Entangled coherent states

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(Received 8 October 1991)

The nonlinear Mach-Zehnder interferometer is presented as a device whereby a pair of coherent states can be transformed into an entangled superposition of coherent states for which the notion of entanglement is generalized to include nonorthogonal, but distinct, component states. Each mode is directed to a homodyne detector. We show that there exist nonclassical intensity correlations at the output ports of the homodyne detectors which facilitate a test of local realism. In contradistinction to previous optical schemes which test local realism, the initial state used here possesses a positive Glauber-Sudarshan representation and is therefore a semiclassical state. The nonlinearity itself is responsible for generating the nonclassical state.

PACS number(s): 42.50.Wm, 42.50.Dv, 03.65.Bz

I. INTRODUCTION

One of the most outstanding features of quantum systems is the property of nonlocal superposition which was elucidated by Einstein, Podolsky, and Rosen [1]. Essentially such a system involves correlated spatially-separated states and such systems have been used as tests of quantum mechanics via the class of theories known as local realism [2]. Thus far experimental evidence appears to support the quantum theory [3]. A particularly simple and dramatic realization of the nonlocal features of quantum systems appears in the two-particle entangled states [4]. Here we generalize the notion of entangled particle states to allow for entangled coherent states and discuss a method for generating such states.

A number of authors have discussed the violation of classical inequalities in a quantum optical context [5–7]. In Ref. [5] it was shown that if the state has a positive Glauber-Sudarshan P representation [8, 9], i.e., the state can be given a semiclassical interpretation, then no nonclassical correlations can arise. In the work of Tan, Walls and Collett [7] the state used was a single-photon entangled state. As this state is derived from a number state it clearly cannot have a positive Glauber-Sudarshan P representation. In this paper we use entangled coherent states. This state is produced by sending a coherent state and a vacuum state into a nonlinear Mach-Zehnder interferometer (NLMZI) which is a Mach-Zehnder interferometer that contains a Kerr nonlinear medium in one (or in both) arms [10, 11]. Even though the input state does possess a Glauber-Sudarshan P representation the Kerr nonlinearity leads to an entangled state for which no Glauber-Sudarshan P representation exists. Thus it is possible to exhibit nonclassical correlations for states which initially possess a Glauber-Sudarshan P representation via a nonlinear interaction.

We calculate the correlation function which is based on intensity correlations from the output ports of homodyne detection for each of the two modes of the entangled state [6, 7] and show that the resulting intensity-correlation function has the same form as that found in Ref. [7], with the intensity of the input coherent state appearing in place of the one-photon intensity. This indicates that the phase sensitivity of these correlation measurements is not due to the phase dependence of the coherent amplitude but rather probes the quantum-mechanical phase of the entangled state.

A two-particle entangled quantum state can be expressed as

$$|\psi> = 2^{-1/2}\left(|\alpha>_1|\beta>_2 + e^{i\varphi}|\gamma>_1|\delta>_2\right) \qquad (1)$$

for $|\alpha>_1$ and $|\gamma>_1$ pure states of particle 1 and similarly for $|\beta>_2$ and $|\delta>_2$ for particle 2. Moreover the state cannot be reduced to the product state $|\epsilon>_1|\eta>_2$ for any ϵ or η. An optical analog to a "particle" corresponds to an excitation of the electomagnetic field in a given mode, i.e., a photon; thus the optical case might involve the entangled photon-number state such as [7]

$$2^{-1/2}(e^{i\pi/4}|0>_a|1>_b + e^{-i\pi/4}|1>_a|0>_b), \qquad (2)$$

which has been discussed in the context of testing Bell's inequalities. It is important to note that the particle (or photon-number) states are orthonormal vectors in the Hilbert space, a condition which must be relaxed when considering entangled coherent states.

