Dispersion cancellation and high-resolution time measurements in a fourth-order optical interferometer

Aephraim M. Steinberg, Paul G. Kwiat, and Raymond Y. Chiao
Department of Physics, University of California at Berkeley, Berkeley, California 94720
(Received 18 October 1991)

Group-velocity dispersion (GVD) places a fundamental limitation on the resolution possible when measuring the propagation time of short optical pulses through dielectric media. We show that due to a nonlocal, quantum-mechanical effect, a surprising cancellation can occur in fourth-order interference, allowing the time interval between photons emitted in spontaneous parametric down-conversion to be measured without significant degradation of the resolution due to the spreading of the individual photon wave packets. This may also prove useful for measuring the higher-order contributions to GVD.

PACS number(s): 42.50.Dv, 42.25.Bs, 07.60.Ly

I. INTRODUCTION

In the process of spontaneous parametric down-conversion, pairs of photons are emitted simultaneously. Although each photon is broadband, energy conservation requires that the frequencies of the two photons in each pair must always sum to the same fixed value. It has been shown that due to this anticorrelation, "the dispersion experienced by one photon can exactly cancel the dispersion experienced by the other in such a way that their coincidence is maintained," [1] if the two photons travel through separate media which are chosen to have dispersive constants equal in magnitude but opposite in sign. We have found a related nonlocal effect in which lowest-order dispersive effects also cancel out, but which does not rely on any such careful choice of materials, and works even if *only one photon* of each pair travels through a dispersive medium. This effect should occur in Hong-Ou-Mandel interference [2], which is known to offer subpicosecond resolution in the comparison of the arrival times of conjugate photons at a beam splitter. We have proposed to exploit it in an experiment to obtain high-precision measurements of the photonic tunneling time [3].

Classically, the propagation of a pulse of light through a dispersive medium engenders a broadening of the temporal profile of the pulse. The broader the bandwidth of the pulse, the larger the effect. This results in a fundamental tradeoff as shorter pulses are produced, since they require larger bandwidths and therefore broaden more quickly under the influence of dispersion than do longer pulses. For example, an optical pulse with a 40-nm bandwidth could be as short as 5 fsec, but after propagating through 1 cm of SF11 glass, it would broaden to over 100 fsec. This group-velocity dispersion (GVD) would seriously degrade the maximum resolution of such time-measurement techniques as direct coincidence detection, standard white-light interferometry, or the use of nonlinear media to determine the overlap of short, intense pulses. Our calculations show, however, that to first order, it should lead to no degradation whatsoever in the resolution of the measurement possible with the Hong-Ou-Mandel interferometer, which is pictured in Fig. 1. (By first order, we mean the *linear* variation of the index of refraction with wavelength, which is the dominant source of dispersive broadening. The next largest order typically contributes less than 10% of the broadening. The cancellation of the first-order GVD may prove useful for making precise measurements of the second-order contributions, which are usually overshadowed.) This effect is nonlocal and fundamentally quantum mechanical in origin.

In this fourth-order interferometer (that is, one in which the interference occurs in coincidence counting; see Fig. 1), an ultraviolet pump photon is down-converted in a $\chi^{(2)}$ nonlinear crystal into two conjugate infrared photons (conventionally denoted "signal" and "idler") whose frequencies ω_s and ω_i are individually broadband,

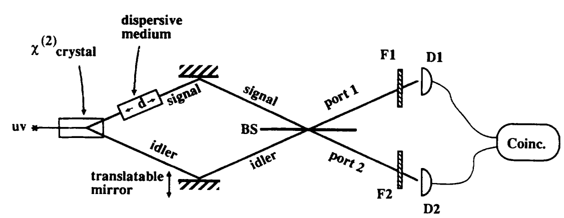

FIG. 1. The Hong-Ou-Mandel interferometer, shown with a dispersive medium inserted in one arm (BS denotes beam splitter). In the absence of this medium, the coincidence rate falls to zero when the path-length difference between the two arms is short relative to the coherence length (determined by the filters $F1$ and $F2$) of the infrared photons reaching detectors $D1$ and $D2$. This has been shown elsewhere to offer subpicosecond resolution in the comparison of the transit times of the photons in the two arms. When the medium is inserted, we expect the dip to be shifted by the corresponding group-velocity time delay for the photon which travels through it. One might also expect group-velocity dispersion to broaden the dip, decreasing the time resolution possible with this interferometer. Our calculations show that, while the dip should indeed be shifted by the expected amount, the leading order of GVD should not contribute any broadening.

