Damping of quantum superpositions

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We study the first-order perturbation contribution of a generic Markovian master equation on a general superposed quantum state. From this we obtain a universal formula describing the decay of quantum superpositions for arbitrary quantum superpositions. For single-photon damping, superposed coherent states appear, naively, to give the smallest damping rate. However, a comparison of damping rates, based on the observability of superposition fringes, shows that superposed squeezed states can do significantly better than their coherent state rivals. The optimum squeezed superposition is shown to have a damping rate (for the zero-temperature Markovian master equation) linear in the phase-space distance between the pieces of the original unsqueezed superposition, as compared with the quadratic dependence for superpositions of coherent states. Finally, we show that superpositions of squeezed states can be generated directly from squeezed states without having to squeeze a superposition.

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I. INTRODUCTION

Quadrature squeezing of quantum states has only one quantitatively significant application in the current literature, and that is the improvement in sensitivity of interferometric measurements [1]. In this paper we present the following application: squeezing a quantum superposition can decrease the rate at which this superposition is destroyed by damping. This reduced damping for squeezed superpositions could prove useful in the experimental search for "macroscopic" superpositions of optical states via homodyne detection [2].

Damping of superpositions due to losses or dissipation is one of the fastest processes in quantum mechanics. This has meant that quantum superpositions of macroscopic states are effectively unobservable [3]. Indeed, some scientists have been actively pursuing the possibility that the laws of quantum mechanics do not even apply at the macroscopic level [4, 5]. They contend that quantum theory applied to macroscopic systems may not be sufficient to explain why our macroscopic world appears to be classical [6].

This paper initiates a study of various damping mechanisms acting on the quantum superposition of arbitrary "well-separated" states. These mechanisms correspond to a model where the system is coupled to an "environment" of many harmonic oscillators at thermal equilibrium. At optical frequencies the spectrum of this environment is usually approximated by a white-noise spectrum. This assumption allows the detailed couplings of system and environment to be replaced by a Markovian "master" equation.

The fast damping of superpositions allows us to make two simplifying assumptions. First we ignore all interactions other than those responsible for damping, and second we assume that only the lowest orders of a perturbation calculation are needed during the short time while the damping is occurring. When the further assumption is made, that in some sense the pieces of the superposition are well separated from each other, then a solution to the master equation can be found which provides a general damping formula.

Here we study the superpositions of quantum states that are single modes of the electromagnetic field. When the environment is at zero temperature and the coupling between it and the system takes the form of so-called "single-photon" damping, then the general damping formula simplifies and admits a simple interpretation of the rate at which superpositions damp. In this case one sees easily that superpositions of coherent states are the most robust over all superpositions for a given coherent amplitude separation between the superpositions' pieces. Nonetheless, they are *not* the optimum choice as far as observability of the superpositions is concerned. In this respect superposed squeezed states can do better.

In Sec. II we derive the damping formula governing the decay of superpositions of well-separated quantum states. In Sec. III we investigate some examples for zero-temperature zero-photon damping and give a mnemonic for calculating the rate at which the superpositions of well-separated quantum states decay. Section IV considers the observability of these superpositions in terms of the statistics of homodyne detection. It is argued that the homodyne statistics at two specially chosen local oscillator phases form an unambiguous signature for identifying a quantum superposition. Furthermore, this signature is independent of scale transformations on the homodyne statistics. In Sec. V explicit numerical calculations of the damping of superposed squeezed and superposed coherent states are carried out. These calculations agree with the damping rate calculated in Secs. II and III. Finally, in Sec. VI we show how to generate superpositions of squeezed states other than by squeezing a superposition of coherent states.

II. SUPERPOSITION DAMPING FORMULA

Damping of a quantum system is described using the master equation approach [7, 8]; an operator $\hat{\theta}$ of the system is coupled to an operator $\hat{B}$ of an environment at temperature T via the interaction Hamiltonian

$$\hat{H}_{\rm int} = \hbar\Gamma(\hat{\theta}\hat{B}^\dagger + \hat{\theta}^\dagger\hat{B}) ;$$

here, Γ is the damping constant. Because the bandwidths over which optical systems couple to their environments are usually very small compared to optical frequencies the white-noise approximation is made for the environment [8]. This corresponds to an environment with a field amplitude that is δ correlated in time, i.e., the correlation times for environments at optical frequencies are usually assumed negligible, so this is often called a Markovian approximation.

This assumption allows us to write down a Markovian master equation describing the evolution of a system $\hat{\rho}$ coupled to an environment [7, 8]; in the interaction picture the master equation is

$$\begin{aligned}\frac{d\hat{\rho}}{d\tau} =& -\tfrac{1}{2}\langle\hat{B}\hat{B}^\dagger\rangle_T\left(\hat{\theta}^\dagger\hat{\theta}\hat{\rho} + \hat{\rho}\hat{\theta}^\dagger\hat{\theta} - 2\hat{\theta}\hat{\rho}\hat{\theta}^\dagger\right) \\ &- \tfrac{1}{2}\langle\hat{B}^\dagger\hat{B}\rangle_T\left(\hat{\theta}\hat{\theta}^\dagger\hat{\rho} + \hat{\rho}\hat{\theta}\hat{\theta}^\dagger - 2\hat{\theta}^\dagger\hat{\rho}\hat{\theta}\right) , \qquad (2.1)\end{aligned}$$

where $\langle\ \rangle_T$ are expectation values over the state of the environment at temperature T, and $\tau = \Gamma t$ is the scaled time.