In Eq. (2) the entanglement involves Fock number states. On the other hand coherent states of the electromagnetic field are considered to be the closest quantum counterpart to states of the classical radiation field [8]. Whereas the Fock number states of the radiation field exhibit highly nonclassical properties and are associated with corpuscular features of the electromagnetic field [12], coherent states are related to the classical wavelike properties of light. However the quasiclassical features of coherent states sometimes give way to distinctly quantum effects and these deviations from classical expectations can be noteworthy. For example a nonlinear medium might cause a coherent state to become a superposition of macroscopically distinct coherent states

[13] which presents an optical analog to "Schrödinger's cat" being in a superposition of being dead or alive [14]. As the philosophical difficulties associated with applying quantum mechanics at the macroscopic level remains a controversial topic, this research continues to be relevant [15].

The study of macroscopic superpositions of distinct coherent states is interesting but does not involve the important nonlocal properties of quantum physics. On the other hand the tests of local realism which involve the generation of states of light from a common source are generally not amenable to simple classical interpretations. Here we introduce a scheme for producing an entangled superposition of coherent states of light. The individual coherent states retain the desirable quasiclassical properties of coherent states but nonlocal features arise due to the entanglement. Here we combine two desirable features: from the first category we desire an entanglement of two essentially classical states of the radiation field and, from the second category, we seek to have the correlated states spatially separated. Moreover the entangled state is obtained from an initially semiclassical state of the electromagnetic field (the coherent state).

II. NONLINEAR MACH-ZEHNDER INTERFEROMETER

The scheme essentially involves a Mach-Zehnder interferometer which possesses two input ports and two output ports. Within one arm of the interferometer a nonlinear Kerr medium is placed which we approximate as a nonlinear oscillator in a single-mode treatment [16] and this interferometer with an internal nonlinear medium is referred to as a nonlinear Mach-Zehnder interferometer [10, 11]. For simplicity we assume that the nonlinear interferometer is lossless and a single-mode treatment is applied. The dynamical description involves two input modes a and b, with corresponding annihilation operators $\hat{a}$ and $\hat{b}$. The input fields for modes a and b are the coherent states $|\alpha>_a$ and $|\beta>_b$, respectively, for the coherent state with the number-state representation [8]

$$<n|\alpha>=\exp(-|\alpha|^2/2)(n!)^{-1/2}\alpha^n. \tag{3}$$

The NLMZI is composed of two beam splitters with a nonlinear Kerr medium in one arm and the corresponding unitary transformation for the input fields is [17]

$$\hat{U}=\hat{B}\hat{K}\hat{B} \tag{4}$$

for

$$\hat{B}=\exp[(i\pi\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger)/4] \tag{5}$$

being the beam-splitter transformation and

$$\hat{K}=\exp[-i\chi(\hat{a}^\dagger\hat{a})^2] \tag{6}$$

the Kerr transformation. In Eq. (6) the nonlinearity coefficient χ is proportional to the nonlinear coefficient $\chi^{(3)}$ of the medium and the interaction length. The initial state which enters the NLMZI is assumed to be the coherent product state $|\alpha>_a|\beta>_b$ and the output state is

$$|\psi>_{\rm out}=\hat{U}|\alpha>_a|\beta>_b\,. \tag{7}$$

The beam-splitter transformation given in Eq. (5) transforms the input product state $|\alpha>_a|\beta>_b$ into

$$\hat{B}|\alpha>_a|\beta>_b=|2^{-1/2}(\alpha+i\beta)>_{\tilde{a}}|2^{-1/2}(\beta+i\alpha)>_{\tilde{b}} \tag{8}$$

for $\tilde{a}$ and $\tilde{b}$ the two beam-splitter output fields. Thus the output state is also a direct product of coherent states at its output given a direct product at the input. There is no entanglement of states at this stage. The effect of the nonlinearity in one arm of the medium is interesting in that it can transform a coherent state into a superposition of two macroscopically distinct coherent states [13]. The Kerr medium transforms the coherent state to [16]

$$\hat{K}|\alpha>=\exp\left(-|\alpha|^2/2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\exp(-i\chi n^2)|n>\,. \tag{9}$$

Henceforth we fix $\chi=\pi/2$; therefore

$$\hat{K}|\alpha>=2^{-1/2}(e^{-i\pi/4}|\alpha>+e^{i\pi/4}|-\alpha>). \tag{10}$$

These superposition states have proved to be very useful in constructing optical analogs to Schrödinger's cat state [14].

