Diagrammatic analysis of atom–squeezed-light interactions

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We study an atomic system interacting with a broadband squeezed vacuum and several applied classical fields. Exact rate equations are derived from the Gardiner-Collett master equation. A diagrammatic analysis is presented, which enables one to write down the contributions in a relatively simple manner and to provide a physical interpretation of the various terms. The method is applied in the following paper [Smart and Swain, Phys. Rev. A **45**, 6863 (1992)] to a study of resonance fluorescence.

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I. INTRODUCTION

Some time ago we showed how *exact* equations of motion for the diagonal elements of the density matrix may be derived which possess a rate-equation structure [1]. The treatment concentrated on the interacting atom–quantized-field system, although the method as presented applies to any quantized system described by a time-independent Hamiltonian. Recently, we have extended the method to deal with the interaction of an atomic system with a classical electromagnetic field [2]. In this paper we generalize the treatment to include the interaction with a broadband squeezed vacuum. Our starting point is the master equation of Gardiner and Collett [3]. (For some recent reviews of squeezing, see Ref. [4]). Since the treatment is exact, the higher-order contributions are complicated, and here we present a diagrammatic approach which enables one to write down the various terms of the solution with much less effort. The diagrammatic analysis greatly facilitates the physical interpretation of the contributing terms. In the following paper [5], the method is applied to obtain some results in the theory of a two-level atom interacting with an applied external field and a broadband squeezed vacuum.

II. THE RATE EQUATIONS FOR AN ATOM IN A SQUEEZED VACUUM

For simplicity, we use a system of units in which $\hbar=1$. Our model is that of an atomic system with nondegenerate energy levels E_i, which are labeled in such a way that $E_i > E_j$ if $i > j$. We consider its interaction with classical fields. The corresponding problem with quantized fields may be dealt with analogously. The Hamiltonian for the interacting system of atom and fields may be written as [2]

$$H=\sum_i E_i|i\rangle\langle i|+V\ ,$$
$$V(t)=\sum_{i,j}\mu_{ij}|i\rangle\langle j|e^{-i\omega_{ij}t}\ , \tag{1}$$

where $\mu_{ij}=\mathcal{P}_{ij}\mathcal{E}_\lambda$ is a coupling constant which is equal to the product of the dipole matrix element $\mathcal{P}_{ij}$ of the atom with the intensity $\mathcal{E}_\lambda$ of the electromagnetic field mode described by the index λ, whose frequency is ω_λ. In general, the mode index λ is a four-dimensional vector consisting of the wave vector and polarization index. As usual, we have made the rotating-wave approximation. We have also assumed that a given mode λ can only cause transitions between one pair of energy levels i and j. We can then uniquely identify the particular field frequency ω_λ with this pair of levels. Henceforth we use the notation $\omega_\lambda=\omega_{ij}$. To be definite, we assume $i>j$: this implies that ω_{ij} is positive. Note that ω_{ij} is not equated to the Bohr frequency (E_i-E_j)—the transition is not necessarily resonant. We adopt the convention that $\omega_{ii}=0$.

The master equation for the density matrix of a nonequally spaced atomic system, including the interaction with squeezed white light, has been given by Gardiner and Collett [3]. It describes the situation in which a separate broadband-squeezed source interacts with each transition. These sources are assumed to be independent and nonoverlapping. Thus the situation in which two distinct transition frequencies ω_{ij} and ω_{kl} are equal is excluded. Assuming the center frequency of the squeezed mode corresponding to the laser frequency ω_{ab} to be equal to that frequency, the master equation has the form

$$\dot\rho_{ab}=-iE_{a,b}\rho_{ab}-i\sum_c[V_{ac}(t)\rho_{cb}-\rho_{ac}V_{cb}(t)]$$
$$+\delta_{ab}\sum_c\gamma_{ca}\rho_{cc}-\Gamma_{ab}\rho_{ab}-\xi_{ab}\rho_{ba}e^{-2i\omega_{ab}t}\ , \tag{2}$$