but must add up to the original sharp ultraviolet frequency ω_p in order to conserve energy. The two photons are emitted on opposite sides of a cone centered on the pump beam, conserving momentum. Two conjugate beams in the horizontal plane constitute the arms of the interferometer. By the use of mirrors, they are brought together on the surface of a beam splitter. Detectors are placed at the two output ports of the beam splitter, and whenever both detect photons within an electronically set time known as a gate window, a coincidence count is recorded. (The width of this window is generally on the order of a nanosecond, which is orders of magnitude larger than the other time scales under consideration.) As the path-length difference between the two arms is slowly scanned, there is no variation in the singles counting rates at these detectors. In general, the coincidence counting rate is also constant. It vanishes, however, as the path-length difference goes to zero such that the two photon wave packets overlap at the beam splitter. This is due to destructive interference between the two Feynman paths leading to coincidence detection, one in which both photons are reflected at the beam splitter, and one in which both photons are transmitted. The width of this coincidence dip is determined by the coherence length of the down-converted light, or loosely speaking, by the width of the photon wave packets. (The coherence length is generally fixed by filters placed either at the output of the crystal or in front of the detectors.) We have already used this effect to demonstrate the phenomenon of quantum erasure [4,5], and Campos *et al.* have presented a general treatment of such interference [6]. One might expect that if a dispersive material were placed in one arm of the interferometer, the dip would be shifted by the resulting time delay for the photon in that arm, and also broadened through dispersion. We show that while the dip is indeed shifted by the group-velocity time delay experienced by the photon traveling through the dielectric, the terms responsible for first-order dispersive broadening (and in general, all odd orders) cancel out due to the frequency anticorrelation of the conjugate photons. We give simple interpretations of this result relying on Feynman's picture of interference and on the collapse viewpoint. We also perform a more general calculation which relaxes the assumption of slow coincidence gating. This leads to the counterintuitive result that the time resolution can *decrease* if the detector system is too *fast*.

II. DISPERSION CANCELLATION

The center frequency of the infrared photon wave packets is selected to be $\omega_0 \equiv \omega_p/2$, and since their bandwidth (typically determined by filters placed at one or both detectors) is relatively small, we can expand the wave number $k(\omega)$ in the dispersive medium in a Taylor series about ω_0 as follows:

$$k(\omega)=k_0+\alpha(\omega-\omega_0)+\beta(\omega-\omega_0)^2+\cdots\ , \tag{1}$$

where we have kept only the lowest order which contributes to broadening. The group velocity is then

$$v_g(\omega)\equiv\frac{d\omega}{dk}\simeq 1/\alpha-2\beta(\omega-\omega_0)/\alpha^2\ . \tag{2}$$

For light of bandwidth $\Delta\omega$ traveling through a length d of this material, group-velocity dispersion contributes a broadening $\Delta\tau\simeq 2\Delta\omega\beta d$.

The state of the light emitted by the crystal can be written as follows:

$$|\Psi\rangle=\int d\omega' f(\omega')|\omega_0+\omega'\rangle_s|\omega_0-\omega'\rangle_i\ , \tag{3}$$

where $f(\omega)=f(-\omega)$ is a bandwidth function describing the spectrum of the down-converted light after the filters, and s and i label the signal and idler modes corresponding to the two arms of the interferometer (see Fig. 1). Let us now place a length d of our dielectric in the signal arm, and some adjustable optical path length δl in the other arm (by simply translating the idler mirror, or by the use of a trombone prism arm, for example). The annihilation operators for the modes 1 and 2 corresponding to the two output ports of the beam splitter are related to those for the signal and idler modes,

$$\begin{aligned} a_1(\omega_1)&=\frac{i}{\sqrt{2}}a_s(\omega_1)e^{ik(\omega_1)d}+\frac{1}{\sqrt{2}}a_i(\omega_1)e^{i\omega_1\delta l/c}\ ,\\ a_2(\omega_2)&=\frac{i}{\sqrt{2}}a_i(\omega_2)e^{i\omega_2\delta l/c}+\frac{1}{\sqrt{2}}a_s(\omega_2)e^{ik(\omega_2)d}\ , \end{aligned} \tag{4}$$

where the factors of i come from the phase shift upon reflection at a beam splitter, and irrelevant overall phase factors have been dropped. These operators obey the canonical commutation relations

$$\begin{aligned} &[a_j(\omega_j),a_k(\omega_k)]=0\ ,\\ &[a_j^\dagger(\omega_j),a_k^\dagger(\omega_k)]=0\ ,\\ &[a_j(\omega_j),a_k^\dagger(\omega_k)]=\delta_{jk}\delta(\omega_j-\omega_k)\quad j,k=1,2\ . \end{aligned} \tag{5}$$

If our coincidence gate window accepts counts for a time T, then the rate of coincidences P_c between detectors 1 and 2 is proportional to

$$\int_0^T dt_1\int_0^T dt_2\langle\Psi|E_1^-(t_1)E_2^-(t_2)E_1^+(t_1)E_2^+(t_2)|\Psi\rangle\ . \tag{6}$$

