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Positronium formation in the $n=1$ and $n=2$ states in e^+-He scattering

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Positronium formation in the $n=1$ and $n=2$ levels in e^+-He collision has been studied using a second-order Born approximation. Differential and integrated capture cross sections are reported in the energy range 100 eV–2 keV. The predicted total positronium-formation cross sections differ from measured values, the theoretical results being lower. The first-order Born results are not expected to be valid in the energy range considered. A Thomas mechanism is found to be valid in the case of the ground as well as the 2s excited state. A similar structure in the differential cross section for 2p-state capture has not been found.

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I. INTRODUCTION

Recently total positronium-formation cross sections in e^+-He scattering have been measured by Fornari, Diana, and Coleman [1], Diana *et al.* [2], and Fromme *et al.* [3]. The agreement between the two sets of results is not very satisfactory. In particular, above the incident energy 100 eV the experimental results of Fromme *et al.* [3] and Diana *et al.* [2] are in conflict and subject to large-percentage errors.

Theoretically, ground-state capture cross sections in e^+-He scattering are reported by Mandal, Guha, and Sil [4], Khan and Ghosh [5], Deb, McGuire, and Sil [6] and Deb, Crothers, and Fromme [7]. The excited-state capture cross sections are predicted by Khan, Majumdar, and Ghosh [8] and Roberts [9]. The different theoretical results are not in good agreement. Moreover, the agreement between theory and experiment is not satisfactory. The present situation demands more elaborate theoretical predictions at intermediate and high energies.

In our earlier paper (Basu and Ghosh [10] referred to hereafter as BG), positronium-formation cross sections in the $n=1$ and $n=2$ states in e^+-H scattering have been reported using a second-order Born approximation (SBA). In the present study, we investigate positronium formation in the $n=1$ and $n=2$ states in e^+-He scattering using a second-order Born method similar to that of Basu and Ghosh [10]. We report the ground- and excited-state capture cross sections in the energy region 100 eV–2 KeV.

II. THEORY

In the conventional perturbative approach, the capture amplitude, retaining up to the second-order term from the ground state (n) of a helium atom, $\Phi_n(r_1,r_2)$, with momentum $\mathbf{K}$, to the final state $\nu'(n'l'm')$ of a Ps atom, $\eta_{\nu'}(r_{12})$, with momentum $\mathbf{K}'$, is given by [Eq. (1) of BG]

$$g_{\nu'n}(\mathbf{K}',\mathbf{K})=g^{B}_{\nu'n}(\mathbf{K}',\mathbf{K})+g^{B_2}_{\nu'n}(\mathbf{K}',\mathbf{K}) , \quad (1)$$

where we have neglected terms arising from electron exchange for computing tractability. Here $g^{B}_{\nu'n}(\mathbf{K}',\mathbf{K})$ and $g^{B_2}_{\nu'n}(\mathbf{K}',\mathbf{K})$ are the first- and second-order capture amplitudes, respectively.

The second-order amplitude $g^{B_2}_{\nu'n}(\mathbf{K}',\mathbf{K})$ may be written as

$$g^{B_2}_{\nu'n}(\mathbf{K}',\mathbf{K}) = \frac{1}{2\pi^2}\sum_{n'}\int \frac{1}{K''^2-K^2_{n'}-i\varepsilon} \times g^{B}_{\nu'n'}(\mathbf{K}',\mathbf{K}'')f^{B}_{n'n}(\mathbf{K}'',\mathbf{K})d\mathbf{K}'' , \quad (2)$$

where $f^{B}_{n'n}(\mathbf{K}',\mathbf{K})$ is the first-order Born amplitude in the direct channel.

In evaluating $g^{B_2}_{\nu'n}(\mathbf{K}',\mathbf{K})$, we have retained three intermediate eigenstates ($1s$, $2\,^1s$, and $2\,^1p$) of the helium atom. In the calculations, we use the target wave functions of Byron and Joachain [11] for the ground and $2\,^1s$ states and that of Eckart [12] for the $2\,^1p$ state.

