Generation of highly squeezed states in a two-photon micromaser

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We consider a lossless two-photon micromaser with atomic polarization. We find that for particular values of the atomic flight time, the field evolves to a pure state which can be made of a superposition of even-photon-number states or odd-number states. These states approach to a perfectly squeezed state as the upper limit of the "trap" increases.

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One-photon micromasers have been the subject of studies in recent years, both experimentally [1,2] and theoretically [3,4]. In particular, in the lossless case, so-called trapping states were found, and if the injected atoms are initially prepared in a coherent superposition of states, the field may evolve to a pure state [5], even with a moderate amount of cavity losses and at low temperature [6]. Also, a value of 52% of squeezing was found in connection with these trapped states [7]. On the other hand, two-photon micromasers have also been studied both in theory [8] and experiment [9].

In the present paper, we analyze a two-photon micromaser consisting of a single lossless-mode cavity into which three-level atoms are injected at a low rate, so that, on the average, there is at most one atom inside the cavity at a given time.

Consider the three-level atom as per Fig. 1. We will assume that the atoms enter the cavity in a coherent superposition of the states $|a\rangle$ and $|c\rangle$, that is,

$$|\psi\rangle_{\rm at}=\alpha|a\rangle+\gamma|c\rangle\ , \tag{1}$$

where the coefficients α and γ are the same for all atoms.

In the absence of losses, the time-evolution operator of the system is [10]

$$U(\tau)=\begin{bmatrix}1-a\hat{C}a^\dagger & 0 & -a\hat{C}a\\ 0 & e^{-i\phi(2\hat{N}+1)} & 0\\ -a^\dagger\hat{C}a^\dagger & 0 & 1-a^\dagger\hat{C}a\end{bmatrix}\ , \tag{2}$$

where $\hat{C}=(1-e^{-i\phi(2\hat{N}+1)})(a^\dagger a+aa^\dagger)^{-1}$, and $\phi\equiv\tau g^2/\Delta$, Δ being the detuning (Fig. 1), g the coupling constant, and τ the flight time of the atom through the cavity. In deriving Eq. (2) [9], we have made the approximation $\Delta/g\gg\sqrt{N}$, which corresponds to a truly two-photon micromaser as opposed to a one-photon cascade system [10].

The evolution of an arbitrary state is given by

$$\begin{aligned}\sum_N S_N|N\rangle(\alpha|a\rangle+\gamma|c\rangle)\to\sum_N S_N\Bigg\{&\left[\alpha\left[1-2i\frac{N+1}{2N+3}e^{i\Omega_{N+1}}\sin\Omega_{N+1}\right]|N\rangle-2i\gamma G(N)e^{-i\Omega_{N-1}}\sin\Omega_{N-1}|N-2\rangle\right]|a\rangle\\ &+\Bigg[-2i\alpha G(N+2)e^{-i\Omega_{N+1}}\sin\Omega_{N+1}|N+2\rangle\\ &\quad+\gamma\left[1-2i\frac{N}{2N-1}e^{-i\Omega_{N-1}}\sin\Omega_{N-1}\right]|N\rangle\Bigg]|c\rangle\Bigg\}\ ,\end{aligned} \tag{3}$$

where $\Omega_l=(2l+1)\phi/2$ and $G(l)=\sqrt{l(l-1)}/(2l-1)$.

The trapping states are immediate from Eq. (3). When $N=N_d$ and

$$\Omega_{N_d}-1=\tfrac{1}{2}(2N_d-1)\phi=q\pi\quad(q\ \text{integer})\ , \tag{4}$$

we have a downward trapping, and for $N=N_u$ such that

$$\Omega_{N_u+1}=\tfrac{1}{2}(2N_u+3)\phi=p\pi\quad(p\ \text{integer})\ , \tag{5}$$

we have an upward trapping and the Fock space is divided into isolated blocks, so that if our initial field distribution is within a given block, we have a closed dynamics leading to pure states.