where $E_{i,j}\equiv E_i-E_j$, $\Gamma_{ab}\equiv(\Gamma_a+\Gamma_b)/2$, and $\Gamma_a=\sum_c\gamma_{ac}$ is the total incoherent transition rate out of level $|a\rangle$. γ_{ac} is the incoherent transition rate between levels $|a\rangle$ and $|c\rangle$. We have

$$\gamma_{ij}=\chi_{ij}\times\begin{cases}(N_{ij}+1) & \text{if } i>j\\ N_{ij} & \text{if } i<j\ ,\end{cases} \tag{3}$$

and

$$\xi_{ij}=\chi_{ij}M_{ij}\ , \tag{4}$$

where χ_{ij} is a damping constant and N_{ij} and M_{ij} are squeezing parameters [3]. We emphasize again that the ij subscripts here label the $|i\rangle \rightarrow |j\rangle$ transition. N is real whilst M may be complex: they are connected by the inequality

$$|M_{ij}|^2 \leq N_{ij}(N_{ij}+1) \ . \tag{5}$$

Denoting the Laplace transform of $\rho(t)$ by $\tilde{\rho}(z)$, and defining

$$\sigma_{ab} \equiv \tilde{\rho}_{ab}(z+i\omega_{ba}) \ , \tag{6}$$

we obtain from the Laplace transform of (2) the following equation for the off-diagonal elements:

$$\varepsilon_{ab}\sigma_{ab} + i \sum_{c\ (\neq b)} \mu_{ac}\sigma_{cb} - i \sum_{c\ (\neq a)} \mu_{cb}\sigma_{ac} + \xi_{ab}\sigma_{ba} = B_{ab} \ , \qquad b \neq a \tag{7}$$

where

$$\varepsilon_{ab}(z) = z + i(E_{a,b} - \omega_{ab}) + \Gamma_{ab} \tag{8}$$

and the quantity

$$B_{ab}(z) = \rho_{ab}(0) - i\mu_{ab}[\tilde{\rho}_{bb}(z) - \tilde{\rho}_{aa}(z)] \tag{9}$$

contains diagonal elements of ρ. Formally, Eq. (7) differs from the corresponding equation in the absence of squeezing only in the last term, which has the interesting effect of connecting an off-diagonal element of the density matrix with its transpose. Squeezing also shows its effects in the expression for the γ_{ij}, where the squeezing parameter N_{ij} plays the role of a photon number, and γ_{ij} describes emission for $i > j$ and absorption for $i < j$.

If we set $\xi_{ij} = 0$ and give the N_{ij} a Bose-Einstein distribution, the master equation (2) describes a system interacting with a thermal heat bath. The essence of our method is to solve the system of Eqs. (7) for the σ_{ab} in terms of the B_{ab}, and then to substitute the result into the equation

$$(z+\Gamma_{aa})\tilde{P}_a = \rho_{aa}(0) - i \sum_{c\ (\neq a)} \mu_{ac}\sigma_{ca} + i \sum_{c\ (\neq a)} \mu_{ca}\sigma_{ac} + \sum_{c\ (\neq a)} \gamma_{ca}\sigma_{cc} \tag{10}$$

for the diagonal matrix elements $\tilde{P}_a(z) \equiv \tilde{\rho}_{aa}(z) = \sigma_{aa}$. The result may be written in the form of a rate equation in Laplace space. We first present and discuss our main results and then outline the derivation. We find

$$z\tilde{P}_a(z) - P_a(0) = \sum_{c\ (\neq a)} [\tilde{W}_{ca}(z)\tilde{P}_c(z) - \tilde{W}_{ac}(z)\tilde{P}_a(z)] + \Delta P_a \ , \tag{11}$$

where to third order in the interactions, the *transition rates* $\tilde{W}(z)$ are given by