Omitting irrelevant normalization constants, which can always be absorbed into the definition of the bandwidth function, the positive- and negative-frequency field operators at detector j are defined by

$$\begin{aligned} E_j^+(t_j)&=\int d\omega_j^+ a_j(\omega_j^+)e^{-i\omega_j^+t_j}\ ,\\ E_j^-(t_j)&=\int d\omega_j^- a_j^\dagger(\omega_j^-)e^{+i\omega_j^-t_j}\ . \end{aligned} \tag{7}$$

These introduce into Eq. (6) four frequency integrals, which can be pulled outside the angle brackets and the time integrals. Since the gate window T is typically much larger than the reciprocal bandwidth of the light or the expected dispersive broadening, the time-integrals yield effective δ functions in $\omega^+-\omega^-$,

$$\lim_{T\to\infty}\int_0^T dt_j e^{it_j(\omega_j^--\omega_j^+)}\propto\delta(\omega_j^--\omega_j^+)\ , \tag{8}$$

lifting two of the frequency-integrals and causing all cross-terms between creation and annihilation operators at different frequencies at the same detector to vanish. The meaning of this is that all interference between different frequency modes washes out over sufficiently large time scales [7]; in the last section of this paper, we will consider the limit where this assumption is no longer valid. Equation (6) reduces to

$$P_c \propto \int d\omega_1 \int d\omega_2 \langle \Psi | a_1^\dagger(\omega_1) a_2^\dagger(\omega_2) a_1(\omega_1) a_2(\omega_2) | \Psi \rangle \ . \tag{9}$$

Defining $\omega' = \omega_1 - \omega_0 = \omega_0 - \omega_2$, and using Eqs. (4), (3), and (1), we evaluate the integrand

$$\begin{aligned}\langle \Psi | a_1^\dagger(\omega_1) a_2^\dagger(\omega_2) a_1(\omega_1) a_2(\omega_2) | \Psi \rangle &= \sum_n \langle \Psi | a_1^\dagger(\omega_1) a_2^\dagger(\omega_2) | n \rangle \langle n | a_1(\omega_1) a_2(\omega_2) | \Psi \rangle \\ &= \langle \Psi | a_1^\dagger(\omega_1) a_2^\dagger(\omega_2) | 0 \rangle \langle 0 | a_1(\omega_1) a_2(\omega_2) | \Psi \rangle \\ &= \| \langle 0 | a_1(\omega_1) a_2(\omega_2) \Psi \rangle \|^2 \\ &= \| \tfrac{1}{2} \langle 0 | [a_i(\omega_1) a_s(\omega_2) e^{i\omega_1 \delta l/c + ik(\omega_2)d} - a_s(\omega_1) a_i(\omega_2) e^{i\omega_2 \delta l/c + ik(\omega_1)d}] | \Psi \rangle \|^2 \\ &= \| \tfrac{1}{2} \delta(\omega_p - \omega_1 - \omega_2) f(\omega') e^{i\omega_0 \delta l/c + ik_0 d} [e^{i\omega' \delta l/c + id(-\alpha\omega' + \beta\omega'^2)} - e^{-i\omega' \delta l/c + id(\alpha\omega' + \beta\omega'^2)}] \|^2 \ .\end{aligned} \tag{10}$$

The factor in brackets on the last line represents the sum of the Feynman amplitudes for the two indistinguishable paths leading to a coincidence event, and we will see that the interference term depends only on the phase difference between these two amplitudes. The essence of the physics is that the requirement that ω_s and ω_i add up to ω_p causes both paths to acquire exactly the same phase from the $\beta(\omega-\omega_0)^2$ term, which is normally responsible for dispersive broadening. The absolute square evaluates to

$$\langle \Psi | a_1^\dagger a_2^\dagger a_1 a_2 | \Psi \rangle = \tfrac{1}{4} |\delta(\omega_p - \omega_1 - \omega_2)|^2 |f(\omega')|^2 \times [2 - 2\,\mathrm{Re}(e^{2i\omega'\delta l/c - 2i\alpha\omega' d})] \ , \tag{11}$$

which when we substitute into Eq. (9) and drop overall numerical factors yields the final coincidence rate

$$P_c \propto \int d\omega' |f(\omega')|^2 \left\{ 1 - \cos\left[2\omega' \left[\frac{\delta l}{c} - \alpha d \right] \right] \right\} . \tag{12}$$

It is apparent that β, the term responsible for group-velocity dispersion, has canceled out in going from (10) to (11), and does not appear in this expression. The dip is centered at $\delta l/c = \alpha d$, as expected from the group velocity $v_g = 1/\alpha$. It has 100% visibility, and its width is determined entirely from the form of $f(\omega)$, corresponding to the coherence times of the photons emitted from the crystal, as if there had not been any dispersive medium present. This is the central result of our paper. Extended to the general case, it says that all GVD due to variation of the index with odd powers of the frequency cancels out exactly. Even orders, on the other hand, persist. If an identical dielectric is placed in the other arm, dispersive effects will cancel out to all orders (odd *and* even).