By combining the Kerr medium and two beam splitters in the NLMZI we obtain the result, for $\chi=\pi/2$,

$$\hat{U}|\alpha>_a|\beta>_b=2^{-1/2}(\,e^{-i\pi/4}|i\beta>_{a'}|i\alpha>_{b'}+e^{i\pi/4}|-\alpha>_{a'}|\beta>_{b'}) \tag{11}$$

for a' and b' the output fields from the NLMZI. At this stage it is worth reflecting on the subtle nature of Eq. (11). The state is essentially a superposition of coherent state $|\alpha>$ in one mode and $|\beta>$ in the other mode (ignoring phases). The state is an entanglement of the quasiclassical product state of a coherent state in each mode. Drawing a distant analogy to Schrödinger's cat, this would be like having two distinct cats, α and β, in two separated boxes, a and b. The state is analogous to a superposition of cat α in box a and cat β in box b with cat α in box b and cat β in box a.

Although the states $|\alpha>$ and $|\beta>$ are not orthonormal we see that the inner product

$$|<\alpha|\beta>|^2=\exp\left(-|\alpha-\beta|^2\right) \tag{12}$$

rapidly tends to zero provided that the distance between α and β becomes large on the complex plane. It is essential here to generalize the notion of entangled states to include entangled coherent states. Earlier it was noted that the entanglement in Eq. (1) consisted of orthonormal states. The state (11) does not satisfy this requirement except in the limit that the coherent field amplitudes α and β are macroscopically separated in the complex plane and Eq. (12) tends to zero. If $\beta=\pm\alpha$ the state is *not* an entanglement. Hence the entanglement of two coherent states is a state which satisfies Eq. (11) and for which expression (12) is negligible. Essentially the overlap between the two coherent states, perhaps considered as the overlap of the Wigner functions for the coherent states in the phase plane, must be small for the states to be distinct. The separation of coherent amplitudes is analogous to the earlier statement that the cats α and β are distinct. Orthonormal states satisfy the requirement (12) as the inner product is trivially zero for distinct states.

As a special case of (11), we fix $\beta = 0$ and obtain

$$\hat{U}|\alpha>_a |0>_b = 2^{-1/2}(e^{-i\pi/4}|0>_{a'} |i\alpha>_{b'} + e^{i\pi/4}|-\alpha>_{a'} |0>_{b'}). \tag{13}$$

which is simpler than expression (11) and permits a simple description of the nonlocal features of entangled coherent states. There is a similarity between this state and the entangled photon-number state in Eq. (2) which has been discussed in the context of testing Bell's inequalities [7]. In order that the states are distinct, the overlap between $|0>$ and $|\alpha>$, given by $\exp(-|\alpha|^2/2)$, must be small; thus we assume that α is large.

III. INTENSITY MEASUREMENT OF ONE MODE

The nonlocal nature of the state in Eq. (13) is revealed by performing an intensity measurement at one of the spatial modes. In this model, an intensity measurement corresponds to photon counting. Thus we take the intensity operator for output mode a' to be the number operator $\hat{I}_{a'} = \hat{a}'^{\dagger}\hat{a}'$. Once again drawing an analogy with a cat in a superposition of being in "box a'" and in "box b'", we expect that detection of the presence (absence) of the "cat" in box a' requires that the cat is absent (present) in box b'. More precisely an ideal photodetection performed at port a' and producing the photon-number value m collapses the a' state to the Fock number state $|m>_{a'}$. A count of m photons at port a' thus prepares the port b' state as

$$\begin{aligned}|\psi>_{b'} &= \sum_{n=0}^{\infty} |n>_{b'}\ {}_{a'}<m|\ {}_{b'}<n|\hat{U}|\alpha>_a |0>_b \\ &= N^{-1}\{e^{-i\pi/4}\delta_{m0}|i\alpha>_{b'} + e^{i\pi/4}\exp(-|\alpha|^2/2) \\ &\qquad \times([-\alpha]^m/\sqrt{m!})|0>_{b'}\}\end{aligned} \tag{14}$$

for

$$N^2 = \exp(-|\alpha|^2)\frac{|\alpha|^{2m}}{m!} + \delta_{m0}. \tag{15}$$

Thus a measurement of $m = 0$ photons at a' produces the state

$$|\psi>_{b'} = \frac{e^{-i\pi/4}|i\alpha>_{b'} + e^{i\pi/4}\exp(-|\alpha|^2/2)|0>_{b'}}{\sqrt{1+\exp(-|\alpha|^2)}}. \tag{16}$$