$$\tilde{W}_{ca}(z) \equiv \gamma_{ca} + \tilde{w}_{ca}(z) = \gamma_{ca} + 2\,\mathrm{Re}\left[\frac{\mu_{ac}\mu_{ca}}{(ca)} + \frac{\mu_{ca}\xi_{ac}\mu_{ca}}{(ca,ac)} - i \sum_{d}^{*} \left[\frac{\mu_{ca}\mu_{ad}\mu_{dc}}{(da,dc)} + \frac{\mu_{ca}\mu_{ad}\mu_{dc}}{(ac,dc)} + \frac{\mu_{ca}\mu_{ad}\mu_{dc}}{(ca,da)} \right] + \cdots \right] \tag{12}$$

and

$$\Delta P_a = -\sum_{b}^{*} \frac{i\mu_{ab}\rho_{ba}(0)}{(ba)} + \sum_{b}^{*} \frac{i\mu_{ab}\xi_{ba}\rho_{ab}(0)}{(ab,ba)} + \cdots + \text{c.c.} \tag{13}$$

The transition rate $\tilde{W}_{ca}(z)$ divides into the sum of an *incoherent* rate γ_{ca} and a *coherent* rate $\tilde{w}_{ca}(z)$. The left-hand side of Eq. (11) is the Laplace transform of $\dot{P}_a(t)$. The final term on the right-hand side, ΔP_a, is the contribution from initial nonzero values of off-diagonal elements of the density matrix. In many applications, the density matrix is assumed to be diagonal at $t=0$, when this term is zero. For simplicity, we assume this to be the case here. We will discuss it in more detail later on. In any case, it describes transient effects which do not contribute to the steady-state solutions for the atomic occupation probabilities.

The definition of the starred summation $\sum^{*}_{i,j,\ldots}$ is that one sums over all values of $i,j,\ldots$ except those values that would make two terms in the $(ab,cd,\ldots)$ functions which appear as denominators in the summand equal or diagonal. That is, terms such as $(\ldots,pq,\ldots,pq,\ldots)$ and $(\ldots,pp,\ldots)$ are excluded. Thus in Eq. (12), the summation is over all values of d except $d=a$ and $d=c$.

Equation (11) is the major result of this paper. It may be considered as a recasting of the master equation of Gardiner and Collett into rate-equation form. It has a simple physical interpretation: in Laplace space, it states that the net rate of change of P_a is equal to the difference between the rate of transitions *into* level $|a\rangle$ from all other levels $|c\rangle$ (the first term on the right-hand side) and the rate of all transitions *out of* atomic level $|a\rangle$ (second term on the right-hand side). This simple structure makes it possible to write down the rate equations for a multilevel system by inspection.

The bulk of the physics of the problem is now encapsulated in the rates $\tilde{w}_{ca}(z)$. Although these quantities are real, we have not been able to show that they are positive definite. Indeed, we have found particular examples where they are zero: this is a manifestation of quantum-mechanical interference effects.

The quantities (ab), (ab,cd), etc. are fully defined mathematically in Eqs. (16)–(18). We conclude this section by giving a brief account of their physical properties. $(ab)^{-1}$ is the Laplace transform of the probability for the density-matrix element σ_{ab} remaining in this state at time

t if it was in this state at time $t=0$. Alternatively, it may be thought of as a quantity whose poles determine the exact energy differences of the interacting atom-field system—that is, it may be considered as a kind of "propagator" in the many-body theory sense. We have to regard the energies as complex because Eq. (2) includes damping effects. For example, we have approximately

$$(ab)=z+i(E_{a,b}-\omega_{ab})+\Gamma_{ab}+\sum_c^*\left[\frac{|\mu_{ac}|^2}{z+i(E_{c,b}-\omega_{cb})+\Gamma_{cb}+\cdots}+\frac{|\mu_{bc}|^2}{z+i(E_{a,c}-\omega_{ac})+\Gamma_{ac}+\cdots}+\cdots\right]. \tag{14}$$

To lowest order the zeros (in iz) of (ab) give the zeroth-order energy differences of the interacting system

$$(E_{a,b}-\omega_{ab})+i\Gamma_{ab}\ ,$$

whilst retaining the next set of terms would give the complex-energy differences of the interacting system to second order in $|\mu|$. In this first-order iteration, the terms quadratic in $|\mu|$ would represent Stark shifts induced by the applied fields.