Let us consider the superposition $\hat{\rho} = |\psi\rangle\langle\psi|$ of N approximately orthogonal pieces

$$|\psi\rangle = f_1|1\rangle + f_2|2\rangle + \cdots + f_N|N\rangle ,$$

where, at this point, the states $|i\rangle$ are arbitrary except for the condition that $\langle i|j\rangle \simeq \delta_{ij}$. We are interested in how the coupling of this state to the environment through Eq. (2.1) destroys the off-diagonal terms in the density matrix, i.e., $\langle i|\hat{\rho}|j\rangle$ for $i \neq j$. If these terms are destroyed almost instantly then the state would become

$$\hat{\rho} \simeq |f_1|^2|1\rangle\langle 1| + |f_2|^2|2\rangle\langle 2| + \cdots + |f_N|^2|N\rangle\langle N| ,$$

which is equivalent to the classical mixture of the N states $|i\rangle\langle i|$, $i = 1,\ldots,N$. The original pure state includes the possibility of the system being in some or all of these states simultaneously, a description which lies beyond the classical "either-or" language.

To find out how quickly the damping destroys the superpositions we need to solve Eq. (2.1). Formally, its solution may be expanded out as a perturbation series in time

$$\hat{\rho}(\tau) = \hat{\rho}_0 + \hat{\rho}_1\tau + \hat{\rho}_2\frac{\tau^2}{2!} + \cdots ;$$

in this paper only the terms up to $\hat{\rho}_1$ will be included. For superpositions of coherent states $|\psi\rangle \propto |\alpha_1\rangle + |\alpha_2\rangle$ the off-diagonal terms damp exponentially quickly; at zero temperature for single-photon damping [9, 10] the logarithmic rate is

$${\rm Re}\frac{d}{d\tau}\ln\langle 1|\hat{\rho}|2\rangle = -\tfrac{1}{2}|\alpha_1 - \alpha_2|^2 ;$$

throughout this paper these rates are written using scaled time. By comparison the coherent amplitudes damp at a logarithmic rate of $-\frac{1}{2}$. This fast decay of superpositions is believed to be the general situation [10]. It is because of this fast damping of the off-diagonal matrix elements that we shall investigate only the solution of Eq. (2.1) to first order. It is expected that, in general, this first term will be valid for times long enough for significant damping of the superpositions to have occurred. Thus we shall have calculated the *effective* rate of decay of the superpositions between the pieces of the wave function.

Let us consider the rate of decay of the off-diagonal term between two of the pieces of the initial state $\hat{\rho}_0 = \sum_{ij}\rho_{ij}|i\rangle\langle j|$; without loss of generality we label these pieces as $|1\rangle$ and $|2\rangle$. Then to first order we have from the approximate orthogonality

$$\begin{aligned}\frac{d}{d\tau}\langle 1|\hat{\rho}|2\rangle \simeq& -\tfrac{1}{2}\langle\hat{B}\hat{B}^\dagger\rangle_T\left(\rho_{i2}\langle 1|\hat{\theta}^\dagger\hat{\theta}|i\rangle + \rho_{1j}\langle j|\hat{\theta}^\dagger\hat{\theta}|2\rangle - 2\rho_{ij}\langle 1|\hat{\theta}|i\rangle\langle j|\hat{\theta}^\dagger|2\rangle\right) \\ &- \tfrac{1}{2}\langle\hat{B}^\dagger\hat{B}\rangle_T\left(\rho_{i2}\langle 1|\hat{\theta}\hat{\theta}^\dagger|i\rangle + \rho_{1j}\langle j|\hat{\theta}\hat{\theta}^\dagger|2\rangle - 2\rho_{ij}\langle 1|\hat{\theta}^\dagger|i\rangle\langle j|\hat{\theta}|2\rangle\right) + O(\tau), \qquad (2.2)\end{aligned}$$

where in this eqution alone repeated indices i and j are summed over. To simplify this we strengthen the condition of approximate orthogonality to what we call approximate "separateness" for the pieces; the additional assumptions are that

$$\begin{aligned}&\langle i|\hat{\theta}|j\rangle \simeq \delta_{ij}\langle i|\hat{\theta}|i\rangle \equiv \delta_{ij}\theta_i, \\ &\langle i|\hat{\theta}^\dagger\hat{\theta}|j\rangle \simeq \delta_{ij}\langle i|\hat{\theta}^\dagger\hat{\theta}|i\rangle, \\ &\langle i|\hat{c}|j\rangle \simeq \delta_{ij}\langle i|\hat{c}|i\rangle ,\end{aligned}$$

with $\hat{c} \equiv [\hat{\theta}, \hat{\theta}^\dagger]$.

These conditions of separateness are seen to depend on which system operator $\hat{\theta}$ is coupled to the environment, therefore they will differ from model to model. Nonetheless, these extra conditions do not appear to significantly affect the general utility of the results presented here. As an example, for single-photon damping [where $\hat{\theta}$ becomes a harmonic-oscillator (HO) annihilation operator $\hat{a}$], these conditions say that the pieces of the superposition must "differ" by more than an annihilation operator and by more than a number operator—not apparently very restrictive assumptions. Imposing these conditions on Eq. (2.2) allows us to calculate the logarithmic time derivative of the off-diagonal matrix element to first order. Only the real part of this will contribute directly to the damping; the imaginary part contributes to the relative phase between the pieces of the superposition. If there are fluctuations in the rate at which the phase is accumulated, then the superpositions can be washed out; however, in this paper such effects will be neglected. The real part of the logarithmic time derivative reduces to

$$\mathrm{Re}\frac{d}{d\tau}\ln\langle 1|\hat{\rho}|2\rangle \simeq -\tfrac{1}{2}\langle\{\hat{B},\hat{B}^\dagger\}\rangle_T\left(|\theta_1-\theta_2|^2+\langle 1|\Delta\hat{\theta}_1{}^\dagger\Delta\hat{\theta}_1|1\rangle+\langle 2|\Delta\hat{\theta}_2{}^\dagger\Delta\hat{\theta}_2|2\rangle\right) - \tfrac{1}{2}\langle\hat{B}^\dagger\hat{B}\rangle_T(\langle 1|\hat{c}|1\rangle+\langle 2|\hat{c}|2\rangle)+O(\tau)\,, \tag{2.3}$$

where $\{\ ,\ \}$ are the anticommutator brackets, and $\Delta\hat{\theta}_i \equiv \hat{\theta}-\theta_i$.