III. RESULTS AND DISCUSSIONS

The convergence of the second-order Born term is required in predicting reliable results. We have tabulated the second-order Born amplitude for ground-state capture at four scattering angles, retaining different sets of intermediate states in the energy region 100 eV–2 keV, in Table I. Convergence of the real part of the scattering amplitude is found to be good at all scattering angles at all incident energies considered here. Imaginary parts of the scattering amplitude, except in the forward direction, are found to be satisfactory at all scattering angles. It may be mentioned that in the forward direction the imaginary part is about one order of magnitude less than the real part. As a consequence, the imaginary part will not contribute appreciably to the cross section. Therefore, we conclude that the inclusion of $1s$, $2\,^1s$, and $2\,^1p$ states

TABLE I. Second-order amplitude (real and imaginary) for Ps formation in the ground state in e^+-He scattering with different sets of intermediate states. (Numbers in square brackets indicate powers of 10.)

Energy	Scattering angle (deg)	1s Re	1s Im	$1s+2\,^1s$ Re	$1s+2\,^1s$ Im	$1s+2\,^1s+2\,^1p$ Re	$1s+2\,^1s+2\,^1p$ Im
100 eV	0	0.2629	0.5143[−1]	0.2510	0.5041[−1]	0.2428	0.5261[−1]
	20	0.2450	0.5676[−1]	0.2362	0.4784[−1]	0.2318	0.3455[−1]
	60	0.1615	0.7384[−1]	0.1594	0.6895[−1]	0.1606	0.6798[−1]
	120	0.8910	0.7236[−1]	0.8864[−1]	0.6984[−1]	0.8811[−1]	0.7085[−1]
300 eV	0	0.5125[−1]	0.2809[−2]	0.4998[−1]	0.2861[−2]	0.4873[−1]	0.3769[−2]
	20	0.4212[−1]	0.5162[−2]	0.4144[−1]	0.5054[−2]	0.4130[−1]	0.4951[−2]
	60	0.1719[−1]	0.8726[−2]	0.1714[−1]	0.8560[−2]	0.1712[−1]	0.8567[−2]
	120	0.7040[−2]	0.6813[−2]	0.7037[−2]	0.6735[−2]	0.7089[−2]	0.6752[−2]
1 keV	0	0.4613[−2]	−0.2267[−3]	0.3951[−2]	−0.2058[−3]	0.3896[−2]	−0.1838[−3]
	20	0.2952[−2]	0.1571[−3]	0.2935[−2]	0.1579[−3]	0.2928[−2]	0.1562[−3]
	60	0.7038[−3]	0.3944[−3]	0.7036[−3]	0.3910[−3]	0.7022[−3]	0.3914[−3]
	120	0.2773[−3]	0.2456[−3]	0.2773[−3]	0.2449[−3]	0.2764[−3]	0.2452[−3]
2 keV	0	0.8103[−3]	−0.7630[−4]	0.8038[−3]	−0.7286[−4]	0.7980[−3]	−0.7063[−4]
	20	0.4396[−3]	0.1589[−4]	0.4384[−3]	0.1591[−4]	0.4306[−3]	0.1572[−4]
	60	0.8332[−4]	0.5129[−4]	0.8332[−4]	0.5096[−4]	0.8320[−4]	0.5099[−4]
	120	0.3493[−4]	0.2937[−4]	0.3494[−4]	0.2925[−4]	0.3487[−4]	0.2927[−4]

as intermediate states is not a bad approximation.

In Fig. 1 the present differential cross section (P) for the ground-state capture along with the first-order Born predictions (B) are shown at three incident energies 100 eV, 500 eV, and 2 keV. As usual, the first-order Born results attain a zero in the cross sections. The second-order Born results show the Thomas peak around 45°. With the increase of energy, the Thomas peak is more prominent, as is evident from Fig. 1. This feature has also been noticed by Basu and Ghosh [10] in the case of the hydrogen atom. At 500 eV and 2 keV (shown in Fig. 1), Deb, McGuire, and Sil [6] have reported differential cross sections using their second-order model, which is essentially valid at high incident energies. There is no structure in their differential cross sections and their results fall very rapidly.

The present differential cross sections (DCS's) for 2s-state capture using the second-order method (P) and the first-order Born predictions (B) are shown in Fig. 2 at three incident energies 200 eV, 500 eV, and 2 keV. As usual, first-order Born approximation (FBA) cross sections attain a zero value around the scattering angle 20°. The structure in the differential cross section using the second-order method is found at all the energies considered here. With the increase of energy, the structure is found to be more prominent.