It is apparent that expressions (12) are complicated in the general case. For this reason, we present diagrammatic methods for their evaluation. It is convenient first to outline the derivation of the rate equations (11).

III. OUTLINE DERIVATION AND DIAGRAMMATIC ANALYSIS

Using the methods previously described [1,2], we may show that the solution of the set of equations (7) is

$$\sigma_{ab}=\frac{B_{ab}}{(ab)}-\frac{B_{ba}\xi_{ab}}{(ab,ba)}-\sum_c^*\left[\frac{i\mu_{ac}B_{cb}}{(ab,cb)}-\frac{i\mu_{cb}B_{ac}}{(ab,ac)}\right]+\sum_c^*\left[\frac{i\xi_{ab}\mu_{bc}B_{ca}}{(ca,ba,ab)}-\frac{i\xi_{ab}B_{bc}\mu_{ca}}{(bc,ba,ab)}+\frac{i\mu_{ac}\xi_{cb}B_{bc}}{(bc,cb,ab)}-\frac{iB_{ca}\xi_{ac}\mu_{cb}}{(ca,ac,ab)}\right]+\sum_{c,d}^*\left[-\frac{\mu_{ac}\mu_{cd}B_{db}}{(ab,cb,db)}+\frac{\mu_{ac}\mu_{db}B_{cd}}{(ab,cb,cd)}+\frac{\mu_{cb}\mu_{ad}B_{dc}}{(ab,ac,dc)}-\frac{\mu_{cb}\mu_{dc}B_{ad}}{(ab,ac,ad)}\right]+\cdots\ . \tag{15}$$

The starred summation sign has the same meaning as in Eq. (12).

We now define the $(ab),(ab,cd),\ldots$ functions. We shall call these quantities propagator denominators (PD's). In general, $(ab)\neq(ba)$, but in those functions with multiple arguments, the order of the arguments in the brackets is unimportant—that is, $(ab,cd,ef)=(cd,ab,ef)=(ef,cd,ab)$, etc. The single-argument PD is defined as

$$(ab)=\varepsilon_{ab}+\sum_c^*\left[\frac{\mu_{ac}\mu_{ca}}{(cb)_{ab}}+\frac{\mu_{bc}\mu_{cb}}{(ac)_{ab}}\right]-\frac{\xi_{ab}\xi_{ba}}{(ba)}-\sum_{c,d}^*\left[\frac{i\mu_{ac}\mu_{cd}\mu_{da}}{(cb,db)_{ab}}-\frac{i\mu_{bc}\mu_{cd}\mu_{db}}{(ac,ad)_{ab}}\right]+\cdots\ . \tag{16}$$

The subscripts on the propagators on the right-hand side impose additional constraints on the variables under the starred summation sign: when the starred summation is carried out, those terms are excluded which would make any of the arguments of the PD's under the summation sign equal to a subscript. Consider, for example, the sum over c in Eq. (16). The restrictions imposed on the allowed values of c by the subscripts are that $c\neq a$ in the first term and $c\neq b$ in the second term. We emphasize that these constraints are additional to those imposed by the exclusion of diagonal or repeated arguments.