Instead of giving here detailed bounds for how long this result remains valid we shall instead try to develop an intuitive feel for its limitations. Let us consider a more general initial state $\hat{\rho}_0 \equiv \sum_{i,j}\hat{\rho}_{ij}$, where now each $\hat{\rho}_{ij}$ need not be a pure state. In this case, Eq. (2.2) takes the form

$$\mathrm{tr}\hat{\rho}_{21}\frac{d}{d\tau}\hat{\rho} \simeq -\tfrac{1}{2}\langle\hat{B}\hat{B}^\dagger\rangle_T\left(\mathrm{tr}\hat{\rho}_{21}\hat{\theta}^\dagger\hat{\theta}\hat{\rho}_{12}+\mathrm{tr}\hat{\rho}_{21}\hat{\rho}_{12}\hat{\theta}^\dagger\hat{\theta}-2\mathrm{tr}\hat{\rho}_{21}\hat{\theta}\hat{\rho}_{12}\hat{\theta}^\dagger\right) -\tfrac{1}{2}\langle\hat{B}^\dagger\hat{B}\rangle_T\left(\mathrm{tr}\hat{\rho}_{21}\hat{\theta}\hat{\theta}^\dagger\hat{\rho}_{12}+\mathrm{tr}\hat{\rho}_{21}\hat{\rho}_{12}\hat{\theta}\hat{\theta}^\dagger-2\mathrm{tr}\hat{\rho}_{21}\hat{\theta}^\dagger\hat{\rho}_{12}\hat{\theta}\right)+O(\tau)\,, \tag{2.4}$$

with the approximate orthogonality and separability conditions now becoming

$$\mathrm{tr}\hat{\rho}_{ij}\hat{\theta}\hat{\rho}_{kl} \simeq \delta_{il}\delta_{jk}\mathrm{tr}\hat{\rho}_{ij}\hat{\theta}\hat{\rho}_{ji},$$
$$\mathrm{tr}\hat{\rho}_{ij}\hat{\theta}^\dagger\hat{\theta}\hat{\rho}_{kl} \simeq \delta_{il}\delta_{jk}\mathrm{tr}\hat{\rho}_{ij}\hat{\theta}^\dagger\hat{\theta}\hat{\rho}_{ji},$$
$$\mathrm{tr}\hat{\rho}_{ij}\hat{\theta}\hat{\theta}^\dagger\hat{\rho}_{kl} \simeq \delta_{il}\delta_{jk}\mathrm{tr}\hat{\rho}_{ij}\hat{\theta}\hat{\theta}^\dagger\hat{\rho}_{ji}\,,$$

so that in some basis the pieces $\hat{\rho}_{ij}$ become well-separated blocks in the matrix representation of $\hat{\rho}_0$. If after some time τ the state $\sum_{i,j}\hat{\rho}_{ij}$ evolves under the action of the master equation into $\sum_{i',j'}\hat{\rho}_{i'j'}$, where the $\hat{\rho}_{i'j'}$ are still well-separated blocks, then Eq. (2.4) will still hold instantaneously in terms of the matrix elements of the new blocks $\hat{\rho}_{i'j'}$. That is, Eq. (2.4) will give the instantaneous rate of decay of the off-diagonal block $\hat{\rho}_{12}$. If we further assume that the matrix elements on the right-hand side of this equation vary slowly in comparison with the rate of decay of the off-diagonal block, then it should be a good approximation to evaluate Eq. (2.4) at $\tau=0$ and take this initial rate as the *effective* rate of decay of the quantum superpositions. A detailed analysis along these lines would be considerably more sophisticated, and more difficult, than simply calculating the next term in our expansion about $\tau=0$. Without a rigourous calculation along these lines we merely conjecture that Eq. (2.4) describes the effective rate at which well-separated superpositions are effectively damped by coupling them to the environment.

Finally, since we are here most interested in the situation where the initial pieces are pure, the above conjecture for the effective rate of decay of superpositions reduces to the expression in Eq. (2.3). In Sec. V the usefulness of Eq. (2.3) as an effective rate is tested through numerical calculations for the case of single-photon damping of superpositions of squeezed states by an environment at zero temperature.

III. ZERO-TEMPERATURE SINGLE-PHOTON DAMPING

For the remainder of this paper we restrict our attention to the situation common in quantum optics: so-called single-photon damping by an environment at zero temperature. In this case the interaction Hamiltonian takes the form

$$\hat{H}_{\mathrm{int}} = \hbar\Gamma\left(\hat{a}\hat{b}^\dagger+\hat{a}^\dagger\hat{b}\right)\,,$$

where $\hat{a}$ is the HO's annihilation operator for the system, and $\hat{b}$ is the annihilation operator for the environment's modes; and the master equation becomes

$$\frac{d\hat{\rho}}{d\tau} = -\tfrac{1}{2}\left(\hat{a}^\dagger\hat{a}\hat{\rho}+\hat{\rho}\hat{a}^\dagger\hat{a}-2\hat{a}\hat{\rho}\hat{a}^\dagger\right)\,. \tag{3.1}$$

It should be noted that this master equation can also be used to calculate the effects of detector losses. If we model an inefficient detector by a beam splitter with an amplitude transmission coefficient of $\sqrt{\eta}$ followed by a 100% efficient detector then we have a detector with effective losses of $1-\eta$. Now for detector losses up to about 25%, Eq. (3.1) will give a good approximation if we equate $\tau=\Gamma t$ with $1-\eta$. Thus, for example, $\tau=0.02$ would produce damping equivalent to detector losses of 2%, and $\tau=0.05$ corresponds to 5% detector losses.