For sufficiently macroscopic coherent states ($|\alpha| \to \infty$), the state in port b' rapidly converges to $|i\alpha>$. Similarly, a measurement of $m \neq 0$ photons results in the b' state being reduced to the vacuum state with an unimportant phase factor.

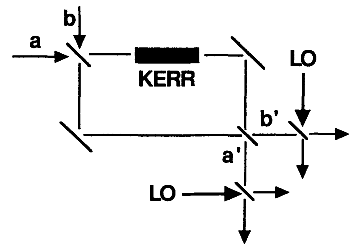

FIG. 1. The nonlinear Mach-Zehnder interferometer with an internal Kerr medium in one arm and two input modes a and b. Each output mode (a' and b') is directed to a separate homodyne detector where the field interferes with a local oscillator (LO). The intensity correlations at the outputs of the homodyne detector are directly measured.

IV. NONCLASSICAL CORRELATION FUNCTIONS

The two output modes of the NLMZI are directed towards two spatially separated homodyne detectors as shown in Fig. 1. Mode a is mixed with a coherent local oscillator of amplitude γ while mode b is mixed with another coherent local oscillator of complex amplitude $\gamma e^{i\phi}$. Thus ϕ is the phase difference between the two local oscillators. This phase difference is analogous to the difference in the local polarization settings in the experiment of Aspect and co-workers [3]. This detection scheme has been discussed by Grangier, Potasek, and Yurke [18] and by Tan and co-workers [6, 7].

Each of the two homodyne detectors has two beam-splitter output modes designated by annihilation operators $\tilde{a}$, $\tilde{c}$ (for mode a) and $\tilde{b}$, $\tilde{d}$ (for mode b). These four output modes are directed onto photoelectron counters. The pure state which is measured by the photoelectron counters is obtained from expression (11):

$$\begin{aligned}\hat{B}_{a'}\hat{B}_{b'}\hat{U}|\alpha>_a|\beta>_b \frac{1}{\sqrt{2}}e^{-i\pi/4}\Bigg(&\left|i\frac{\beta+\gamma}{\sqrt{2}}\right\rangle_{\tilde{a}}\left|i\frac{\alpha+\gamma e^{i\phi}}{\sqrt{2}}\right\rangle_{\tilde{b}}\left|\frac{\gamma-\beta}{\sqrt{2}}\right\rangle_{\tilde{c}}\left|\frac{\gamma e^{i\phi}-\alpha}{\sqrt{2}}\right\rangle_{\tilde{d}} \\ &+i\left|\frac{i\gamma-\alpha}{\sqrt{2}}\right\rangle_{\tilde{a}}\left|\frac{\beta+i\gamma e^{i\phi}}{\sqrt{2}}\right\rangle_{\tilde{b}}\left|\frac{\gamma-i\alpha}{\sqrt{2}}\right\rangle_{\tilde{c}}\left|\frac{\gamma e^{i\phi}+i\beta}{\sqrt{2}}\right\rangle_{\tilde{d}}\Bigg).\end{aligned} \tag{17}$$