If, in a steady state, the field evolves to a pure state as a superposition of number states, restricted between a lower and an upper bound, N_d and N_u, respectively, and if one more atom crossed the cavity, the state of the atom-field system would be modified only by a global phase factor and a different atomic superposition [5], that is,

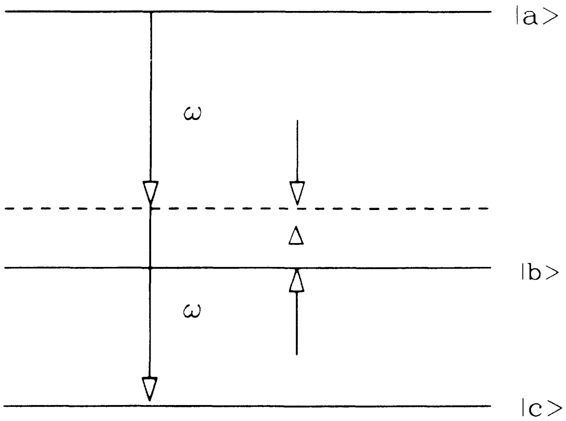

FIG. 1. Energy levels relevant to the two photon laser with atomic polarization.

$$\sum_{N=N_d}^{N_u} S_n|N\rangle(\alpha|a\rangle+\gamma|c\rangle)$$

$$\rightarrow e^{i\phi}\sum_{N=N_d}^{N_u} S_n|N\rangle(\alpha'|a\rangle+\gamma'|c\rangle)\ , \qquad (6)$$

where ϕ, α' and γ' are independent of N. By comparing the right-hand side of Eqs. (3) and (6), we readily get

$$e^{i\phi}\alpha' S_N=\alpha\left[1-2i\frac{N+1}{2N+3}e^{-i\Omega_{N+1}}\sin\Omega_{N+1}\right]S_N$$
$$-2i\gamma G(N+2)e^{-i\Omega_{N+1}}\sin\Omega_{N+1}S_{N+2}\ , \qquad (7a)$$

$$e^{i\phi}\gamma' S_N=-2i\alpha G(N)e^{-i\Omega_{N-1}}\sin\Omega_{N-1}S_{N-2}$$
$$+\gamma\left[1-2i\frac{N}{2N-1}e^{-i\Omega_{N-1}}\sin\Omega_{N-1}\right]S_N\ . \qquad (7b)$$

It is simple to see that for Eqs. (7a) and (7b) to be consistent, it is necessary that

$$\alpha=\alpha' e^{i\phi}\ ,$$
$$\gamma=\gamma' e^{i\phi}\ . \qquad (8)$$

If the above conditions are satisfied, then the two recursion relations (7a) and (7b) become a single one, namely,

$$S_{N+2}=-\frac{\alpha}{\gamma}\left[\frac{N+1}{N+2}\right]^{1/2}S_N\ . \qquad (9)$$

From the recursion relation (9), we can readily generate an even or odd superposition of N states:

$$|f\rangle_{\rm even}=S_0\sum_{N=N_{d/2}}^{N_u/2}(-1)^N\left[\frac{\alpha}{\gamma}\right]^N\times\left[\frac{(2N-1)!!}{(2N)!!}\right]^{1/2}|2N\rangle\ , \qquad (10a)$$

$$|f\rangle_{\rm odd}=S_l\sum_{N=(N_d-1)/2}^{(N_u-1)/2}(-1)^N\left[\frac{\alpha}{\gamma}\right]^N\times\left[\frac{(2N)!!}{(2N+1)!!}\right]^{1/2}|2N+1\rangle\ , \qquad (10b)$$

where S_0 and S_1 are normalization constants.

We define as usual the two Hermitian quadratures of the field:

$$a_1=\tfrac{1}{2}(a+a^\dagger)\ ,$$
$$a_2=\tfrac{1}{2i}(a-a^\dagger)\ . \qquad (11)$$

From these definitions and making use of the $|f\rangle_{\rm even}$ states (10a), *with* $N_d=0$, we readily get

$$(\Delta a_2)^2=\frac{1}{4}\left[1+\frac{2}{M}\sum_{N=0}^{N_u/2}(2N)\left[\frac{\alpha}{\gamma}\right]^{2N}\frac{(2N-1)!!}{2N!!}\times\left[1-\left|\frac{\gamma}{\alpha}\right|\right]\right]\ , \qquad (12)$$

where

$$M=\sum_{N=0}^{N_u/2}\left[\frac{\alpha}{\gamma}\right]^{2N}\frac{(2N-1)!!}{2N!!}\ , \qquad (13)$$

and we have chosen a relative phase between α and γ equal to π.