Replacing $\hat{\theta}$ by $\hat{a}$, θ_i by α_i, and $\hat{B}$ by $\hat{b}$ in Eq. (2.3), and setting the environment temperature to zero gives

$$\mathcal{R}_{12} \equiv \mathrm{Re}\frac{d}{d\tau}\ln\langle 1|\hat{\rho}|2\rangle\Big|_{\tau=0} \simeq -\tfrac{1}{2}(|\alpha_1-\alpha_2|^2+\langle 1|\Delta\hat{a}_1{}^\dagger\Delta\hat{a}_1|1\rangle +\langle 2|\Delta\hat{a}_2{}^\dagger\Delta\hat{a}_2|2\rangle)\,. \tag{3.2}$$

This equation describes the effective rate (now denoted $\mathcal{R}_{12}$) for the damping of superpositions via zero-temperature single-photon damping. For superposed coherent states $f_1|\alpha_1\rangle+f_2|\alpha_2\rangle$ this yields the rate

$$\mathcal{R}_{12} = -\tfrac{1}{2}(|\alpha_1-\alpha_2|^2)\,, \tag{3.3}$$

and for superposed number states $f_1|n_1\rangle+f_2|n_2\rangle$ the decay rate is

$$\mathcal{R}_{12} = -\tfrac{1}{2}(n_1+n_2)\,, \tag{3.4}$$

both of which agree with known solutions for these problems [12]. It is worth noting that Eq. (3.4) has a simple physical interpretation [11]: as soon as a single photon from either $|n_1\rangle$ or $|n_2\rangle$ decays to the environment, the superposition between these pieces will be destroyed. This same interpretation works for the superposition of coherent states only when one of them is the vacuum. Surprisingly, no more general physical interpretation is known which explains Eq. (3.3).

Now if we note that

$$\langle i|\Delta\hat{a}_i^\dagger \Delta\hat{a}_i|i\rangle = \langle i|\hat{a}^\dagger \hat{a}|i\rangle - |\alpha_i|^2$$

is just the excess number of photons in piece $|i\rangle$ beyond those of a coherent state of the same amplitude α_i, then we can give Eq. (3.2) a simple interpretation, or more correctly a mnemonic for the result. The effective rate of decay of superpositions for zero-temperature single-photon damping is due to "two" processes: first, the decay due to the separation of the coherent amplitudes of the pieces as in Eq. (3.3), and second, the decay due to the "excess" number of photons each piece must carry to be other than a coherent state as in Eq. (3.4). This interpretation of the superposition damping is not unique, however, it appears to be the simplest interpretation that treats the pieces of the superposition in a symmetric manner.

Now that we have a simple picture let us consider a couple of examples.

(i) Superposed displaced number states:

$$f_1\hat{D}(\alpha_1)|n_1\rangle + f_2\hat{D}(\alpha_2)|n_2\rangle ,$$

with $\hat{D}(\alpha)^\dagger\hat{a}\hat{D}(\alpha) = \hat{a}+\alpha$, then

$$\mathcal{R}_{12} = -\tfrac{1}{2}\left(|\alpha_1-\alpha_2|^2 + n_1 + n_2\right) .$$

(ii) Superposed squeezed states:

$$f_1\hat{D}(\alpha_1)\hat{S}(r_1;\varphi_1)|0\rangle + f_2\hat{D}(\alpha_2)\hat{S}(r_2;\varphi_2)|0\rangle ,$$

where $\hat{S}(r;\varphi)^\dagger\hat{a}\hat{S}(r;\varphi) = \hat{a}\cosh(r) + \hat{a}^\dagger e^{2i\varphi}\sinh(r)$, then

$$\mathcal{R}_{12} = -\tfrac{1}{2}\left[|\alpha_1-\alpha_2|^2 + \sinh^2(r_1) + \sinh^2(r_2)\right] .$$

It is easy to see that our mnemonic for the effective rate of damping of superpositions can be immediately applied to many examples. It should be further noted that in each of the above cases it has been implicitly assumed that the conditions of approximate orthogonality and separateness of the pieces of the superposition apply. If they do not, then in a sense the pieces of the state are not well separated so their labeling as individual pieces will be somewhat arbitrary.

IV. OBSERVATION VIA HOMODYNE DETECTION

Yurke and Stoler [2] have proposed that the photocurrent statistics from homodyne detection give an unambiguous experimental signature for identification of quantum superpositions. They describe how the same device can both "see" the interference features of the superposition and "see" that there are two separate things involved in making those fringes; by changing the phase of the local oscillator one or other of these features would be available.

In balanced homodyne detection a local oscillator and signal field are combined at a 50-50 beam splitter; the sum and difference fields produce beats in the currents of a pair of photodetectors. These currents are subtracted to give a "difference photocurrent" that forms the output for the device. For an intense local oscillator this differenced photocurrent is approximately proportional to the operator

$$\hat{x}_\theta = \tfrac{1}{2}\left(\hat{a}e^{i\theta} + \hat{a}^\dagger e^{-i\theta}\right) .$$

The variance in the differenced photocurrent has been successfully used [13, 14] as an experimental signature for squeezing. In the application discussed here, however, the *full* statistics of $\hat{x}_\theta$ are required, not just the lowest order moments; fortunately, a sufficiently intense local oscillator allows an ideal balanced homodyne detector to achieve this [15].