It is worth remarking that the integral over the slow-coincidence gate windows was a crucial step in this derivation. Generally, the higher the time resolution available for coincidence counting, the more accurate an experiment can be. In this case, it turns out that as $T \to 0$, one can no longer neglect the cross-terms we have thrown away, and the full integral yields a complicated function including beat notes between the different frequency components of the photons. The envelope of this function does not show the same sharp behavior as the archetypical Hong-Ou-Mandel dip, but instead has a large width on the order of the classical GVD prediction. Thus, it is essential that the coincidence counter operate on a *slower* time scale than the envelopes of the broadened photon wave packets in order for the broadening not to manifest itself in the width of the coincidence dip. (As pointed out earlier, this is always true in practice.) The reasons for this become clear with a more intuitive discussion of the phenomenon.

III. INTERPRETATION

A simplified explanation of the Hong-Ou-Mandel dip is that destructive interference occurs only when the two photons overlap at the beam splitter. This picture needs to be modified when frequency-dependent dispersion comes into play. Specifically, the two photons are not interfering with each other, and therefore need not overlap in time. The interference takes place between the alternate Feynman paths which lead to the same final state, i.e., the same coincidence detection event. (In this case, the term "event" must be taken in a strict sense to refer to *in principle* indistinguishable outcomes only. A slow coincidence counter cannot render two nonoverlapping photoelectrons indistinguishable in the quantum-mechanical sense.) What is necessary in order for interference to occur is that the different Feynman paths both lead to the *same* pair of detections. Thus, interference between two paths will occur only if both paths make detector 1 go off at a unique time t_1 and detector 2 go off at a unique time t_2, such that while $t_1 - t_2$ need not be zero, it is the same for both paths.

The photon which has propagated through the dispersive medium can be pictured as spread out into a frequency-chirped wave packet. Under normal dispersion, for example, the redder part of the wave packet would lead and the bluer part of the wave packet would trail. We can then speak loosely of whether the redder part or the bluer part of the photon wave packet excited a given detector, knowing that for energy conservation to hold, the conjugate part of the other wave packet must excite its detector. Energy conservation must also hold locally, so that as long as there is no uncontrollable disturbance (for example, resulting from rapid shutters in front of the detector), the energy of the photoelectron is in principle a means of distinguishing different-frequency photons. For this reason, a path in which detector 1 is excited by a "blue" photon will interfere only with other paths in which detector 1 is excited by "blue" photons. (We use quotation marks because, in practice, all of the photons are infrared.) In the Feynman path where both photons are transmitted, the idler photon reaches detector 1 at a time determined by free-space propagation, which we will consider our reference point. Suppose that detector 1 absorbs the "blue" part of this photon wave packet. Then the "red" part of the signal-photon wave packet, which reaches detector 2 *sooner* than the center of the pulse, excites detector 2. (See Fig. 2.) In the alternate Feynman path, where both photons are reflected at the beam splitter, it is the idler photon which reaches detector 2, with no delay, but it is the "red" part which must be registered in order for there to be interference. The "blue" part of the signal travels through the dielectric, and reaches detector 1 a little *later* than the center of the pulse. Thus our two paths are as follows: one in which detector 2 is excited a little bit early, and one in which detector 1 is excited a little bit late. In both cases, the time lag is exactly the same. As the time of emission of the photon pairs is fundamentally unknowable [8], these two final states are equivalent, and can interfere. The meaning of this is that even though only one photon travels through the dielectric at a time, *which* photon does so is undetermined. Hence the effect is a nonlocal one. For this reason, two photons can sample two