We can calculate M approximately as follows. The sum appearing in Eq. (13) can be split as $\sum_{N=0}^{N_u/2}=\sum_{N=0}^{\infty}-\sum_{N=1+N_{u/2}}^{\infty}$. The first term can be readily done, and the second one can be approximated by the Stirling formula. The result is

$$M=\frac{1}{(1-x^2)^{1/2}}-\frac{x^{N_u+2}}{1-x^2}\ , \qquad (14)$$

where $x=|\alpha/\gamma|$. We can find the extremum of M that would minimize $(\Delta a_2)^2$, which occurs in the neighborhood of $x=1$ (from below) for large N_u. In a simple calculation, one finds that $x\,\min=1-\epsilon$, for $\epsilon=k/N_u$, and also $(\Delta a_2)^2=k'/N_u$, k and k' being constants of the order of 1.

We performed a numerical calculation of the fluctuations of a_2 using the $|f\rangle_{\rm even}$ states given in Eq. (7a). Figure 2 shows $(\Delta a_2)^2$ and $(\Delta a_1)(\Delta a_2)$ as a function of α/γ

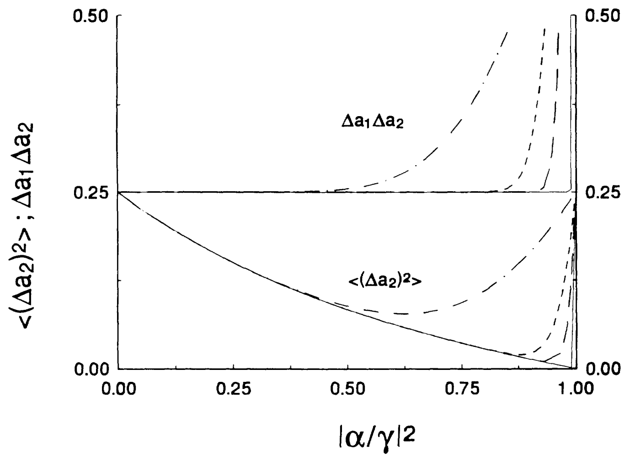

FIG. 2. Variance of $(\Delta a_2)^2$ and $(\Delta a_1)(\Delta a_2)$ vs $|\alpha/\gamma|$ for $N_u=6$ (dot-dashed line), 40 (short-dashed line), 100 (long-dashed line), and 1000 (solid line). We note that for $N_u=1000$ and $|\alpha/\gamma|=1-\epsilon$, we get a nearly perfect squeezed state, which is not a minimum-uncertainty state.

for various values of N_u.

We note the dramatic reduction of $(\Delta a_2)^2$ for large values of N_u and α/γ in the neighborhood of 1 (from below). It can be also observed that at the minimum of each curve the product $(\Delta a_1)(\Delta a_2)$ acquires a value larger than 1/4, implying that we are generating, in this system, perfectly squeezed states which are not minimum-uncertainty states (MUS's).

To summarize the results presented in this paper, we find from both numerical and analytical results that in a two-photon micromaser without electromagnetic losses, one can approach a perfectly squeezed state, when one approaches $x\rightarrow 1$ from below and N_u is large. So far, we have calculated the properties of the even states, without actually proving their existence. We present here a numerical proof.

The master equation for the reduced field density, after k atoms pass through the cavity, is given by $\rho_f^k=\mathrm{Tr}_{\mathrm{at}}(U\rho_f^{k-1}\rho_{\mathrm{at}}U^\dagger)$, and it leads to a steady state that is an even state [Eq. (10a)], if the initial state of the field is a superposition of even n-states in the corresponding block. As a matter of fact, the initial state of the field does not have to be a pure state. We only require that the density matrix elements ρ_{NN} should be nonzero only for N even within the corresponding block.

If we define the entropy as

$$S_l=-\mathrm{Tr}\rho_f^{(l)}\ln\rho_f^{(l)} ,$$

where l is the label for the lth atom. If when $l\rightarrow\infty$, we approach $S\rightarrow 0$, then we have a pure state as a steady state for the field.

In Fig. 3 we show $(\Delta a_2)^2$ and S vs l (or time) for the pure initial vacuum state of the field. We note that after 4500 atoms have passed through the cavity, $S=0$, and therefore the steady state of the field is a pure state (even state) and the above arguments are correct. On the other hand, $(\Delta a_2)^2$ converges to 0.0194, which agrees with the