Of interest is the correlation function

$$E(\phi) = \frac{<:(\hat{I}_{\tilde{b}} - \hat{I}_{\tilde{d}})(\hat{I}_{\tilde{a}} - \hat{I}_{\tilde{c}}):>}{<:(\hat{I}_{\tilde{b}} + \hat{I}_{\tilde{d}})(\hat{I}_{\tilde{a}} + \hat{I}_{\tilde{c}}):>}, \tag{18}$$

where I_λ is the intensity of mode λ. We calculate this correlation function for the state in Eq. (13). For simplicity we take α as real and write γ as $\gamma e^{i\theta}$ for γ real and θ represents the phase difference between the coherent amplitude of the input state and the amplitude of the homodyne detector for mode a. The result is

$$E(\phi) = \frac{-\alpha^2}{\alpha^2 + \gamma^2} \cos\phi. \tag{19}$$

We note that this result is independent of θ and depends only on the phase difference between the two spatially separated local oscillators. When $\alpha = 1$ this result is identical to that obtained by Tan, Walls, and Collett for the entangled one-photon state (apart from an unimportant phase shift) [7]. The fact that this correlation function depends only on the phase difference between the two local oscillators indicates that this detection scheme directly probes nonlocal quantum correlations in the entangled state.

If the coefficient of $\cos\phi$ in the correlation function $E(\phi)$ has a magnitude greater that $1/\sqrt{2}$, then a violation of the classical Bell inequality will occur [2, 6]. The entangled quantum state discussed here will thus violate the Bell inequality for an intensity ratio which satisfies the inequality $(\gamma/\alpha)^2 < \sqrt{2} - 1$.

Suppose that inefficient photodetection is allowed into the calculation. A photodetector can be represented by a beam splitter followed by a perfectly efficient photodetector [19]. The signal field enters one port of the beam splitter and a vacuum state enters the other port. One output port of the beam splitter is detected by the perfect photodetector. For the coherent state $|\alpha>$ being detected by a photodetector with efficiency η, the result is equivalent to a perfect photodector which counts photons for the coherent state $|\sqrt{\eta}\alpha>$. Thus inefficient photodetection of the state (17) does not change expression (19): the η parameter cancels out of the expression. Hence the violation of Bell's inequality is independent of the detector efficiency. Of course it is implicitly assumed in the model that the photons which are not counted are randomly selected by the photodetection process.

V. CONCLUSION

The model presented here presents a possible simple scheme for producing entangled quasiclassical radiation states. Moreover the entanglement consists of coherent states and vacuum states which can be thought of in a classical way: both are quadrature-phase minimum-uncertainty states which possess a Glauber-Sudarshan P-representation. Thus we have essentially a superposition of two spatially separated semiclassical states. By producing states which are easily identified with classical notions, an experimental realization of this scheme would present a dramatic demonstration of quantum features.

We have shown that this state exhibits nonclassical (nonlocal) intensity correlations which generalized a result for entangled single photon states previously obtained [7]. In the case of single photon states there is no semiclassical interpretation to begin with and the entangled state is simply produced with a beam splitter. In the case considered here the input state does have a semiclassical interpretation (in the sense of a Glauber-Sudarshan P representation), and conversion of this state to a nonclassical state requires a nonlinear device. Rather than producing an entangled state from a nonclassical state with a linear device (beam splitter) in the case of Ref. [7], we produce an entangled state from a semiclassical state via a nonlinear interaction in the NLMZI. Of course it must be emphasized that it is easier to produce single-photon states than to find a large Kerr nonlinearity as is required in the scheme of this paper [20]. An optimistic estimate of χ which uses a large but realistic value of $\chi^{(3)}$ suggests that over 1000 km of interaction would be required to allow $\chi = \pi/2$ as required in these calculations [11, 21]. For this interaction length, assumptions involving the single-mode treatment and of no loss require further attention. On the other hand the experimental determination of nonclassical correlations should be easier due to the large photon number in the input state. In contrast to the measurements for single-photon entangled states, which generally involve approximately one photon in the apparatus at any time, this scheme can involve large numbers of photons which reduces the integration time necessary for the photodetectors.

ACKNOWLEDGMENTS

I thank S. J. Carter and R. McDuff for helpful discussions about nonlinear interferometers, S. L. Braunstein for discussions about Bell inequalities, and J. D. Cresser for his suggestions and comments. Finally G. J. Milburn has provided many valuable suggestions and constructive criticisms which I gratefully acknowledge.

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