The PD with two arguments is defined by the relation

$$(ab,cd)=(ab)(cd)_{ab} \tag{17}$$

[or equivalently, as $(cd)(ab)_{cd}$], which splits it up into the product of two single argument PD's. The quantity $(ab)_{ij}$ is defined analogously to Eq. (16)

$$(ab)_{ij}=\varepsilon_{ab}+\sum_c^*\left[\frac{\mu_{ac}\mu_{ca}}{(cb)_{ab,ij}}+\frac{\mu_{bc}\mu_{cb}}{(ac)_{ab,ij}}\right]-\frac{\xi_{ab}\xi_{ba}}{(ba)_{ij}}-\sum_{c,d}^*\left[\frac{i\mu_{ac}\mu_{cd}\mu_{da}}{(cb,db)_{ab,ij}}-\frac{i\mu_{bc}\mu_{cd}\mu_{db}}{(ac,ad)_{ab,ij}}\right]+\cdots\ . \tag{18}$$

That is, in the definition of $(ab)_{ij}$ the subscripts ij appears in every PD. Higher-order members are similarly defined. For example, $(ab,cd,ef)\equiv(ab)(cd)_{ab}(ef)_{ab,cd}$. For a finite system of equations (i.e., a finite number of atomic levels), the restrictions on the sums cause the series to terminate, and the exact solution is obtained.

The terms in Eq. (15) may be interpreted as follows. We have argued that the quantities $(ab)^{-1}$ may be loosely thought of as propagators—that is, $(ab)^{-1}$ is (the Laplace transform of) the conditional probability that the density-matrix remains σ_{ab} at time t if it was σ_{ab} at time zero. More briefly, we may say that it is the probability that the system remains in the state ab at time t if it was in state ab at time zero. To interpret Eq. (15) we regard B_{ab} as the absolute probability that the system is in the state ab at time zero. Thus the first term in Eq. (15) may be regarded as the probability that the system is in state ab initially times the probability that it remains there. The four factors in the second term [writing (ab,ba) as $(ab)(ba)_{ab}$] are interpreted as the probability that the sys-

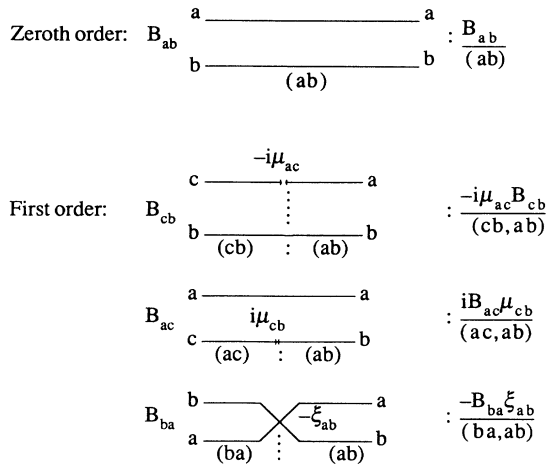


FIG. 1. The diagrams for the off-diagonal elements

$$\sigma_{ab}=\frac{B_{ab}}{(ab)}-\frac{B_{ba}\xi_{ab}}{(ab,ba)}-\sum_{c}^{*}\left[\frac{i\mu_{ac}B_{cb}}{(ab,cb)}-\frac{i\mu_{cb}B_{ac}}{(ab,ac)}\right]+\cdots$$

up to and including first-order terms, together with their contributions.