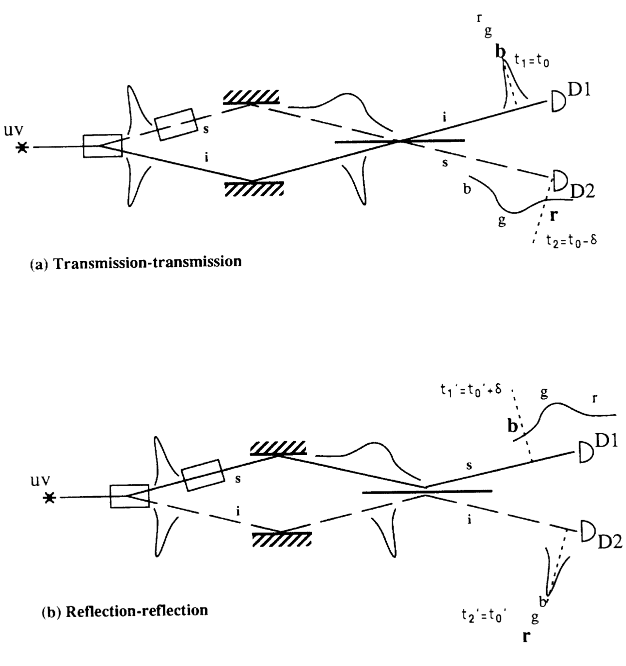

FIG. 2. The two Feynman paths leading to coincidence detection of a "blue" photon at $D1$ and a "red" photon at $D2$. The letters r, g, and b label different frequency components of the photon wave packets; the photon which has traveled through the dispersive medium exhibits a positive frequency chirp. In (a), both photons are transmitted at the beam splitter. In (b), both photons are reflected. Despite the dispersive broadening in the signal arm, these two paths are fundamentally indistinguishable since in both cases, the detection event at $D1$ lags the detection event at $D2$ by the same amount δ.

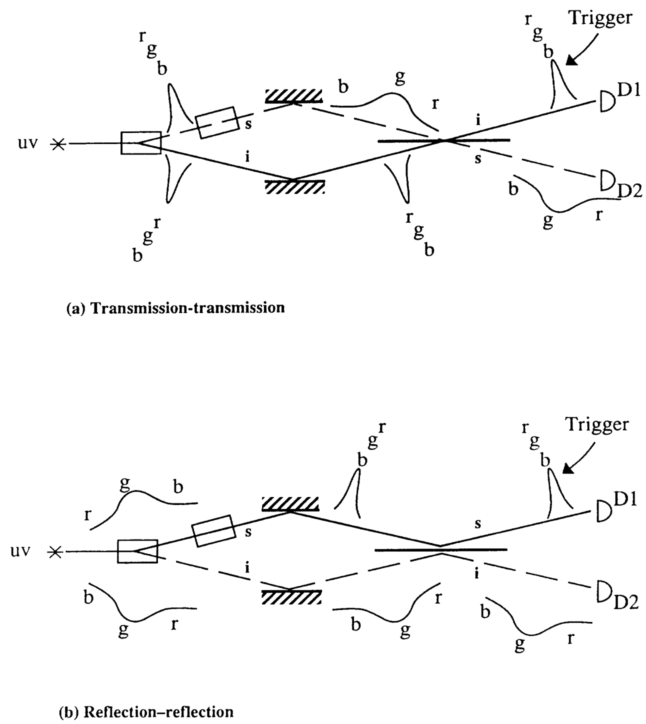

FIG. 3. The collapse picture applied to the two component product states present in the output of the interferometer which lead to nonzero amplitudes for coincidence detection. We use $D1$ as our trigger, projecting the state of the light incident on it onto a minimum-uncertainty wave packet. We can now determine our coincidence rates simply by considering what the fields are at $D2$ contingent upon this trigger detection event. In (a), where both photons are transmitted at the beam splitter, this leaves the light incident on $D2$ in a wave packet which displays positive frequency chirp, such that the "blue" trails the "red." In (b), both photons are reflected, but as explained in the text, the light incident on $D2$ still collapses onto a state with positive frequency chirp. The wave packets from (a) and (b) have total overlap when the optical path-lengths are properly balanced, and interfere destructively because of the phase shift at the beam-splitter. We show that this overlap should fall to zero when the path-length imbalance is of the order of the coherence length of the photons, regardless of the amount of first-order GVD.

different values of a frequency-dependent index simultaneously, in such a way that any linear dependence on frequency (or more generally, any odd order) has no effect on their relative times of arrival, nor consequently on their ability to interfere upon coincidence detection.

An alternative description of the same phenomenon involves the collapse picture. Frequently, coincidence-counting experiments can be simply understood by postulating a collapse of the wave function upon detection of the "trigger" photon, and then calculating the singles rate at the other detector, contingent upon this first event [9]. In this way, fourth-order interference reduces to the more familiar second-order problem. The system under consideration is linear. Therefore we can postulate a von Neumann projection of the state of the system onto one in which the photon is in a minimum-uncertainty wave packet just prior to detection. Furthermore, since we are dealing with an entangled state, we can perform this projection separately on each of the component product states and then form the superposition of their separate detection amplitudes at the second detector.