To understand what this proposal for observing superpositions suggests, let us consider a signal that is a superposition of coherent states $|\psi\rangle = f_1|\alpha_1\rangle + f_2|\alpha_2\rangle$. Defining the variables

$$x_j(\theta) = \tfrac{1}{2}\left(\alpha_j e^{i\theta} + \alpha_j^* e^{-i\theta}\right) ,$$
$$y_j(\theta) = \tfrac{i}{2}\left(\alpha_j^* e^{-i\theta} - \alpha_j e^{i\theta}\right)$$

for $j=1,2$ so that $\alpha_j e^{i\theta} = x_j(\theta) + iy_j(\theta)$, and also the variable

$$z(\theta) = \tfrac{1}{2}\left(\alpha_1 e^{i\theta} + \alpha_2^* e^{-i\theta}\right) ,$$

allows us to write the homodyne statistics as

$$\begin{aligned}P_\theta(x) = \sqrt{2/\pi}[\ & |f_1|^2\exp\{-2[x-x_1(\theta)]^2\} \\ &+|f_2|^2\exp\{-2[x-x_2(\theta)]^2\} \\ &+2\,\mathrm{Re}(f_2^* f_1\langle\alpha_2|\alpha_1\rangle \\ &\qquad\times\exp\{-2[x-z(\theta)]^2\})] . \qquad (4.1)\end{aligned}$$

This expression shows a "lump" at $x = x_1(\theta)$ with probability $|f_1|^2$, a lump at $x = x_2(\theta)$ with probability $|f_2|^2$, and an interference term when $z(\theta)$ is complex.

In order to visualize how this expression behaves at various local oscillator settings we start by giving $x_j(\theta)$ and $y_j(\theta)$ simple geometric meanings: if we plot the complex number α_j on a complex plane, then in the coordinate system rotated clockwise by θ radians $x_j(\theta)$ and $y_j(\theta)$ are the projections of α_j onto the real and imaginary axes, respectively. Now writing $z(\theta)$ as

$$z(\theta) = \tfrac{1}{2}[x_1(\theta)+x_2(\theta)] + \tfrac{i}{2}[y_1(\theta)-y_2(\theta)] ,$$

we may pull out the two types of complementary behavior displayed by Eq. (4.1). If the complex plane is rotated by an angle (call it θ_R) so that its real axis is parallel to the line joining α_1 and α_2, then clearly the imaginary projections of α_1 and α_2 will be equal, i.e., $y_1(\theta_R) = y_2(\theta_R)$. In this case $z(\theta)$ is real and the third term is proportional to $|\langle\alpha_2|\alpha_1\rangle| = \exp(-\tfrac{1}{2}|\alpha_1-\alpha_2|^2)$; when $|\alpha_1-\alpha_2| \gtrsim 2$ this third term can be neglected at $\theta=\theta_R$. Thus

$$\begin{aligned}P_{\theta_R}^{\mathrm{coh}}(x) \simeq \sqrt{2/\pi}(\ & |f_1|^2\exp\{-2[x-x_1(\theta_R)]^2\} \\ &+|f_2|^2\exp\{-2[x-x_2(\theta_R)]^2\}) .\end{aligned}$$

If we rotate this coordinate system $\pi/2$ rad further (i.e., advance the local oscillator phase by this amount) to $\theta=\theta_I\equiv\theta_R+\pi/2$, then the real axis projections are now equal, i.e., $x_0\equiv x_1(\theta_I) = x_2(\theta_I)$ and $y_1(\theta_I)-y_2(\theta_I) = \pm|\alpha_1-\alpha_2|$, the sign depending on the sense of the axis. This rotated complex plane gives $z(\theta)$ its maximum imaginary component, and hence the maximum interference; here

$$z(\theta_I) = x_0 \pm \tfrac{i}{2}|\alpha_1 - \alpha_2|$$

and the homodyne statistics take the form

$$P^{\rm coh}_{\theta_I}(x) = \sqrt{2/\pi}\exp[-2(x-x_0)^2] \times ((|f_1|^2 + |f_2|^2) + 2\,\mathrm{Re}\{f_2^* f_1 e^{i\Theta} \times \exp[\pm 2i(x-x_0)|\alpha_1-\alpha_2|]\}) ,$$

where Θ is the phase of the inner product $\langle\alpha_2|\alpha_1\rangle$. These two distributions of homodyne statistics [i.e., $P_{\theta_R}(x)$ and $P_{\theta_I}(x)$] make up the "experimental signature for superpositions."

Let us now look at the homodyne statistics of the squeezed superposition of coherent states

$$|\psi\rangle = \hat{S}(-r,\theta_R)(f_1|\alpha_1\rangle + f_2|\alpha_2\rangle) ,$$

where we have specially chosen the orientation of the squeeze operator to coincide with the line between the coherent amplitudes of the states $|\alpha_1\rangle$ and $|\alpha_2\rangle$. In this case the homodyne statistics take a simple form when the local oscillator phase is again chosen along either $\theta = \theta_R$ or θ_I:

$$P^{\rm sq}_{\theta_R}(x) = e^r P^{\rm coh}_{\theta_R}(e^r x),$$

$$P^{\rm sq}_{\theta_I}(x) = e^{-r} P^{\rm coh}_{\theta_I}(e^{-r} x) .$$

That is, the lumps of $P^{\rm coh}_{\theta_R}$ are brought closer together by the factor e^{-r} in $P^{\rm sq}_{\theta_R}$, and the interference pattern of $P^{\rm coh}_{\theta_I}$ is spread out by the factor e^r in $P^{\rm sq}_{\theta_I}$.