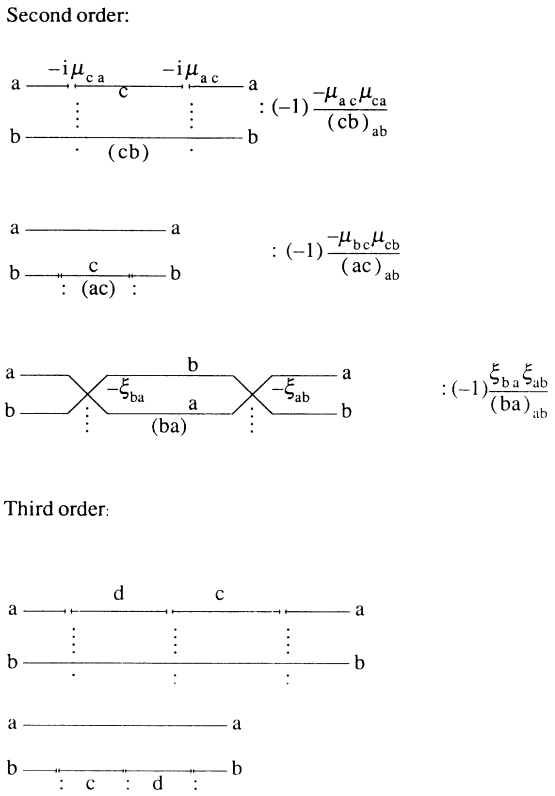


FIG. 2. The diagrams for the propagators (ab), showing second- and third-order terms. The contributions of the second-order terms are shown explicitly. Note that for this quantity, the product of the factors has to be multiplied by -1 to obtain the contribution.

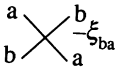
tem begins in the state ba, propagates in this state for a time, then is switched to the state ab by the squeezing interaction, and finally propagates in the state ab. The third term has a similar interpretation, but here the switching from state cb or state ac is brought about by the dipole interaction.

Expression (15), whilst exact, is not simple. However, the various terms in the series and their contributions can be written down relatively easily by making use of a diagrammatic interpretation, which we present in Fig. 1 for the case of σ_{ab}. For a particular diagram, we draw two lines, labeling the right-hand ends with a and b to represent the final "state" σ_{ab}, and labeling the left-hand ends to represent the initial state—say with c and d if the initial state is σ_{cd}. We obtain a diagram for each possible initial state. The amplitude for the system starting in the initial state is B_{cd}. Time is assumed to run from left to right. For the zeroth-order contribution, Fig. 1(a), the situation is trivial: the system begins in the state σ_{ab} and remains in that state. The contribution is thus $B_{ab}/(ab)$ [$(ab)^{-1}$ represents the probability for the system to remain in the state σ_{ab}]. In the first of the first-order diagrams shown in Fig. 1(b), the system begins in the state σ_{cb}, but the dipole interaction, marked by a cross in the upper line, causes a transition from the state c to the state a. With a cross on the *upper* line marking a transition from a to c, we associate the *Hermitean conjugate* interaction matrix element $-i\mu_{ca}$ as a factor

a $\xrightarrow{-i\mu_{ca}}$ c (upper line) .

If a cross occurs in the *lower* line we associate with it the interaction matrix element $i\mu_{ac}$,

a $\xrightarrow{i\mu_{ac}}$ c (lower line) .

The squeezing interaction causes a switch of states, represented by a crossover diagram  .

The sign and nature of these matrix elements arises from the form of the equation for σ_{ab}, which is given in the caption for Fig. 1. Propagators are associated with each state σ_{ij} that appears in the diagram. For the first of the first-order diagrams the propagators are $(cb)^{-1}$ and $(ab)^{-1}$. The contribution of each diagram is obtained by taking the product of all these factors, and the total up to the order being considered is obtained by adding the contribution of the diagrams corresponding to all possible initial states.

A similar diagrammatic interpretation of the terms in the series for (ab) is given in Fig. 2. The rules are the same as for Fig. 1, except that (i) the initial and final states ab are the same; (ii) propagators are only associated with the intermediate states; and (iii) each contribution must be multiplied by the factor (-1). For such a quantity as $(ab)_{cd}$ the rules are the same, except that diagrams which would give rise to the factor $(cd)\ldots$ are excluded.

Equation (15) gives the solution for the off-diagonal elements in terms of the diagonal elements and the initial coherences $\rho_{ab}(0)$ through the factors B_{ab}. We substitute these solutions into Eq. (10) for $\sigma_{aa}\equiv\rho_{aa}(z)\equiv\tilde{P}_a(z)$. We obtain