Let us use detector 1 as our trigger. (See Fig. 3.) Then in the first of the two interfering paths ("transmission-transmission"), an idler photon arrives without having traveled through the dielectric. Projecting this photon onto a minimum-uncertainty packet collapses the system into a state in which the photons were both emitted in minimum-uncertainty wave packets from the crystal. The signal photon, having traveled through the dielectric, will thus have developed a positive frequency chirp by the time it reaches detector 2, such that the blue trails the red. In the second of the two paths ("reflection-reflection"), on the other hand, it is the signal photon which triggers detector 1, after having traveled through the dispersive medium. However, we still collapse the state onto a *minimum-uncertainty* wave packet. Since the dielectric contributes a positive frequency chirp, this involves collapsing the state onto one where the signal photon was emitted from the crystal with a *negative* frequency chirp which exactly compensates the dispersive broadening. (We can see this by considering the time-reversed path of the photon's propagation through the dielectric.) The conjugate idler photon must then have been emitted with a positive chirp (such that the sum of the signal and idler frequencies is constant, and equal to the pump frequency). It retains this chirp upon propagating to detector 2. Thus, when it arrives at the detector, it has perfect overlap with the signal photon from the first Feynman path, allowing complete destructive interference to occur, undiminished by the spreading of the wave packets. By perfect overlap, we mean that each frequency component of one wave packet coincides in time with the corresponding frequency component of the other wave packet. This overlap falls to zero when the chirped wave packets are displaced relative to one another by an amount comparable to the coherence length. This is because the portions of the wave packets which coincide in time no longer coincide in frequency. For this reason, the width of the interference dip is still determined by the coherence length, regardless of the amount of dispersion.

IV. INFINITELY FAST DETECTORS

Let us now reexamine Eq. (6), making the assumption that we have access to detectors and a coincidence box which are fast relative to the other time scales in the problem. (At present, this is experimentally unrealistic.) A coincidence event is then labeled by two times t_1 and t_2 corresponding to the detection times at each detector,

$$P_c(t_1,t_2)\propto \|\langle 0|E_1^+(t_1)E_2^+(t_2)|\Psi\rangle\|^2 \; . \tag{13}$$

We write this matrix element M

$$\int\int d\omega_1 d\omega_2 e^{-i\omega_1 t_1 - i\omega_2 t_2}\langle 0|a_1(\omega_1)a_2(\omega_2)|\Psi\rangle \; . \tag{14}$$

Without loss of generality, let us set $t_1=0$ and drop the subscript on t_2. As before, we define $\omega'=\omega_1-\omega_0$ and instead of Eq. (10), we find

$$\begin{aligned} M &\propto \int d\omega' e^{-i\omega_2 t} f(\omega') e^{i\omega_0\delta l/c + ik_0 d}\left(e^{i\omega'\delta l/c + id(-\alpha\omega'+\beta\omega'^2)} - e^{-i\omega'\delta l/c + id(\alpha\omega'+\beta\omega'^2)}\right) \\ &\propto -2i\int d\omega' f(\omega') e^{i\omega_0\delta l/c - i\omega_0 t + i\omega' t - ik_0 d + i\beta d\omega'^2}\sin\left[\omega'\left[\frac{\delta l}{c}-\alpha d\right]\right] \; . \end{aligned} \tag{15}$$

We define the path-length difference

$$\tau=\delta l/c-\alpha d \tag{16}$$

and neglect the overall phase factor $-ie^{i\omega_0\delta l/c - i\omega_0 t - ik_0 d}$. In addition, let us assume the spectrum to be a Gaussian shape with rms width σ so that

$$f(\omega')\propto e^{-\omega'^2/4\sigma^2} \; . \tag{17}$$

Substituting Eq. (17) into Eq. (15) and integrating, we find

$$P_c(0,t)\propto \|M\|^2 \propto e^{-2a(t+\tau)^2}+e^{-2a(t-\tau)^2} - 2e^{-2at^2}e^{-2a\tau^2}\cos(4bt\tau) \; , \tag{18}$$

where we have defined

$$\begin{aligned} a &\equiv \mathrm{Re}\frac{1}{\frac{1}{\sigma^2}-4i\beta d} = [\sigma^{-2}+(4\sigma\beta d)^2]^{-1} \; , \\ b &\equiv \mathrm{Im}\frac{1}{\frac{1}{\sigma^2}-4i\beta d} = (4\sigma^2\beta d)a \; . \end{aligned} \tag{19}$$

The width w' of the Gaussian functions is just

$$w'=\frac{1}{\sqrt{4a}}=\left[\left[\frac{1}{2\sigma}\right]^2+(2\sigma\beta d)^2\right]^{1/2} \; , \tag{20}$$

which is exactly what one expects for a wave packet of bandwidth σ which at first has a minimum-uncertainty width and then undergoes GVD broadening. Note that b is greater than a by the ratio of the GVD broadening to