Although these patterns for the squeezed superposition appear on different scales from the original superposition, they give just as good an unambiguous signal for a quantum superposition. First, consider the distance between the lumps; to be able to tell that the two lumps are far enough apart to be distinguishable we require that the distance between their peaks be larger than the peak widths. In fact, it is only sensible to quote their separation in terms of units of peak width (i.e., a signal-to-noise measure). Thus the lumps of the squeezed superposition are as well separated as for the original state. In a similar way, when an experimentalist measures the contrast of the interference pattern, a bin size will be chosen to optimize the contrast; when the new pattern for the squeezed superposition is used, this measure of contrast will be unchanged for the new optimized bin size (now e^r times larger than originally).

We have argued that squeezed superpositions offer the same unambiguous experimental signatures for quantum superpositions as the original unsqueezed superposition. In the following section we will consider the affects of damping for these two situations. In Sec. VI we show how this superposition can be generated in a way other than by squeezing a quantum superposition.

V. DAMPING OF SQUEEZED SUPERPOSITIONS

From the preceding section we saw that a family of states like

$$|\psi\rangle_r \equiv \hat{S}(-r,\theta_R)(f_1|\alpha_1\rangle + f_2|\alpha_2\rangle) \tag{5.1}$$

has experimental signatures for observing quantum superpositions which are independent of the squeeze parameter r (up to the irrelevant scale factors $e^{\pm r}$). The response to damping of these states is, however, sensitive to r. From Eq. (3.2), the effective rate at which the superpositions in Eq. (5.1) are destroyed by damping is given by

$$\mathcal{R}_{12} \simeq -\tfrac{1}{2}\left(e^{-2r}|\alpha_1-\alpha_2|^2 + 2\sinh^2 r\right) . \tag{5.2}$$

The "coherent" damping rate decreases as e^{-2r}, while the rate due to introducing "excess photons" into each piece increases roughly as e^{2r} with r. In general there will be a minimum to this rate at some nonzero value of squeezing. In particular, the minimum occurs at

$$e^{4r} = 2|\alpha_1-\alpha_2|^2 + 1 ,$$

with an effective rate of

$$\mathcal{R}_{12} \simeq -\tfrac{1}{2}\sqrt{2}|\alpha_1-\alpha_2| ; \tag{5.3}$$

this produces a rate only linear in the phase-space distance between the original pieces—not quadratic.

In order to test this improvement we consider a special case of the family of states represented by Eq. (5.1); the state

$$|\psi\rangle_r \equiv f\hat{S}(-r,\theta_R)(|\alpha\rangle - |-\alpha\rangle) \tag{5.4}$$

is obtained by taking $\alpha = \alpha_1 = -\alpha_2$ to be real so $\theta_R = 0$, and taking $f = f_1 = -f_2$ (also real) so f becomes purely a normalization constant dependent only on α. We calculate numerically the effects of damping on the experimental signature for superpositions in two cases: $r = 0$ with $\alpha = 5$, displayed in our figures as solid lines; and $r = -1$ with $\alpha = 5$, displayed as dashed lines. In order to immediately compare our results to previous work [2] we consider the effects of damping after $\tau = 0.02$ and 0.05 (corresponding to the damping produced by detector losses of 2% and 5%, respectively).

Figures 1(a) and 1(b) show the unscaled homodyne statistics $P_{\pi/2}(x)$ and $P_0(x)$, respectively, for the state $f(|5\rangle - |-5\rangle)$ (solid line), and the state $f\hat{S}(-1,0)(|5\rangle - |-5\rangle)$ (dashed line). The dashed line patterns for the squeezed superposition when compared to the solid lines for the original superposition differ only by a scale factor of $e^{\pm 1}$.

The master equation from Eq. (3.1) was solved by iteration on a number state basis truncated at $n = 50$. Time steps were taken to be $\Delta\tau = \Gamma\Delta t = 0.001$. Figure 2 shows the homodyne statistics of the above states after they have been evolved by the master equation for a time $\tau = 0.02$; the dashed lines of the damped squeezed superposition have been explicitly scaled here (and in Fig. 3) by the factors $e^{\pm 1}$ so as to match the scale of the unsqueezed superposition. Figure 3 shows the same plot as Fig. 2, but after the longer damping time of $\tau = 0.05$. As can be seen, the interference features of the superposed coherent states are strongly damped at much shorter times than those of the squeezed superposition.

In Figs. 2(b) and 3(b) the lumps representing the two separated pieces of the squeezed superposition have successively broadened slightly due to the reduction in squeezing in each piece. This effect was not taken into account by the damping formula of Eq. (5.2); but so long as the initial squeezing is not too strong this will not significantly reduce the well separatedness of the lumps (in units of peak width). Unfortunately, if one uses the optimum squeezing based on the observabilty of the interference fringes alone, as was done to obtain Eq. (5.3), then one cannot guarantee that the pieces will remain separated when $P_{\theta_R}(x)$ is observed.

To estimate this effect we start by calculating the rate at which the squeezed variance increases; for short times the master equation determines the rate to be

$$\frac{d}{d\tau}\ln\langle \hat{x}_{\rm sq}^2\rangle = 2\sinh^2 r \simeq \frac{e^{2r}}{2} ,$$

where $\hat{x}_{\rm sq}$ is the quadrature operator for the squeezed quadrature. If we place a limit on the variance so that it does not increase by more than the fraction ϵ then we must limit the damping to last for a time shorter than

$$\tau \simeq 2\epsilon e^{-2r} ,$$

which is valid for $\epsilon \lesssim 1$. With this bound there is no point in squeezing the quantum superposition so hard as to decrease the damping of the superpositions beyond

$$|\mathcal{R}_{12}| \simeq \tau^{-1} \simeq \frac{e^{2r}}{2\epsilon} .$$

That is, we wish to match the maximum time before the peak widths increase by the fraction ϵ to the inverse damping rate $\mathcal{R}_{12}$.