$$(z+\Gamma_{aa})\sigma_{aa}=\rho_{aa}(0)+\Delta P_a+\sum_{c\,(\neq a)}\gamma_{ca}\sigma_{cc}$$
$$-i\sum_{c\,(\neq a)}\mu_{ac}\left[\frac{-i\mu_{ca}(\sigma_{aa}-\sigma_{cc})}{(ca)}-\sum_d{}^{*}\frac{\mu_{cd}\mu_{da}(\sigma_{aa}-\sigma_{dd})}{(ca,da)}+\sum_d{}^{*}\frac{\mu_{da}\mu_{cd}(\sigma_{dd}-\sigma_{cc})}{(ca,cd)}+\cdots\right]$$
$$+i\sum_{c\,(\neq a)}\mu_{ca}\left[\frac{-i\mu_{ac}(\sigma_{cc}-\sigma_{aa})}{(ac)}-\sum_d{}^{*}\frac{\mu_{ad}\mu_{dc}(\sigma_{cc}-\sigma_{dd})}{(ac,dc)}+\sum_d{}^{*}\frac{\mu_{dc}\mu_{ad}(\sigma_{dd}-\sigma_{aa})}{(ac,ad)}+\cdots\right]$$
$$+i\sum_{c\,(\neq a)}\left[\frac{\mu_{ac}^2\xi_{ca}(\sigma_{aa}-\sigma_{cc})}{(ca,ac)}+\frac{\mu_{ca}^2\xi_{ac}(\sigma_{aa}-\sigma_{cc})}{(ac,ca)}+\cdots\right], \tag{19}$$

where ΔP_a is given by Eq. (13). The diagrammatic series for this term is shown in Fig. 3. Here we list all possible diagrams which take one from any initial off-diagonal state ij to the final state aa.

We relabel and rearrange terms in the above series to obtain our main result, the rate equation (11). We have used the properties

$$(ij,kl,\ldots)=(ji,lk,\ldots)^{*}\ ,$$
$$(ij,kl)_{ab,cd,\ldots}=(ji,lk)^{*}_{ba,dc,\ldots}\ , \tag{20}$$

which are easily inferred from the definitions (16)–(18).

The contributions to $\tilde{w}_{ca}$ may be obtained using the diagrammatic analysis illustrated in Fig. 4, where we show the diagrams that contribute up to third order. The rules are the same as in earlier figures, with the initial state being cc and the final state aa, and only intermediate-state propagators being taken into consideration. In addition, we omit all the mirror-image diagrams of those shown in Fig. 4 because their contribution is just the Hermitean conjugate of the contribution of the original diagrams. The total contribution is thus twice the real part of the contributions of the diagrams shown.

The quantity

$$w_a(z)\equiv\sum_{c\,(\neq a)}w_{ac}(z) \tag{21}$$

is the total rate of coherent transitions out of level a. One can show that, to third order, it is given by

$$w_a=2\,\mathrm{Re}\left[\sum_c\frac{\mu_{ac}\mu_{ca}}{(ac)}-i\sum_{c,d}\frac{\mu_{cd}\mu_{da}\mu_{ac}}{(ca,da,ea)}-\sum_c{}^{*}\frac{\mu_{ca}\xi_{ac}\mu_{ca}}{(ca,ac)}+\cdots\right]. \tag{22}$$

It is represented by the particularly simple set of diagrams we show in Fig. 5 (omitting mirror images). Here both lines begin and end with the state a, and we must multiply the contribution of each diagram by (-1).

Frequently, one is interested only in the steady-state solutions. These can be found from the Laplace transforms without making a Laplace inversion. One way is to solve the algebraic equations (11) and then use the identity