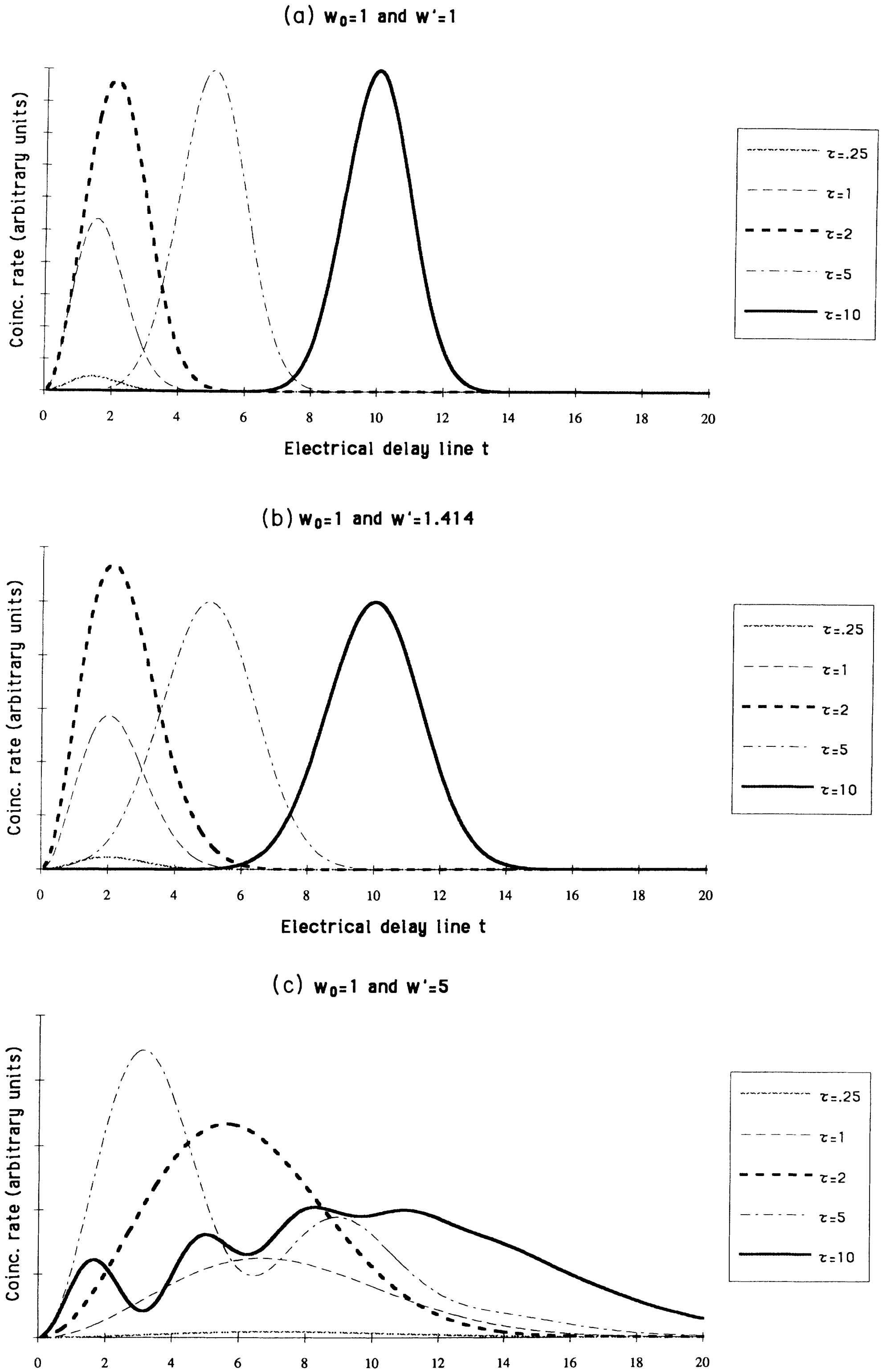

FIG. 4. These graphs show the predicted coincidence rate P_c as a function of the electrical delay setting t of the coincidence box, for several different values of the optical path-length difference τ inside the interferometer. (As explained in the text, they can equally well be seen as graphs of P_c vs τ for several values of t.) All times are in units of the coherence length w_0 of the infrared photons, and the count rates are in arbitrary units. In (a), there is no GVD and the "broadened width" w' of the photon wave packets is equal to w_0. We see that the temporal width of the coincidence distribution is unity, as expected, and that the overall coincidence rate falls to zero for small τ. In (b) and (c), the GVD is equal to and 5 times greater than the coherence length, respectively. The patterns broaden, but also show temporal beating. It is less obvious, but the *integrated* coincidence rates still fall to zero as sharply as in (a).