This match occurs at $e^{2r} \simeq \sqrt{\epsilon}|\alpha_1 - \alpha_2|$, which means that the effective rate of

$$\mathcal{R}_{12} \simeq -\tfrac{1}{2}\frac{|\alpha_1 - \alpha_2|}{\sqrt{\epsilon}}$$

for $\epsilon \lesssim 1$, allows the observation of both sets of homodyne statistics while still only having a superposition damping rate linear in the phase-space separation of the original pieces from the superposition of coherent states.

Finally, we note that there have been other proposals [16, 17] for helping preserve superposition states by replacing the vacuum fluctuations of the reservoir by the fluctuations of squeezed modes. Such schemes are only viable when the phase of the fluctuations from the reser-

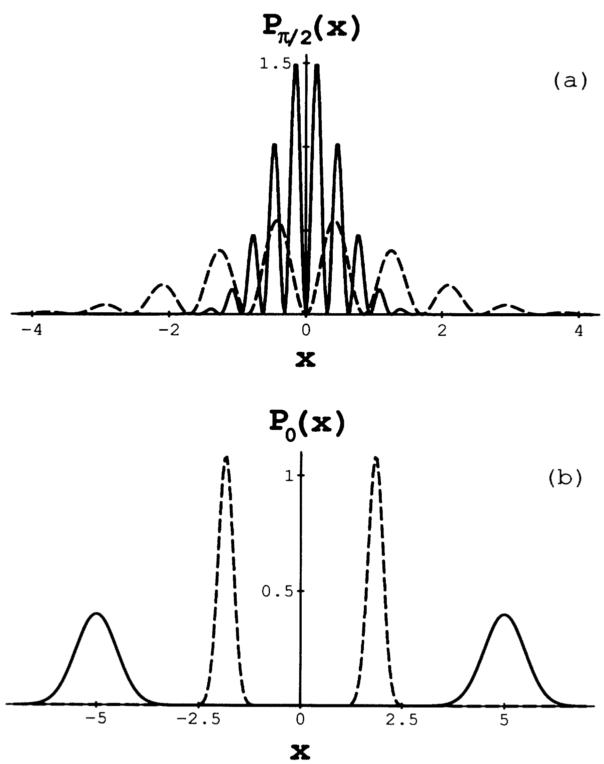

FIG. 1. Plots of the homodyne statistics of (a) $P_{\pi/2}(x)$ and (b) $P_0(x)$ vs x for the states $f(|5\rangle - |-5\rangle)$ (solid line) and $f\hat{S}(-1,0)(|5\rangle - |-5\rangle)$ (dashed line). In each figure the plots are of exactly the same functions up to the scale factors $e^{\pm 1}$.

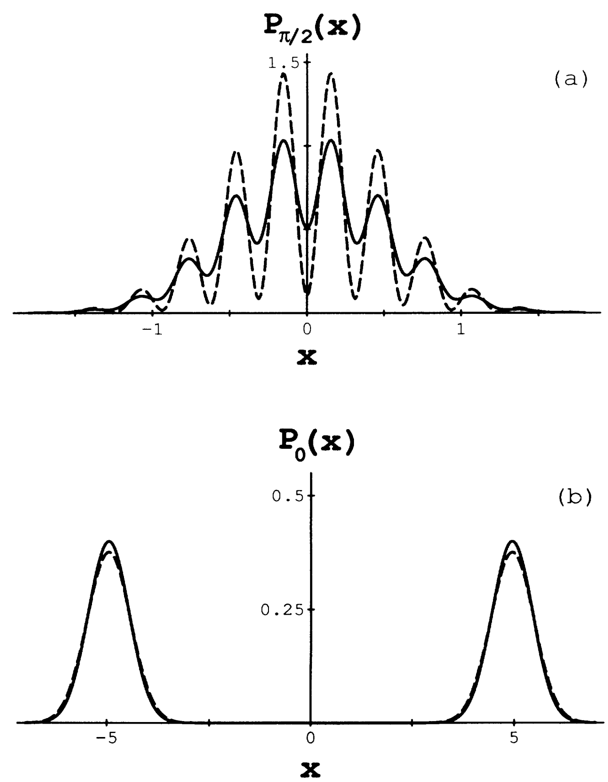

FIG. 2. Effect of damping for a scaled time $\tau = \Gamma t = 0.02$. Plots of the homodyne statistics of (a) $P_{\pi/2}(x)$ and (b) $P_0(x)$ vs x for the damped versions of the states $f(|5\rangle - |-5\rangle)$ (solid line) and $f\hat{S}(-1,0)(|5\rangle - |-5\rangle)$ (dashed line). The dashed plots have been scaled by the factors $e^{\pm 1}$ so as to coincide with the scales of the solid line plots.

voir can be controlled; as a consequence, these schemes cannot be used to help preserve superposition states against losses due to scattering, absorption, or detector inefficiencies. By contrast, the squeezed superposition states discussed here do not derive their improved immunity to damping from the careful control of the reservoir modes; thus they remain robust when coupled to any kind of losses, whether they be due to detector inefficiency, scattering, absorption or whatever. Figures 3(a) and 3(b), for example, show that squeezed superpositions can survive a 5% detector inefficiency.