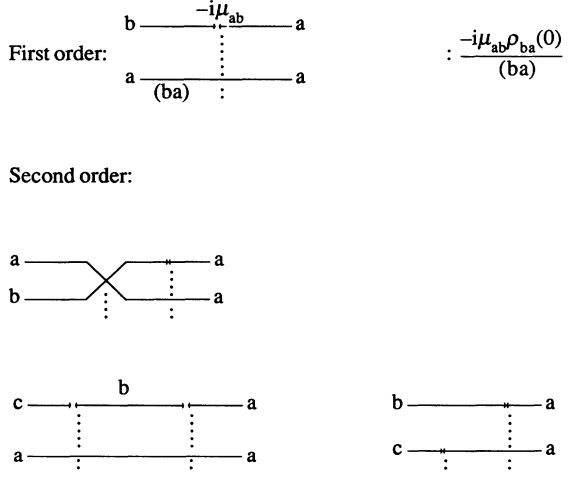


FIG. 3. The diagrams for the quantity $\Delta P_a(0)$ (which incorporates the initial values of the coherences) up to and including second-order terms. For all orders there are also mirror-image diagrams (the mirror being below and parallel to the diagram) which contribute the complex conjugate of these terms. These mirror-image diagrams have not been shown.

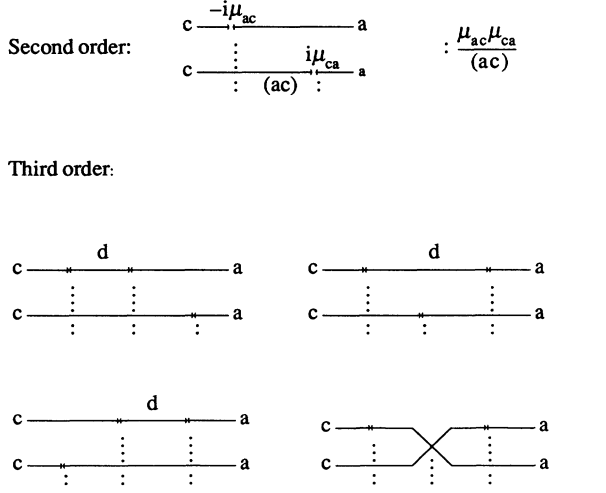


FIG. 4. The diagrams for the generalized transition rate $\tilde{w}_{ca}(z)$, up to and including the third-order terms. The mirror-image diagrams, which contribute the complex conjugate of these terms, have not been shown.

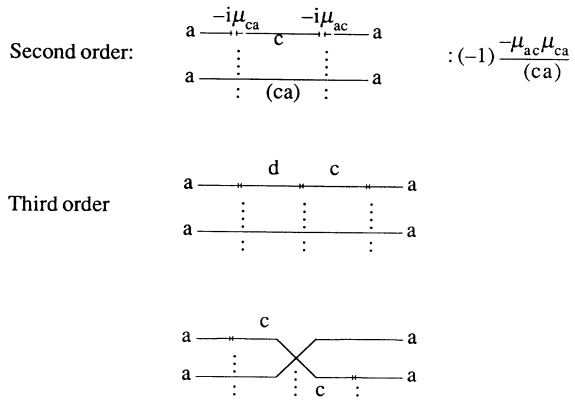


FIG. 5. The diagrams for the quantity $\tilde{w}_a(z)$, up to and including the third-order terms. Note that for this quantity the product of the factors has to be multiplied by (-1) to obtain the contribution. The mirror-image diagrams, which contribute the complex conjugate of these terms, have not been shown.

$$\lim_{t\to\infty} P(t)=\lim_{z\to 0} z\tilde{P}(z) \ . \tag{23}$$

Alternatively, one may solve the homogeneous system of equations

$$\dot{P}_a(\infty)=0=\sum_{c\ (\neq a)}\{[\gamma_{ca}+\tilde{w}_{ca}(0)]P_c(\infty) -[\gamma_{ac}+\tilde{w}_{ac}(0)]P_a(\infty)\} \ , \tag{24}$$

where z has been set to zero in the transition rates.

IV. CONCLUSION

We have used the master equation of Gardiner and Collett, which describes the interaction of an atomic system with a broadband squeezed vacuum, to derive rate equations for such a system interacting with applied classical light fields. The resulting equations, being exact, are complicated—at least, in the higher orders. We have presented a diagrammatic analysis which enables one to write down the contributions much more simply than using the algebraic expressions for the solutions. The diagrams also assist in the physical interpretation of the various terms.

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