Figure 3 shows the present (P) differential cross sections for 2p-state capture along with the first-order Born predictions (B) at the three incident energies 200 eV, 500 eV, and 2 keV. The second-order Born method does not provide the structure in the cross section; however, at the scattering angle of about 20°, there is a change of slope in the DCS. As the Thomas mechanism is valid at high energies, this feature is not unlikely. It may be mentioned that in the case of the helium atom, FBA results for 2p-

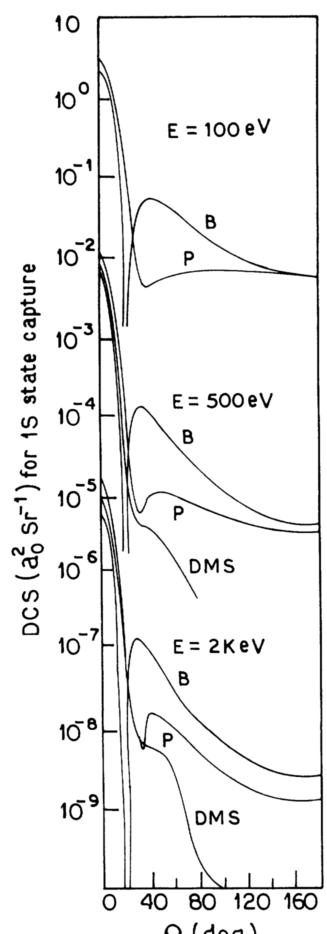

FIG. 1. Differential cross sections (a_0^2 sr^{-1}) for ground-state capture in e^+-He scattering: P, present; B, FBA; DMS, Deb, McGuire, and Sil (Ref. [6]).

state capture attain zero in the DCS unlike in hydrogen.

Figure 4 contains the first-order and second-order Born integrated capture cross sections in e^+-He scattering along with the predictions of Roberts [9] using the second-order Faddeev-Watson approximation (FWA) and Deb, Crothers, and Fromme [7] who employed a first-order target-continuum distorted-wave (TCDW1) approximation. The measured data due to Diana *et al.* [2] and Fromme *et al.* [3] are compared with the theoretical predictions in the same figure. All the theoretical total cross sections except those of Deb, Crothers, and Fromme [7] refer to capture into 1*s*, 2*s*, and 2*p* states. Experimental results refer to capture into all states. Results using the second-order FWA lie well above the measured data as well as other theoretical predictions. However, their first-order FWA results (not shown) are in good agreement with the measured data of Fromme *et al.* [3] above 200 eV. This agreement may be accidental because second-order results are expected to be more sound theoretically. Moreover, the use of the peaking approximation in evaluating the Rutherford phase term by Roberts may be responsible for high cross sections. The ground-state results of Deb, Crothers, and Fromme [7] are in fair agreement with those of Fromme *et al.* [3] above 200 eV. However, the theoretical soundness of TCDW1 is not beyond question. Our second-order results for the ground- as well as excited-state capture (Table II) are greater than the corresponding FBA predictions. The difference between the present second-order Born and the first-order Born results suggests that the FBA is not valid for charge-transfer processes up to 2 keV (Table II). In the case of ion-atom collisions, this is

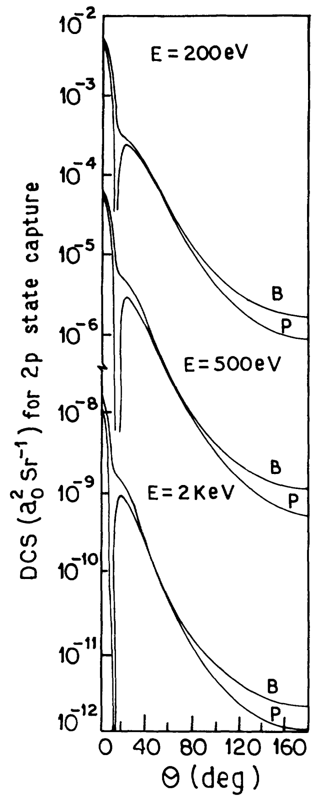

FIG. 3. Differential cross sections (a_0^2 sr^{-1}) for 2*p*-state capture in e^+-He scattering: P, present; B, FBA.

