig. 1(b)]).

IV. DISCUSSIONS AND CONCLUSIONS

The reverse absorption process to the plasma maser comes from the quasilinear interactions between electrons and the Langmuir turbulence [6], and is given by $\frac{1}{2}[\partial^2\epsilon_0(\mathbf{K},\Omega)/\partial\Omega\,\partial t)]$,

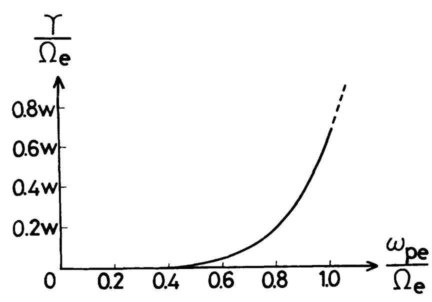

FIG. 2. Normalized growth rate of the Bernstein mode [Eq. (35)] vs (ω_{pe}/Ω_e) for $T/T_b=9$, $\delta=0.1$, $K_\perp a_e=0.5$, $K_\perp\rho_e=1.5$. Here, $W[=\sum_{\mathbf{k},\omega}|E_l(\mathbf{k},\omega)|^2/4\pi NT]$ is the normalized Langmuir-turbulence energy density.

$$\frac{\partial^2\epsilon_0(\mathbf{K},\Omega)}{2\partial\Omega\,\partial t}=\left[\frac{\omega_{pe}}{K_\perp}\right]^2\sum_{n=1}^{\infty}\frac{2\Omega n^2 I_n(K_\perp^2\rho_e^2)e^{-K_\perp^2\rho_e^2}}{(\Omega^2-n^2\Omega_e^2)\rho_e^2}\times\int\frac{\partial f_{0e}(v_\parallel)}{\partial t}dv_\parallel , \quad (36)$$

where $f_{0e}(v_\parallel)$ is the electron distribution function parallel to the external magnetic field. In deriving Eq. (36), we use Eq. (11). The slow time change of the electron distribution function comes from the quasilinear interaction between resonant beam electrons and Langmuir turbulence as

$$\frac{\partial f_{0e}(v_\parallel)}{\partial t}=\left[\frac{e}{m}\right]^2\frac{\partial}{\partial v_\parallel}\sum_{\mathbf{k},\omega}|E_l(\mathbf{k},\omega)|^2\pi\delta(\omega-k_\parallel v_\parallel)\frac{\partial}{\partial v_\parallel}f_{0e} . \quad (37)$$

Inserting Eq. (37) into Eq. (36), we find the reverse absorption process to the plasma maser vanishes in addition to the plasma-maser contribution from the direct coupling [Eq. (12)].

We now consider the momentum (energy) sources of the plasma maser. Both the electron beam [Eq. (1)] and the Langmuir turbulence carry momentum only in the z direction. Thus they provide no free momentum (energy) source for the ES Bernstein mode with perpendicular propagation considered here. The momentum source exists in the external magnetic field itself because the magnetic field carries momentum $(e/c\,\mathbf{A})$, where $\mathbf{A}$ is the vector potential. For the present case $(\mathbf{B}_0=\mathbf{z}B_0)$, we get $A_x=(-yB_0/2)$, $A_y=(xB_0/2)$, and $A_z=0$. Accordingly, the external magnetic field carries momentum only in perpendicular direction. Indeed, the growth rate $\gamma_p(\mathbf{K},\Omega)$ vanishes for $B_0\to 0$. This consideration clearly shows that the primary momentum (energy) source of the plasma maser lies in the external environment (magnetic field), which is consistent with recent analysis [4]. In other words, the plasma maser vanishes for a closed system without an external magnetic field because

$\mathrm{Im}\epsilon_d(\mathbf{K},\Omega)+(\frac{1}{2})\partial^2\epsilon_0(\mathbf{K},\Omega)/\partial\Omega\,\partial t=0$ with $\mathrm{Im}\epsilon_p(\mathbf{K},\Omega)=0$ [6]. The plasma maser is effective only in an open system with momentum (energy) sources from outside systems. This point is markedly different from the two standard mode-mode coupling processes in plasmas [Figs. 1(a) and 1(b)].

Historically, the plasma-maser process from the direct-coupling term (ϵ_d) was pointed out by Tsytovich and co-workers for an ion-sound turbulence in an unmagnetized plasma. The contribution from the direct-coupling term often cancels out with the reverse absorption process due to the conservation of adiabatic invariant (plasmon number) [6] in the quasilinear process. Both processes vanish identically for the present case. On the other hand, the plasma-maser interaction from the polarization term (ϵ_p) for magnetized plasma was pointed out in Ref. [1]. For a magnetized plasma, the dominant contribution to the plasma maser comes from the polarization term. The primary momentum (energy) source of the plasma maser exists in the external magnetic field in addition to the low-frequency turbulence.

The plasma-maser interactions between Langmuir turbulences and the ES Bernstein modes are studied for a strongly magnetized plasma. The enhanced growth rate of the ES Bernstein mode comes only from the polarization term [Eq. (35)]. The direct-coupling contribution vanishes identically, as does the reverse-absorption process due to the quasilinear process between Langmuir waves and beam electrons. In contrast to the parametric-resonance processes, the plasma-maser interaction does not require any matching condition between the nonresonant high-frequency mode and the resonant low-frequency mode. Accordingly, the investigation of plasma-maser processes may open a branch in plasma physics. It is left for experimentalists to check the finer details of the theoretical predictions. One of the most interesting applications of the plasma-maser instability might be to pulsars, which have superstrong dc fields. The full QED calculation is necessary to investigate whether the plasma maser has any interesting properties in a relativistic electron-positron plasma.

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[11] M. Nambu, Phys. Rev. A **23**, 3272 (1981) (the contribution from the polarization term [Eq. (38)] should vanish for an unmagnetized plasma as shown in Appendix B of Ref. [7]. Furthermore, the diagram [Fig. 2(c)] is not complete. The correct diagram for the plasma maser (vertex correction) is given by Fig. 1(c) in the present paper).