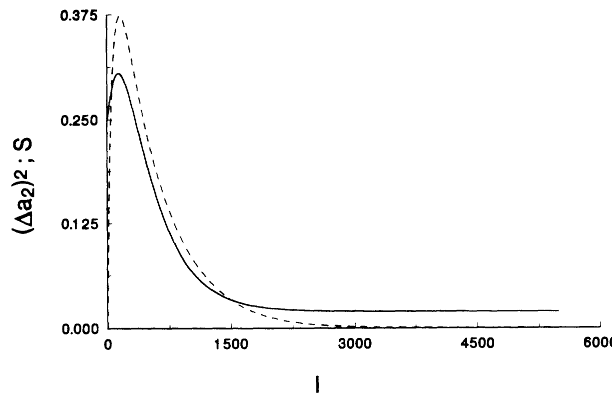

FIG. 3. $(\Delta a_2)^2$ (solid line) and S (dashed line) vs the number of injected atoms l. For $l\geq 4500$, S vanishes and $(\Delta a_2)^2=0.0194$. The initial state of the field is the vacuum state, and $N_d=0$, $N_u=40$, and $|\alpha/\gamma|=0.88$.

steady even-state value of Fig. 2.

In Fig. 4 we show basically the same result, but when the initial state of the field is in a mixed state, namely, $\rho_{NM}=\frac{1}{2}(\delta_{N0}\delta_{M0}+\delta_{N2}\delta_{M2})$. The other parameters are the same as in Fig. 3.

We end this paper with five observations.

(1) Although a model with a lossless cavity might seem academic, actual experiments in micromasers have been performed with extremely high Q values, and so this model might not be unrealistic [2].

(2) It has been numerically observed that for $|\alpha/\gamma|<1$, the maximum photon distribution occurs for $N=0$. However, this maximum moves abruptly to a nonzero value as we cross the $|\alpha/\gamma|=1$ line. Therefore $|\alpha/\gamma|=1$ is a "critical point." As a matter of fact, for $N_u\rightarrow\infty$, $(\Delta a_2)^2$ suffers a sudden jump, at $x=1$, from 0 to 0.25.

(3) A valid question is, how can we generate one of

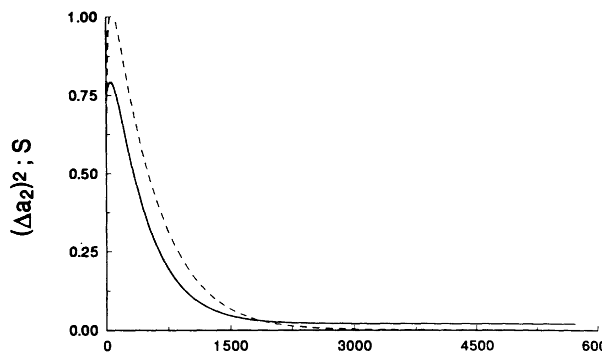

FIG. 4. $(\Delta a_2)^2$ (solid line) and S (dashed line) vs the number of injected atoms l. For $l\geq 4500$, S vanishes and $(\Delta a_2)^2=0.0194$. The initial state of the field is a *mixed state* with $\rho_{NM}=\frac{1}{2}(\delta_{N0}\delta_{M0}+\delta_{N2}\delta_{M2})$, $N_d=0, N_u=40$, and $|\alpha/\gamma|=0.88$.

these states? For example, in order to generate an even state first, we have to prepare an initial state of the field in the corresponding block. This could be done by sending first a beam of atoms excited in the upper state with an appropriate speed as to generate a pure even- n state (N_i). The second beam will have atomic polarization and an interaction time (or ϕ) that will correspond to a pair of integers N_d and N_u, such that $N_d \le N_i \le N_u$. As for the odd states, the procedure could be same, except that the initial preparation of $|N_i\rangle$, $N_i=$ odd integer number, could be done with a different atom with the same level separation and a one-photon transition.

(4) An experimental verification of these results, as in the one-photon case, requires an extremely low temperature, such that the number of thermal photons is much less than 1, which can be achieved experimentally [1,2]. On the other hand, one would expect, as in the one-photon case, these states to be robust to cavity damping [5]. In any event, with present-day technology, very-high-Q cavities are available [1,2] ($Q \simeq 10^{11}$). The existence of the trapping states does not seem to be very sensitive to small velocity fluctuations in the beam [11].

(5) From Eq. (5) we see that $\phi/2\pi=\tau g^2/2\pi\Delta = p/(2N_u+3)$, and so the trapping condition implies that $\phi/2\pi$ is a rational number. The limit where the large squeezing occurs (N_u large) does not imply that the coupling constant or τ has to be very small or Δ very large. It may simply imply that $\phi/2\pi$ approaches an irrational number. This suggests a connection between this problem and number theory and chaos [3].

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