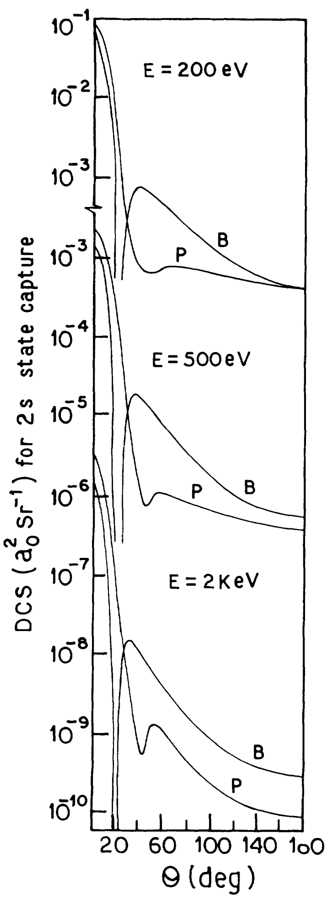

FIG. 2. Differential cross sections (a_0^2 sr^{-1}) for 2*s*-state capture in e^+-He scattering: P, present; B, FBA.

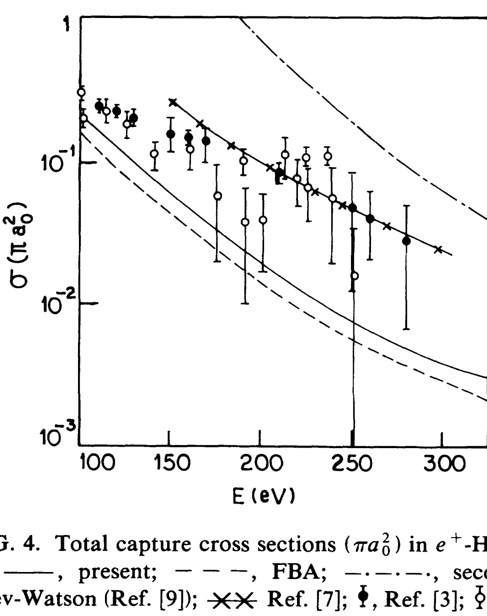

FIG. 4. Total capture cross sections (πa_0^2) in e^+-He scattering: ——, present; — — —, FBA; —·—·—·, second-order Fadeev-Watson (Ref. [9]); -×-×- Ref. [7]; ●, Ref. [3]; ○, Ref. [2].

TABLE II. Integrated ground- and excited-state capture cross sections (in units of πa_0^2) in e^+-He scattering. (Numbers in square brackets are powers of 10.)

Energy	1s		2s		2p	
(eV)	FBA	SBA	FBA	SBA	FBA	SBA
100	1.54[−01]	1.68[−01]	2.22[−02]	2.41[−02]	4.37[−03]	4.50[−03]
200	1.47[−02]	1.68[−02]	2.08[−03]	2.63[−03]	2.62[−04]	2.78[−04]
300	2.89[−03]	3.41[−03]	3.97[−04]	5.64[−04]	3.67[−05]	4.12[−05]
500	2.99[−04]	4.07[−04]	3.98[−05]	6.49[−05]	2.43[−06]	3.09[−06]
1000	1.01[−05]	1.51[−05]	1.32[−06]	2.89[−06]	4.40[−08]	6.08[−08]
2000	2.65[−07]	6.55[−07]	3.39[−08]	8.91[−08]	5.77[−10]	8.52[−10]

also true. Our second-order Born results for the total positronium-formation cross sections lie below the measured values of two groups in this energy region. The present calculations neglect the contribution of higher excited states ($n > 2$) and the continuum as intermediate states. We believe that this contribution may not reduce the difference between the theoretical predictions and measured values appreciably. Reasons for the differences between theoretical results and experimental data are not very clear to us. Our theoretical model originates from our experience in ion-atom charge-transfer processes. In our model we implicitly assumed that Ps formation is basically a two-step process, but this model may require modifications. In other words, the dynamics of the system may not be properly understood. On the other hand, the two sets of measured data differ appreciably from each other. Uncertainties in each measurement are also very large. More theoretical and experimental work is essential to understand the process of Ps formation by high velocity positrons.

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