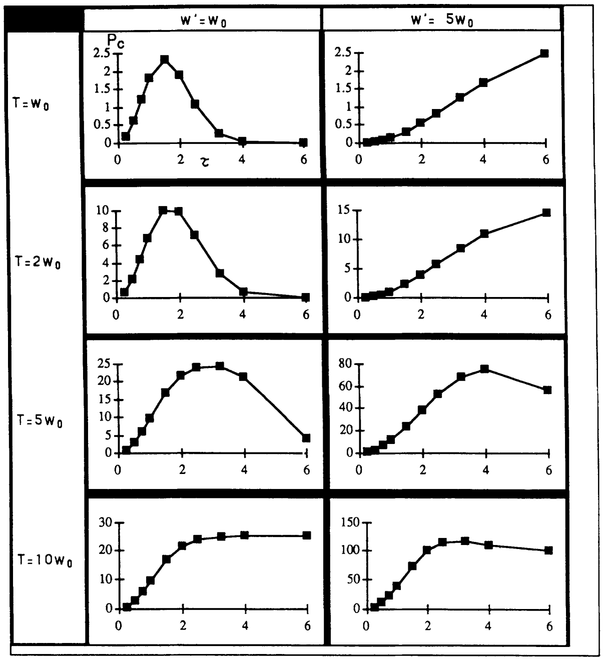

FIG. 5. These eight graphs are of the coincidence rate P_c in arbitrary units vs the optical delay τ, integrated over several different hypothetical electronic gate windows. They are numerically integrated from the formula graphed in Fig. 4. The left-hand column shows the usual Hong-Ou-Mandel dip, in the absence of dispersion, and the right-hand column shows the corresponding curves for strong GVD ("broadened width" $w'=5$, in units of the coherence length $w_0=1$). Notice that once the gate window T is long compared to the width of the broadened photon wave packets, the sharp behavior of P_c is regained.

the unbroadened width.

The meaning of Eq. (18) is that broadened wave packets are visible at $t=\pm\tau$ and that when they begin to overlap each other (i.e., when t and τ are both near 0), interference terms show up. Figures 4 and 5 demonstrate this. The frequency of the interference terms is determined by τ and by the group-velocity dispersion. In essence, the time delay between the two chirped pulses translates into a frequency shift and thus a beat note. It is interesting to observe that the equations are entirely symmetric under exchange of t and τ, that is, of the delay setting of the coincidence box and of the optical path-length difference inside the interferometer. One simple consequence of this is that in analogy with the original Hong-Ou-Mandel experiments, infinitely fast detectors would *never* register perfect coincidences at zero electronic delay, regardless of the path-length difference.

Because of this mathematical similarity, Fig. 4 can either represent the coincidence rate versus electrical delay for several values of the path-length difference, or the coincidence rate versus path-length difference for several different values of the electrical delay. In general, these plots all show broadening from GVD. When a real experiment is carried out, however, using relatively slow detectors, the data taken at each value of the path-length difference corresponds to the *integral* of one of these curves. The surprising cancellation effect is less obvious from these curves than from the simplified calculation, but for $T\rightarrow\infty$, the integral yields

$$P_c \propto 1-e^{-2a\tau^2}e^{-(4b\tau)^2/8a}=1-e^{-\tau^2/2w_0^2}, \qquad (21)$$

where $w_0=1/2\sigma$ is the unbroadened width of the photon wave packets. This is exactly the same result we get by substituting Eq. (17) into Eq. (12). For values of T other than 0 and ∞, the integral must be evaluated numerically, and in Fig. 5 we show that once T is large compared to the GVD broadening the coincidence rate does in fact show an unbroadened dip at zero path-length difference, and remains constant once τ is greater than the coherence length, regardless of the amount of dispersion.

V. CONCLUSION

We have shown that even with only one dispersive medium, the Hong-Ou-Mandel interferometer should allow a peculiar cancellation effect to take place between the dispersion experienced by different photons with anticorrelated frequencies. This is a fundamentally quantum-mechanical effect with no classical analog, depending as it does on nonlocal correlations. This phenomenon makes the use of such interferometers even more promising for carrying out high-precision measurements of the time intervals between photons traveling through various media, regardless of the classical group-velocity dispersion of the materials in question. It also appears to offer a way of measuring the higher-order contributions to GVD.

Note added in proof. Since the original submission, this prediction has been experimentally verified [A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, Phys. Rev. Lett. (to be published)].

ACKNOWLEDGMENTS

This work was supported by the U.S. Office of Naval Research under Grant No. N00014-90-J-1259. We thank I. H. Deutsch for helpful discussions.

[1] J. D. Franson, Phys. Rev. A **45**, 3126 (1992).
[2] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[3] R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg, Physica B **175**, 257 (1991).
[4] P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, in Proceedings of the Second Santa Fe Meeting on the Foundations of Quantum Mechanics, edited by M. O. Scully *et al.* (World Scientific, Singapore, in press).
[5] P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A (to be published).
[6] R. A. Campos, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **42**, 4127 (1990).
[7] G. Magyar and L. Mandel, Nature **198**, 255 (1963).
[8] J. D. Franson, Phys. Rev. Lett. **62**, 2205 (1989).
[9] See, for example, P. G. Kwiat and R. Y. Chiao, Phys. Rev. Lett. **66**, 588 (1991).