VI. GENERATION OF SUPERPOSITIONS OF SQUEEZED STATES

We have shown in the preceding sections that superpositions of squeezed states have reduced damping when compared to superpositions of coherent states having the same experimental signature for quantum superpositions. The obvious way of generating these squeezed superpositions, i.e., by squeezing superpositions of coherent states, is unlikely to be a successful scheme; it involves first generating the more sensitive state before transforming it into the less sensitive one.

Instead, here we consider starting with a squeezed state having a nonzero coherent amplitude and using a nonlinear interaction to convert it into a superposition of the type we want. To do this we consider the interaction Hamiltonian

$$\hat{H}_{\rm int} = \hbar\chi(\hat{a}^\dagger\hat{a})^2 ,$$

which is the description of an idealized Kerr medium. In the interaction picture this interaction generates an evolution operator

$$\hat{U}(\chi t) = \exp[-i\chi t(\hat{a}^\dagger\hat{a})^2] .$$

It is known [2, 18] that $\hat{U}(\chi t)$ can generate superpositions of coherent states if χt takes on special values; for instance, when $\chi t = \pi/2$ we have

$$\hat{U}(\pi/2)|\alpha\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle\right) .$$

We wish to show that its action on a squeezed state with nonzero coherent amplitude has a similar effect. Since the squeeze operator is built of pairs of creation and annihilation operators we have

$$\hat{U}(\pi/2)\hat{S}(r,\phi)\hat{U}(\pi/2)^\dagger = \hat{S}(r,\phi) , \tag{6.1}$$

and putting this result together with the action of $\hat{U}(\pi/2)$ on the coherent state $|\alpha\rangle$ gives

$$\hat{U}(\pi/2)\hat{S}(r,\phi)|\alpha\rangle = \frac{1}{\sqrt{2}}\hat{S}(r,\phi)(e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle) .$$

This now is exactly the state in Eq. (5.4) used in our numerical calculations except for a relative phase factor which merely shifts the interference fringes one quarter of a cycle. This procedure still involves starting with a state sensitive to losses (the squeezed state); however, it is an improved scheme over taking a superposition and squeezing it.

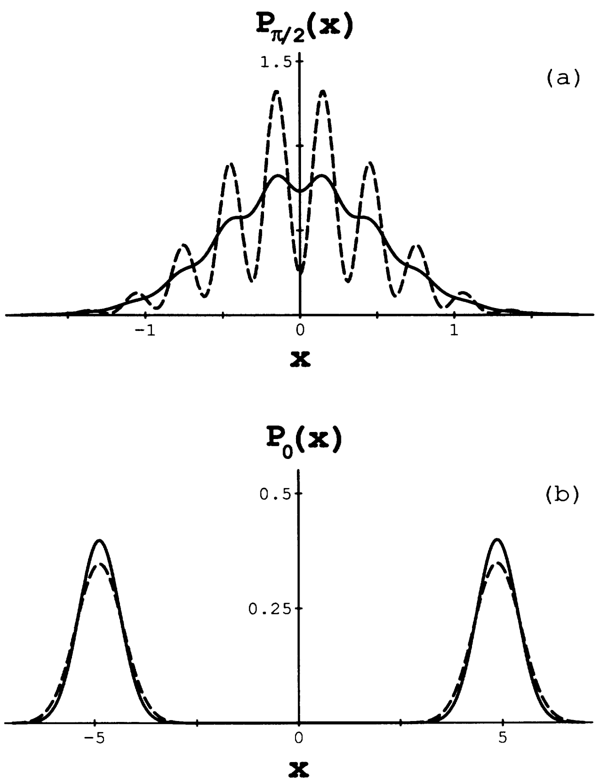

FIG. 3. Effect of damping for a scaled time $\tau = \Gamma t = 0.05$. Plots of the homodyne statistics of (a) $P_{\pi/2}(x)$ and (b) $P_0(x)$ vs x for the damped versions of the states $f(|5\rangle - |-5\rangle)$ (solid line) and $f\hat{S}(-1,0)(|5\rangle - |-5\rangle)$ (dashed line). The dashed plots have been scaled by the factors $e^{\pm 1}$ so as to coincide with the scales of the solid line plots.

VII. CONCLUSION

The damping of superpositions is fast and may be treated adequately through first-order perturbation theory. For suitable separability assumptions the *effective* damping rate of an arbitrary quantum superposition may be calculated. For the zero-temperature Markovian master equation this damping rate may be thought of as coming from two "distinct" processes: one damping term from the phase-space separation of the pieces of the superposition, and one term from the excess number of photons each piece needs to be other than a coherent state with its own coherent amplitude.

We have taken two sets of homodyne statistics to be *the* experimental signature for a quantum superposition: one set having the optimum contrast for observing interference fringes, and one set showing the optimum separation between the lumps of the superposition (measured in terms of their peak widths). Based upon optimizing this signature we have shown that superpositions of squeezed states are less sensitive to damping than superpositions of coherent states. The optimum damping rate for a squeezed version of the superposition of coherent states $f(|\alpha_1\rangle + |-\alpha_2\rangle)$ is approximately

$$\mathcal{R}_{12} \simeq -\tfrac{1}{2}\frac{|\alpha_1 - \alpha_2|}{\sqrt{\epsilon}}$$

for $\epsilon \lesssim 1$, instead of

$$\mathcal{R}_{12} \simeq -\tfrac{1}{2}|\alpha_1 - \alpha_2|^2 \ ,$$

if the state were left unsqueezed; here the fraction ϵ is the increase in peak widths of the lumps we are willing to tolerate and still consider the state a combination of sufficiently separated pieces. Finally, we have shown that superpositions of squeezed states can be generated directly from squeezed states by an ideal Kerr interaction without the necessity of first generating superpositions of coherent states.

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